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# Criteria of Determining the P/T Upper Limits of GR&R in MSA

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**Abstract.** The MSA combines estimates of the variations of repeatability and reproducibility and is mainly analyzed by experimental design. The variations of personnel, measurement equipment, and the part itself can be analyzed via the data obtained to improve the capability of the measurement system. MSA of both QS9000 and ISO/TS16949 defines GR&R acceptable criteria. GR&R of the MSA in QS9000 is determined by the Precision-to-Tolerance (P/T) value, the percentage of the measurement system variations to the deviation during the manufacturing process or to the part tolerance. If the P/T value is less than 10%, the accuracy of the measurement system is acceptable. If the P/T value falls between 10 and 30%, acceptance of the accuracy of the measurement system is up to the company. When the P/T value is greater than 30%, precision of the measurement system will not be accepted. The aforementioned GR&R acceptance criteria were established by three major automobile companies of the US according to their past experiences. As the capability index $C_{pm}$ reflects both process yield and process loss, we use $C_{pm}$ to set a proper range of GR&R acceptable criteria. If the P/T value is not within the acceptable range, the measurement system is required for modification. If the P/T value is within the acceptable range, The process capability can be enhanced by improving the manufacturing process.

**Key words:** capability index, measurement system analysis, measurement system repeatability and reproducibility.

## 1. Introduction

Measurement errors caused by measurement instruments, measuring personnel, objects to be measured, and the environment. Measurement data become inaccurate without a precise measurement system for assessment, evaluation, and monitoring, which leads to inaccurate calculations of process capabilities. MSA of both QS9000 and ISO/TS16949 defines GR&R acceptable criteria. David (2002) GR&R acceptance standards specified in

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the MSA manual edited by three major automobile companies, GM, Ford, and Chrysler in America are based on the Precision-to-Tolerance P/T value. Plenty of documents and literature on the scope of P/T values were published by many experts like Tsai (1988–1989), Montgomery and Runger (1993), Levinson (1996), and Jheng (2001). The criteria of acceptable P/T values were established by three major automobile companies of the US according to their past experiences. When the P/T value is less than 10%, the accuracy of the measurement system is acceptable. If the P/T value falls between 10 and 30%, acceptance of the accuracy of the measurement system is up to the company. When the P/T value is greater than 30%, precision of the measurement system will not be accepted. However, Pan and Jiang (2002) indicated the range from 10 to 30% was established by the three major automobile companies in the US in accordance with their past experiences.

To solve this problem, we will be evaluated via the capability index $C_{\mathrm{pm}}$. Besides, $\alpha$ and $\gamma$ based P/T criteria will be used to set the tolerable P/T ceiling limits of various $C_{\mathrm{pm}}$ values for two types of risks. The main reason of selecting the $C_{\mathrm{pm}}$ in this study is because it is a convenient and efficient tool to evaluate process capabilities of products. In addition, a number of scholars working on certain properties of the $C_{\mathrm{pm}}$ facilitate easier and correct application of it. Among the scholars are Kane (1986), Chan et al. (1988), Boyles (1991), Cheng (1992), Johnson (1992), Pearn et al. (1992), Fred (1997), Chen et al. (1999), and Chen and Chen (2004a, b) and so on. Nevertheless, the above researches misjudged actual process capabilities, as they did not consider whether the $C_{\mathrm{pm}}$ values would be affected by the precision of measurement instruments or not. Furthermore, Chen et al. (1999) claimed the $C_{\mathrm{pm}}$ could be applied to industries extensively to evaluate process capabilities of products and the process loss could be reflected completely as the denominator of the $C_{\mathrm{pm}}$ value was the expected value $[\sigma^2 + (\mu - T)^2]$ of the Taguchi Loss Function. In addition, Pan and Jiang (2002) also indicated a sufficient $C_{\mathrm{pm}}$ value might present the process yield rate, leading to a formula of Yield $\geqslant 2\Phi(3C_{\mathrm{pm}}) - 1$. As a result, shortcomings of P/T criteria established by past experiences will be corrected in this research and a proper range of P/T acceptable values will be set up using the relationship between the capability index $C_{\mathrm{pm}}$ and GR&R so as to increase the process yield and reduce the process loss.

The $C_{\mathrm{pm}}$ values of GR&R variations will be defined next. Meanwhile, a relation of GR&R variations considered is established (to locate $C_0$). Next, $\gamma$ and reasonable measures will be decided. At last, a rational range of GR&R ceiling limits will be concluded and a summary of contributions made by this article will be made.

## 2. GR&R $C_{pm}$ Evaluation

As the process capability index $C_{pm}$ is an unknown parameter, sampling examination is required. Most of the scholars do not take the GR&R errors into account while calculating the capability index, leading to the existence of GR&R variations in the $C_{pm}$ values observed. Therefore, GR&R variations need to be removed for more precise $C_{pm}$ values. Generally speaking, observation includes variations of the actual part and measurement deviations. As a whole, a variance can be expressed as the sum of part variation and gage variation as the following:

$$\sigma_x^2 = \sigma_{\text{part}}^2 + \sigma_{\text{gage}}^2, \tag{2.1}$$

where $\sigma_x^2$ refers to the variance observed, $\sigma_{\text{part}}^2$ as the variation of actual part, and $\sigma_{\text{gage}}^2$ as measurement variation.

In the formula of $\sigma_{\text{gage}}^2 = \sigma_{\text{repeatability}}^2 + \sigma_{\text{reproducibility}}^2$, $\sigma_{\text{gage}}^2$ refers to the measurement variation, $\sigma_{\text{repeatability}}^2$ as the repeatability variation and $\sigma_{\text{reproducibility}}^2$ as the reproducibility variation. It is obvious only GR&R errors are left if errors of measurement instruments are adjusted by reference values in advance. Accordingly, the process capability index of GR&R is called $\mathrm{OC}_{pm}$ in this article, which is the capability index observed and expressed as follows:

$$\mathrm{OC}_{pm} = \frac{d}{3\sqrt{\sigma_x^2 + (\mu_x - T)^2}} = \frac{d}{3\sqrt{\left(\sigma_{\text{part}}^2 + \sigma_{\text{gage}}^2\right) + (\mu_x - T)^2}}, \tag{2.2}$$

where $d$ is half of the tolerance and $\mu_X = \mu'_X - B$ is the mean of actual parts (an average after correction), $\mu'_X$ as the observation, $B$ as the mean measurement bias, and $T$ as the target. After deducting $\sigma_{\text{gage}}^2$ or when $\sigma_{\text{gage}}^2$ converges towards 0, $\sigma_x^2 = \sigma_{\text{part}}^2$. The index is called the actual process capability index, $\mathrm{AC}_{pm}$ and expressed as the following:

$$\mathrm{AC}_{pm} = \frac{d}{3\sqrt{\sigma_{\text{part}}^2 + (\mu_x - T)^2}}. \tag{2.3}$$

According to formula (2.2), it is known that

$$\sigma_{\text{part}}^2 = \left(\frac{d}{3OC_{pm}}\right)^2 - \sigma_{\text{gage}}^2 - (\mu_x - T)^2. \tag{2.4}$$

Therefore,

$$\mathrm{AC}_{pm} = \frac{1}{\sqrt{\left(\frac{1}{OC_{pm}}\right)^2 - \left(\frac{3\sigma_{\text{gage}}}{d}\right)^2}}. \tag{2.5}$$

Currently, the P/T acceptance criterion of GR&R in QS9000 is used mostly and is defined as follows: $\mathrm{P/T} = \frac{5.15\sigma_{\mathrm{gage}}}{\mathrm{Tolerance}} \times 100\% = \frac{5.15\sigma_{\mathrm{gage}}}{2d} \times 100\%$. Obviously, as $3\sigma_{\mathrm{gage}}/d = 6\,(\mathrm{P/T})\,/5.15$,

$$\mathrm{AC}_{\mathrm{pm}} = \frac{1}{\sqrt{\left(\frac{1}{OC_{\mathrm{pm}}}\right)^2 - \left(\frac{6(P/T)}{5.15}\right)^2}}. \tag{2.6}$$

According to formula (2.6), the P/T value determines the difference between $\mathrm{AC}_{\mathrm{pm}}$ and $\mathrm{OC}_{\mathrm{pm}}$. If the P/T value is smaller, the observation capability index $\mathrm{OC}_{\mathrm{pm}}$ will be more similar to the actual capability index $\mathrm{AC}_{\mathrm{pm}}$.

For estimation of the required $\mathrm{AC}_{\mathrm{pm}}$, no measurement error exists if the process capability equals the actual part process capability. As a matter of fact, the alternative hypothesized process capability index is less than the actual part process capability index ($\mathrm{AC}_{\mathrm{pm}}$) since $\mathrm{OC}_{\mathrm{pm}}$ contains repeatability and reproducibility variations. The accuracy of the measurement system here is defined as $\alpha$, which means the probability of being the observation process capability index ($\mathrm{OC}_{\mathrm{pm}}$) when the process capability index ($\mathrm{C}_{\mathrm{pm}}$) is the actual part process capability index ($\mathrm{AC}_{\mathrm{pm}}$) and GR&R variations do not exist; whereas, $\gamma$ refers to the probability of being the actual process capability index ($\mathrm{AC}_{\mathrm{pm}}$) when the process capability index ($C_{\mathrm{pm}}$) is the observation process capability index ($\mathrm{OC}_{\mathrm{pm}}$) Pan and Jiang (2002) deem the process capability should be as close to the actual part process capability as possible. In reality, a higher $\gamma$ means the observation of the process capability index is closer to the true value when $\alpha$ is steady.

The MSA manual for QS9000 divides GR&R variations into equipment variation (EV) by David (2002), appraiser variation (AV), and part variation (PV). It is presumed that a total of $n$parts are sampled and measured $r$ times repeatedly by $h$appraisers. Next, measurement data will be based on for the estimation of $C_{\mathrm{pm}}$, which can be expressed as follows:

$$\hat{C}_{\mathrm{pm}} = d/3\sqrt{(nrh)^{-1}\sum_{i=1}^{h}\sum_{j=1}^{r}\sum_{k=1}^{n}\left(X_{ijk} - T\right)^2}.$$

Apparently, $(nrh)^{-1}\sum_{i=1}^{h}\sum_{j=1}^{r}\sum_{k=1}^{n}\left(X_{ijk} - T\right)^2$ is the unbiased estimator of $\sigma^2 + (\mu - T)^2$. The critical value $C_0$ may be calculated with a condition of $\alpha$-*risk* while evaluating if the $C_{\mathrm{pm}}$ is the actual $C_{\mathrm{pm}}$ ($C_{\mathrm{pm}} = \mathrm{AC}_{\mathrm{pm}}$) by the following equation:

$$\alpha = P(\hat{C}_{\mathrm{pm}} \leqslant C_0 | C_{\mathrm{pm}} = AC_{\mathrm{pm}}).$$

As $Y = (nrh)\,(1+L)\,\hat{C}_{\mathrm{pm}}^2 = \sum_{i=1}^{h}\sum_{j=1}^{r}\sum_{k=1}^{n}\left(X_{ijk} - T\right)^2/\sigma^2$ on $N$ degrees of freedom with noncentral chi-square distribution [marked as $Y \sim \chi'^2$

$(N;\lambda = NL)]$ of no central parameter $\lambda = NL$ and $L = (u-T)^2/\sigma^2$, $N = nrh$, the above equation can be expressed as follows:

$$\alpha = P\left(N(1+L)\left(\frac{\mathrm{AC}_{\mathrm{pm}}}{\hat{C}_{\mathrm{pm}}}\right)^2 \geqslant N(1+L)\left(\frac{\mathrm{AC}_{\mathrm{pm}}}{C_0}\right)^2\right)$$
$$= p\left(\chi'^2(N;\lambda = NL) \geqslant N(1+L) \times \left(\frac{\mathrm{AC}_{\mathrm{pm}}}{C_0}\right)^2\right). \tag{2.7}$$

So,

$$\chi'^2_{1-\alpha}(N;\lambda) = N(1+L)\left(\frac{\mathrm{AC}_{\mathrm{pm}}}{C_0}\right)^2, \tag{2.8}$$

$$C_0 = \sqrt{\frac{N(1+L)}{\chi'^2_{1-\alpha}(N;\lambda)\left(\left(\frac{1}{\mathrm{OC}_{\mathrm{pm}}}\right)^2 - \left(\frac{6\mathrm{P/T}}{5.15}\right)^2\right)}}, \tag{2.9}$$

where $\chi^2_{1-\alpha}(N;\lambda)$ are the quintile/fractal of $1-\alpha$ on $(N;\lambda)$ degrees of freedom in chi-square distribution.

Next, the minimum $C_0$ will be calculated with P/T as 5, 10, 15, and 20%, respectively. As $L$ is unknown, there will be different $L = (u-T)^2/\sigma^2$ values with different $C_0$. Therefore, the $L$value will affect the $C_0$ value. $C_0$ values corresponding to $\mathrm{OC}_{\mathrm{pm}}$ of 1.00, 1.33, 1.50, and 2.00 with $L$=0.00 (0.05) 2.00 are listed (Figures 1–4). It is observed $L = 0$ results in the minimum $C_0$. It goes without saying that when $\hat{C}_{\mathrm{pm}} \leqslant C_0' = \min_L C_0(L)$, $\hat{C}_{\mathrm{pm}} \leqslant C_{\mathrm{o}}$ expressed as follows:

$$C_0' = \min_{L=0}^{2} C_0(L) = \sqrt{\frac{N}{\chi^2_{1-\alpha}(N)\left(\left(\frac{1}{\mathrm{OC}_{\mathrm{pm}}}\right)^2 - \left(\frac{6\mathrm{P/T}}{5.15}\right)^2\right)}}. \tag{2.10}$$

Therefore, the $C_0'$ value can be served as the critical value. When $C_{\mathrm{pm}}$ is less or equivalent to $C_0'$, $C_{\mathrm{pm}}$ is the true $C_{\mathrm{pm}}$; on the contrary, when $C_{\mathrm{pm}}$ is greater than $C_0'$, $C_{\mathrm{pm}}$ will be the observed $C_{\mathrm{pm}}$.

## 3. Determine $\gamma$ and Reasonable Measures

As what is said above, the estimated critical value $C_0'$ is located by probability $\gamma$ while evaluating if the process capability index $C_{\mathrm{pm}}$ is the actual

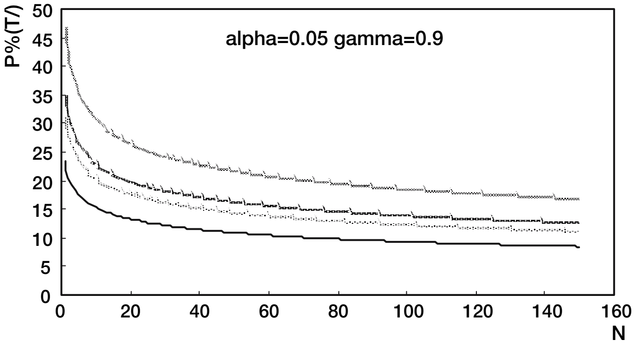


*Figure 1.* P/T as 5% of $C_0$(L).

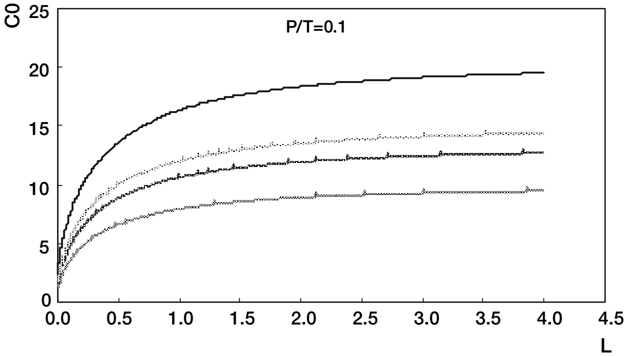


*Figure 2.* P/T as 10% of $C_0$(L).

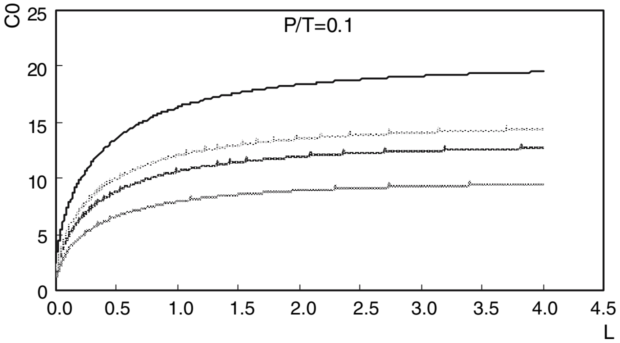


*Figure 3.* P/T as 15% of $C_0$(L).

capability index ($C_{pm} = AC_{pm}$) and $\gamma$ represents the probability without GR&R variation despite of the existence of GR&R variation. The tolerable P/T upper limits of various $C_{pm}$ values will be established in compliance with probability $\gamma$ and expressed as the following:

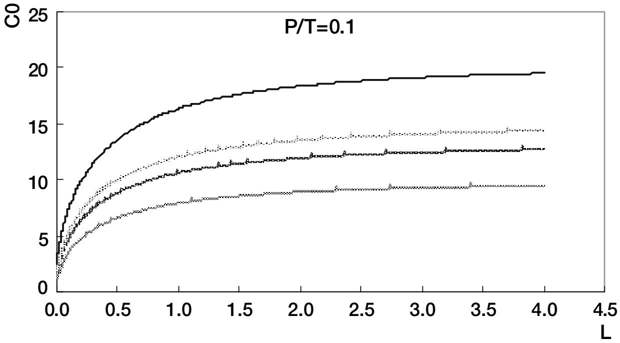


*Figure 4.* P/T as 20% of $C_0$(L).

$$\begin{aligned}\gamma &= P\left(\hat{C}_{\text{pm}} > C_0' | C_{\text{pm}} = \text{OC}_{\text{pm}}\right)\\ &= P\left(\left(\frac{C_{\text{pm}}}{\hat{C}_{\text{pm}}}\right)^2 < \frac{\chi^2_{1-\alpha}(N)}{N}\left(\left(\frac{1}{\text{OC}_{\text{pm}}}\right)^2 - \left(\frac{6\text{P/T}}{5.15}\right)^2\right) \times C^2_{\text{pm}} | C_{\text{pm}}^{=}\text{OC}_{\text{pm}}\right)\\ &= P\left(\chi^2_N < \chi^2_{1-\alpha}(N) \times \left(1 - \text{OC}^2_{\text{pm}}\left(\frac{6\text{P/T}}{5.15}\right)^2\right)\right).\end{aligned}$$

So,

$$\chi^2_\gamma(N) = \chi^2_{1-\alpha}(N) \times \left(1 - \text{OC}^2_{\text{pm}}\left(\frac{6\text{P/T}}{5.15}\right)^2\right). \tag{3.1}$$

Conversion of Equation (3.1) results in the acceptable P/T of GR&R as follows:

$$\text{P/T} = \sqrt{\left(1 - \frac{\chi^2_\gamma(N)}{\chi^2_{1-\alpha}(N)}\right) \times \frac{1}{\text{OC}^2_{\text{pm}}}} \times \frac{5.15}{6}, \tag{3.2}$$

where $\chi^2_\gamma(N)$ are the quintile/fractal $\gamma$ on $N-1$ degrees of freedom in chi-square distribution.

With a fixed $\alpha$, a greater $\gamma$ means a higher probability of regarding the $C_{\text{pm}}$ with GR&R as the actual $C_{\text{pm}}$ (for more correct decision-making) leading to a lower P/T upper limit (stricter). Consequently, P/T upper limits and $\gamma$ distribution with $\alpha = 0.05$, and a process capability index of 1.00, 1.33, 1.50, and 2.00 will be examined here (Equation (3.1) resulting in Figure 5).

According to Figure 5, a smaller P/T leads to a greater $\gamma$, which means the measurement systems judges more rigidly and the observed process

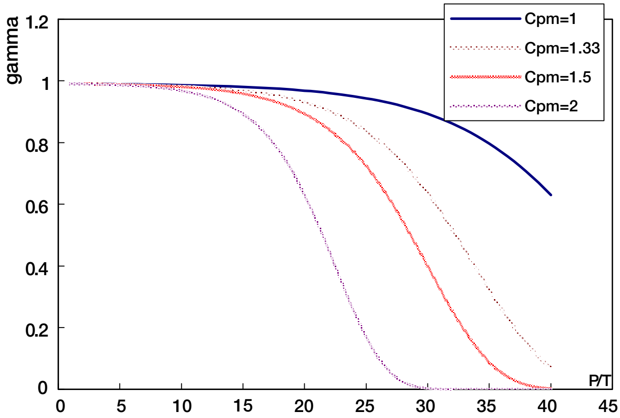


*Figure 5.* P/T Upper Limits & $\gamma$ Distribution with $\alpha = 0.05$ and a $C_{pm}$ of 1, 1.33, 1.50, and 2.

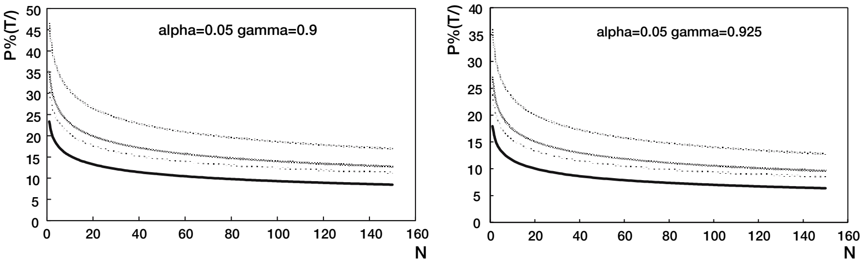


*Figure 6.* Chart of $N$ and P/T with $\alpha$ as 0.05 and $\beta$ as 0.9 and 0.925.

capability is closer to the actual process capability. As a result, the P/T has to be less when a required $C_{pm}$ is greater to achieve the target.

The chart (Figure 6) of P/T upper limits and the number of measurements ($N$) with two different $\gamma$ is rearranged according to Table I.

It is obvious to see that P/T decreases and becomes steady with an increase in $N$; i.e., the slope/curvature converges toward zero (shown as Figure 6). If the slope/curvature $\left[\left(C_{pm(i)} - C_{pm(i+1)}\right)/(n_i - n_{i+1})\right]$ is less than 0.1, the number of measurements ($N$) and P/T upper limits corresponding to various $C_{pm}$ values will be able to locate (indicated as Table II) and a reasonable number of samples will be obtained. For example, in Appendix 2, $C_{pm} = 1.33$, $\alpha = 0.05$ and $\gamma = 0.925$ are required, resulting in a slope/curvature of 0.0912, a P/T upper limit of 7.41 and an $N$ of 18 (the principle of determining P/T and $N$to be discussed next). According to Figure 6,

*Table I.* P/T Corresponding to 4 $C_{pm}$ (1.00, 1.33, 1.50, 2.00) values and 2 $\gamma$ (0.9, 0.925) values with $\alpha = 0.05$ for each $N$

| $N$ | $C_{pm} = 1.00$ | Slope 1 | $C_{pm} = 1.33$ | Slope 1.33 | $C_{pm} = 1.5$ | Slope 1.50 | $C_{pm} = 2$ | Slope 2.00 |
|---|---|---|---|---|---|---|---|---|
| 1 | 17.4830 | | 13.1451 | | 11.6553 | | 8.7415 | |
| 10 | 11.1548 | 0.2442 | 8.3871 | 0.1836 | 7.4365 | 0.1629 | 5.5774 | 0.1221 |
| 11 | 10.9369 | 0.2179 | 8.2233 | 0.1638 | 7.2913 | 0.1452 | 5.4685 | 0.1089 |
| 12 | 10.7406 | 0.1963 | 8.0757 | 0.1476 | 7.1604 | 0.1309 | 5.3703 | 0.0982 |
| 13 | 10.5622 | 0.1784 | 7.9415 | 0.1342 | 7.0415 | 0.1189 | 5.2811 | 0.0892 |
| 14 | 10.3988 | 0.1634 | 7.8187 | 0.1228 | 6.9325 | 0.1090 | 5.1994 | 0.0817 |
| 15 | 10.2483 | 0.1505 | 7.7055 | 0.1132 | 6.8322 | 0.1003 | 5.1242 | 0.0752 |
| 16 | 10.1089 | 0.1394 | 7.6007 | 0.1048 | 6.7393 | 0.0929 | 5.0545 | 0.0697 |
| 17 | 9.9792 | 0.1297 | 7.5032 | 0.0975 | 6.6528 | 0.0865 | 4.9896 | 0.0649 |
| 18 | 9.8580 | 0.1212 | 7.4120 | 0.0912 | 6.5720 | 0.0808 | 4.9290 | 0.0606 |
| 19 | 9.7443 | 0.1137 | 7.3266 | 0.0854 | 6.4962 | 0.0758 | 4.8722 | 0.0568 |
| 20 | 9.6374 | 0.1069 | 7.2461 | 0.0805 | 6.4249 | 0.0713 | 4.8187 | 0.0535 |

*Table II.* P/T upper limits and slopes/curvatures with $\alpha = 0.05$ and $\gamma = 0.945$ for different $C_{pm}$ and $N$

| $\alpha = 0.05$ | $\gamma = 0.9$ | | | | $\gamma = 0.925$ | | | |
|---|---|---|---|---|---|---|---|---|
| $N$ | $C_{pm} = 1$ | $C_{pm} = 1.33$ | $C_{pm} = 1.5$ | $C_{pm} = 2$ | $C_{pm} = 1$ | $C_{pm} = 1.33$ | $C_{pm} = 1.5$ | $C_{pm} = 2$ |
| 1 | 46.67 | 35.09 | 31.12 | 23.34 | 35.88 | 26.98 | 23.92 | 17.94 |
| 10 | 30.55 | 22.97 | 20.37 | 15.28 | 23.18 | 17.43 | 15.46 | 11.59 |
| 20 | 26.52 | 19.94 | 17.68 | 13.26 | 20.08 | 15.09 | 13.38 | 10.04 |
| 30 | 24.33 | 18.29 | 16.22 | 12.16 | 18.40 | 13.83 | 12.27 | 9.20 |
| 40 | 22.86 | 17.18 | 15.24 | 11.43 | 17.27 | 12.99 | 11.51 | 8.64 |
| 50 | 21.76 | 16.36 | 14.51 | 10.88 | 16.43 | 12.36 | 10.96 | 8.22 |
| 60 | 20.89 | 15.71 | 13.93 | 10.45 | 15.77 | 11.86 | 10.52 | 7.89 |
| 70 | 20.18 | 15.17 | 13.45 | 10.09 | 15.23 | 11.45 | 10.15 | 7.62 |
| 80 | 19.58 | 14.72 | 13.05 | 9.79 | 14.78 | 11.11 | 9.85 | 7.39 |
| 90 | 19.06 | 14.33 | 12.71 | 9.53 | 14.38 | 10.81 | 9.59 | 7.19 |
| 100 | 18.61 | 13.99 | 12.41 | 9.30 | 14.04 | 10.55 | 9.36 | 7.02 |

a higher $\gamma$ value leads to a lower P/T upper limit (stricter) and the number of samples may be reduced. The P/T upper limit has to be lower when the required $C_{pm}$ is higher for more precise measurement instruments.

*Table III.* Table of standardized GR&R with different $C_{pm}$ Indexes

| $C_{pm}$ | 1.00 | | | 1.33 | | | 1.50 | | | 2.00 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | $N$ | P/T | | $N$ | P/T | | $N$ | P/T | | $N$ | P/T | |
| | $n$ | $h$ | $r$ | $n$ | $h$ | $r$ | $n$ | $h$ | $r$ | $n$ | $h$ | $r$ |
| 0.900 | 60 | 20.89 | | 40 | 17.18 | | 36 | 15.59 | | 30 | 12.16 | |
| | 10 | 3 | 2 | 10 | 2 | 2 | 6 | 3 | 2 | 5 | 3 | 2 |
| 0.925 | 40 | 17.27 | | 30 | 13.83 | | 28 | 12.45 | | 24 | 9.65 | |
| | 10 | 2 | 2 | 5 | 3 | 2 | 7 | 2 | 2 | 6 | 2 | 2 |
| 0.945 | 24 | 9.26 | | *18* | *7.41* | | 16 | 6.74 | | 12 | 5.37 | |
| | 6 | 2 | 2 | 3 | 3 | 2 | 4 | 2 | 2 | 3 | 2 | 2 |

*Note*: $N$ refers to number of measurements, $n$ as number of sampling, $h$ as appraisers, and $r$ as number of repeated measurements.

## 4. Establish a Reasonable Range of GR&R Upper Limits

To provide reasonable criteria of judgment while implementing measurement system analysis, $\alpha = 0.05$ is presumed in this research and various GR&R acceptance criteria. According to Pan and Jiang (2002), it is suggested appraisers and the number of samples be as many as possible and the number of repeated measurements be as few as possible for the best combination of the number of samples, appraisers and repeated measures ($r$). Table III is thus arranged for reference of users. For examples, if $C_{pm}$ is required over 1.33 with $\alpha$ as 0.05 and $\gamma$ as 0.945, then 18 measures (three appraisers measuring three samples two times, respectively) in accordance with Table III and the last example in Section 3. Meanwhile, the P/T upper limit is 7.41%, which means the measurement system is acceptable with the GR&R variation ratio under 7.41%.

## 5. Conclusion

Currently, most of the proprietors comply with GR&R acceptance principles in QS9000 when conducting a measurement system analysis for accuracy judgment of a measurement system. However, these acceptance principles are established in accordance with previous process variations or tolerances without taking the standard bias into consideration. The P/T upper limit of GR&R is determined by the process capability index ($C_{pm}$) that can reflect the process yield and the process loss in this article. In addition, a table of reasonable criteria for deciding GR&R is established for the reference of GR&R in MSA for industries. When the actual P/T is lower than this upper limit, the measurement system is acceptable; on the contrary, if the actual P/T exceeds this upper limit, the measurement system will not be accepted. As the criteria are based on dichotomy, the ambiguous range between 10 and 30% in the MSA manual will be excluded. Last, the suggestion of many appraisers and numbers of samples with a limited number of few repeated measurements proposed by Pan and Jiang (2002) is complied for the best combination of the number of sampling ($n$), appraisers ($h$), and the number of repeated measurements ($r$). As for the destructive inspection measurement system, repeatability, and reproducibility cannot be obtained via repetitive measurements. As a result, the accuracy evaluation model of this type of measurement system will become an interesting subject for further study in the future.

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