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Possibilistic programming in production planning of assemble-to-order environments

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Abstract

For the sake of alleviating the influences of demand uncertainty in assemble-to-order (ATO) environments, the strategies of regulating dealers' forecast demands, determining appropriate safety stocks, and deciding the numbers of key machines are usually adopted by manufacturers. In this paper, we propose a possibility linear programming model to manage these production planning problems. The proposed model accomplishes forecasting adjustments, material management, and production activities. Because of price fluctuations, material obsolescence, and the time value of capital, the ambiguity of cost is considered in the objective function of the model. We substituted the fuzzy objective function with three crisp objectives: minimizing the most possible cost, maximizing the possibility of obtaining lower cost, and minimizing the risk of obtaining higher cost. Zimmermann's fuzzy programming method is then applied for achieving an overall satisfactory compromise solution. Finally, an example is given to illustrate our model.

Keywords: Assemble-to-order; Forecast demands; Safety stock; Possibility programming

1. Introduction

A manufacturing strategy where materials and subassemblies are made or acquired according to forecasts, while the final assembly of products is delayed until customer orders have been received is commonly referred to as assemble-to-order (ATO) [4–6, 8, 13]. In such environments, a rolling schedule method is generally applied for supervising the newest market information, satisfying customer requirements, and maintaining the lowest inventory [1]. In a rolling schedule process, at period t, dealers are usually requested to place their firm order FD_{tp0} and perform their demand forecasts for the next few periods; e.g. 2 periods, $FD_{(t+1)p1}$ and $FD_{(t+2)p2}$ as shown in Table 1. $FD_{(t+1)p1}$ and $FD_{(t+2)p2}$ are used as references for ordering materials with acquisition lead time $l_c = 1$ and 2 for final assembly that will be performed at period $t + 1$ and $t + 2$, respectively. These forecasting demands are temporary and will be updated in the next forecasting cycle. That is, at period $t + 1$, the dealer places a firm order $FD_{(t+1)p0}$ and presents forecasts $FD_{(t+2)p1}$ and $FD_{(t+3)p2}$,

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Table 1
The rolling schedule with demand forecasts for the next 2 periods

Current period \ Forecast period	t	$t+1$	$t+2$	$t+3$	$t+4$	…
t	FD_{tp0}	$FD_{(t+1)p1}$	$FD_{(t+2)p2}$			
$t+1$		$FD_{(t+1)p0}$	$FD_{(t+2)p1}$	$FD_{(t+3)p2}$		
$t+2$			$FD_{(t+2)p0}$	$FD_{(t+3)p1}$	$FD_{(t+4)p2}$	
⋮				⋮	⋮	⋮

where $FD_{(t+1)p1}$ is updated by firm order $FD_{(t+1)p0}$, and $FD_{(t+2)p2}$ is updated to $FD_{(t+2)p1}$. The remainder can be deduced in the same manner. The production schedule is named a "rolling schedule" because demand forecasts are updated periodically. Fig. 1 depicts the relative activities based on the time horizon in ATO circumstances.

However, forecasts are rarely accurate. The accuracy of forecasts does affect the performance of the production system. Underestimating causes material deficiencies that may create final product stockouts. On the other hand, overestimating will result in excess of materials which may lead to increases in inventory holding costs. To compensate for forecasting inaccuracies, two material management processes are generally executed by manufacturers. The first involves preparation of appropriate levels of material safety stocks to absorb the influences of demand uncertainty. The second involves regulating forecast demands by modifying forecasting activities. Besides, the numbers of key machines should be decided for forecasted production [2, 3]. However, three managerial decisions are commonly determined subjectively in practice. Shieh [16] developed a semi-analytical model to solve such situations. Considering the characteristics of product life cylce, Hsu and Wang [9] modified Shieh's model to reflect the dynamic nature of the market. All of the models take into account crisp cost parameter values.

The managerial decisions in material management are essentially conditioned by product stockout costs and inventory holding costs. In general, the profit rate is the decisive factor for the former, while material price and the inventory holding rate influence for the latter. For capacity analysis, the idleness of key machines has a vital impact upon investment utilization. Because of price fluctuation in a dynamic market, material obsolescence, and the time value of capital, assigning a set of crisp values for parameters is no longer appropriate for dealing with such ambiguous decision problems. Fortunately, possibility distribution offers an effectual alternative for proceeding with inherent ambiguous phenomena in determining cost parameters [7, 10, 12, 14, 17, 20]. Therefore, in this study we constructed a possibilistic linear programming model to determine appropriate safety stocks of materials, regulation of forecast demands and the numbers of key machines. Next, we transformed the possibilistic linear programming model into a crisp multiple objective linear programming model. Finally, Zimmermann's fuzzy programming method [21] is applied to obtain a composite single objective.

The remainder of this paper is organized as follows. In Section 2 we describe the production system. In Section 3 we formulate and explain the production possibilistic programming model and the approach for solving the proposed model. In Section 4 a numerical illustration is presented. Finally, Section 5 provides some concluding remarks.

2. Problem statement

The production system considered in this paper is a single stage assemble-to-order factory. That is, all materials are procured from suppliers, and the final assembly of the products is initiated after receiving the

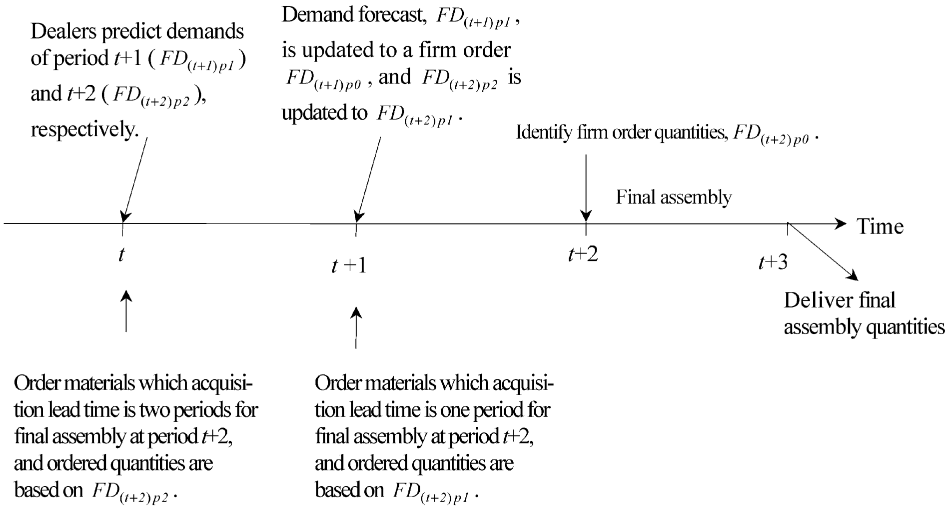

Fig. 1. Activities of material acquisition for final assembly at period $t + 2$.

dealers' firm orders. Some of the materials are unique to specific final products, while other materials are common to two or more final products. The rolling schedule method is adopted. Demand forecasts provide important managerial references for material acquisition for the future final assembly. Some other assumptions in the production planning scenario are given as follows:

(1) The actual production quantities are determined by the status of on-hand material inventories and available capacity sizes.
(2) Firm orders cannot be withdrawn if accepted by the manufacturer.
(3) If the system cannot completely produce the quantities ordered, backorder is not considered.
(4) Once material purchasing orders are released to suppliers, they cannot be revoked. Moreover, all purchased materials are delivered on schedule and without shortage.
(5) The acquisition lead time for each material is constant.

The shortage penalty costs for final products not only comprises the explicit profit loss, but also includes the implicit loss of a firm's goodwill, customers or market share. The inventory holding costs contain the cost of capital tied up, insurance, price variation, and so on. The roughly estimated idle capacity penalty cost is affected by the excess supply of key machines with respect to the actual production volume at each period. In this study the shortage penalty costs for products, the holding costs per each unit of material and idle capacity penalty costs are represented by triangular possibility distributions. The parameters of a triangular possibility distribution are given as the optimistic, the most possibility, and the pessimistic values, which were both estimated by experts.

3. Construction of production possibilistic programming (PPP) model

A production possibilistic programming model was built for regulating forecast demands reasonably, determining appropriate material safety stock levels, and deciding the numbers of key machines. The objective function considered takes into account the costs of product stockout, inventory holding, and capacity idleness. These are the critical concerns when evaluating the performance of ATO implementation. Several balance equations and necessary constraints are conceived to reflect the production system.

3.1. Notations and the formulation

In order to formulate the problem mathematically, the notations below are introduced.

Parameters and indices

t the planning horizon time period, $t=1,2,\ldots,T$
c the type of materials, $c=1,2,\ldots,C$
l_c acquisition lead time of material c
$\tilde{\omega}_c$ inventory holding cost per unit of material c per unit time period
p the type of products, $p=1,2,\ldots,P$
$\tilde{\gamma}_p$ stockout penalty cost per unit of unsatisfied demand for product p per unit time period
j the type of key machines, $j=1,2,\ldots,J$
$\tilde{\delta}_j$ idle capacity penalty cost per unit of key machine j per unit time period
CL_j available capacity level per unit of key machine j
u_{pc} number of units of material c required for each unit of product p
FD_{tpl_c} demand forecast for product p at period t, performed by dealers at period $t-l_c$; when $l_c=0$, it expresses actual order quantities of product p at period t

Process variables

β_p service level of product p
SO_{tp} stockout quantities of product p at period t
I_{tc} inventory level of material c at the end of period t
$EI_{(t-1)c}$ estimated inventory level of material c at the end of period $(t-1)$ performed at period $(t-l_c)$
TU_{tpc} total usage of material c for product p at period t
Q_{tcl_c} order quantities of material c at period $(t-l_c)$ and the volume will be received at period t
AP_{tp} actual production quantities of product p at period t

Decision variables

SS_c safety stock level of material c
α_{pl_c} regulating factor applied to adjust the demand forecast for an uncertain order of product p, FD_{tpl_c}, performed by dealers at period $(t-l_c)$
K_j the optimal number of key machine j

The production system, formulated as a linear program, can be expressed as follows:

$$\text{Min.} \quad \tilde{Z}=\sum_{t=1}^{T}\sum_{p=l}^{P}\tilde{\gamma}_p * SO_{tp}+\sum_{t=1}^{T}\sum_{c=1}^{C}\tilde{\omega}_c * I_{tc}+\sum_{t=1}^{T}\sum_{j=1}^{J}\tilde{\delta}_j * \left(CL_j * K_j-\sum_{p=1}^{P}AP_{tp}\right). \tag{1}$$

$$\text{Subject to} \quad AP_{tp}\geqslant \beta_p * FD_{tp0}, \tag{2}$$

$$FD_{tp0}\geqslant AP_{tp}, \tag{3}$$

$$\sum_{p=1}^{P}AP_{tp}\leqslant CL_j * K_j, \tag{4}$$

$$SO_{tp}=FD_{tp0}-AP_{tp}, \tag{5}$$

$$I_{tc} = I_{(t-1)c} + Q_{tcl_c} - \sum_{p=1}^{P} TU_{tpc}, \tag{6}$$

$$Q_{tcl_c} = SS_c + \sum_{p=1}^{P} FD_{tpl_c} * u_{pc} * \alpha_{pl_c} - EI_{(t-1)c}, \tag{7}$$

$$EI_{(t-1)c} = I_{(t-l_c)c} + \sum_{i=1}^{l_c-1} Q_{[t-(l_c-i)]ci} - \sum_{i=1}^{l_c-1}\sum_{p=1}^{P} FD_{[t-(l_c-i)]pi} * u_{pc} * \alpha_{pl_c}, \tag{8}$$

$$TU_{tpc} = AP_{tp} * u_{pc}, \tag{9}$$

$$I_{(t-1)c} + Q_{tcl_c} \geqslant \sum_{p=1}^{P} TU_{tpc}, \tag{10}$$

$$0 \leqslant \alpha_{pl_c} \leqslant 2, \tag{11}$$

β_p, SO_{tp}, I_{tc}, $EI_{(t-1)c}$, TU_{tpc}, Q_{tcl_c}, AP_{tp}, $SS_c \geqslant 0$ and K_j are integers,

where $\tilde{\gamma}_p = (\gamma_p^m, \gamma_p^o, \gamma_p^p)$, all p; $\tilde{\omega}_c = (\omega_c^m, \omega_c^o, \omega_c^p)$, all c; and $\tilde{\delta}_j = (\delta_j^m, \delta_j^o, \delta_j^p)$, all j. All of these parameters have triangular possibility distributions shown in Fig. 2.

The objective function is to minimize the sum of the product stockout costs, the material inventory holding costs, and idle capacity penalty costs.

The relative constraints of production activities in each period are explained as follows. Eq. (2) guarantees demand fulfillment greater than the required service level for product p at period t. Eqs. (3) and (4) represent the characteristics of no end-products held in the ATO situation and restrict actual production quantities according to the capacity of key machines in the system. Eq. (5) expresses stockouts derived strictly from the difference between actual production quantities and firm order quantities of product p at period t.

Material management in an ATO firm is represented by the following constraints. Eq. (6) is the inventory equilibrium equation that balances the inventory levels of material c at period $t-1$, quantities received and actual usage of material c at period t. $\sum_{p=1}^{P} FD_{tpl_c} * u_{pc} * \alpha_{pl_c}$ represents the forecasted gross demand of material c in period t. Consequently, Eq. (7) computes the order quantity of material c, Q_{tcl_c}, purchased at period $t-l_c$, derived from its safety stocks, gross demand, and estimated one period advanced ending inventory level. At period $t-l_c$, the estimated inventory level of material c at the end of period $t-1$ is determined by the ending inventory at period $t-l_c$, delivered quantities and forecasted gross demands from period $t-l_c$ to period $t-1$. Eqs. (9) and (10) express the precision and the availability of material usage, respectively.

3.2. Approach to solving PPP model

With respect to the techniques for solving a linear programming problem with imprecise coefficients in the objective function, Rommelfanger [14] states that a fuzzy objective function should be interpreted as a multiobjective demand. In general, an ideal solution to this problem does not exist. The first method to obtain a compromise solution was proposed by Tanaka et al. [19]. They adopt a weighted average as a substitute for the fuzzy objective with a special crisp compromise objective. The extreme values of the parameters will have an impact on the effect of the weighting sum. An α-Pareto-optimal solution proposed by Sakawa and Yano [15] restricts the fuzzy coefficients to α-level-sets. Luhandjula's [12] β-possibility efficient solution is a similar concept. As these authors do not explain the specifications of the levels α or β and use several

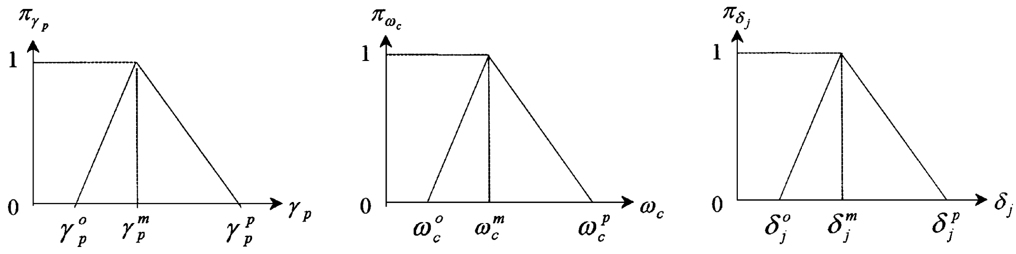

Fig. 2. The triangular possibility distributions of $\tilde{\gamma}_p$, $\tilde{\omega}_c$, and $\tilde{\delta}_j$.

restrictive assumptions, the application of these approaches may be limited in practice. Besides, when the goal of the objective can be given, Tanaka and Asai [18] considered the objective function as a fuzzy constraint. However, a given goal for the objective function is always difficult for managers to decide.

Lai and Hwang [11] referred to portfolio theory and converted the fuzzy objective with a triangular possibility distribution into three crisp objectives. According to their method, we represent Eq. (1) of our model as Min. $\tilde{Z} = (\mathbf{z}^m\mathbf{x}, \mathbf{z}^o\mathbf{x}, \mathbf{z}^p\mathbf{x})$ where $\mathbf{x}$ is a feasible solution for the proposed PPP model. Geometrically, the three critical points $(\mathbf{z}^m\mathbf{x}, 0), (\mathbf{z}^o\mathbf{x}, 1)$, and $(\mathbf{z}^p\mathbf{x}, 0)$ in Fig. 3 fully describe a fuzzy objective. We therefore proceed to minimize the fuzzy objective by pushing the three points toward the left. Solving the fuzzy objective becomes the process of minimizing $\mathbf{z}^m\mathbf{x}$, $\mathbf{z}^o\mathbf{x}$ and $\mathbf{z}^p\mathbf{x}$ simultaneously.

However, there may exist a conflict in the simultaneous optimization process. We substitute minimizing $\mathbf{z}^m\mathbf{x}$, maximizing $(\mathbf{z}^m\mathbf{x} - \mathbf{z}^o\mathbf{x})$, and minimizing $(\mathbf{z}^p\mathbf{x} - \mathbf{z}^m\mathbf{x})$ for minimizing $\mathbf{z}^m\mathbf{x}$, $\mathbf{z}^o\mathbf{x}$, and $\mathbf{z}^p\mathbf{x}$. That is, we minimize the most possible value of imprecise cost, $\mathbf{z}^m\mathbf{x}$. Meanwhile, we maximize the possibility of obtaining lower cost, $\mathbf{z}^m\mathbf{x} - \mathbf{z}^o\mathbf{x}$, and minimize the risk of obtaining higher cost, $\mathbf{z}^p\mathbf{x} - \mathbf{z}^m\mathbf{x}$. The last two objectives are actually relative measures from $\mathbf{z}^m\mathbf{x}$. The three replaced objective functions still guarantee the above declaration of pushing the possibility distribution toward the left in Fig. 3. In this way, our problem can be transformed into a multiple objective linear programming (MOLP) as follows:

$$\text{Min.}\quad z_1 = \sum_{t=1}^{T}\sum_{p=1}^{P}\gamma_p^m * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C}\omega_c^m * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J}\delta_j^m * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right),$$

$$\text{Max.}\quad z_2 = \sum_{t=1}^{T}\sum_{p=1}^{P}(\gamma_p^m - \gamma_p^o) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C}(\omega_c^m - \omega_c^o) * I_{tc}$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}(\delta_j^m - \delta_j^o) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right),$$

$$\text{Min.}\quad z_3 = \sum_{t=1}^{T}\sum_{p=1}^{P}(\gamma_p^p - \gamma_p^m) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C}(\omega_c^p - \omega_c^m) * I_{tc}$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}(\delta_j^p - \delta_j^m) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right).$$

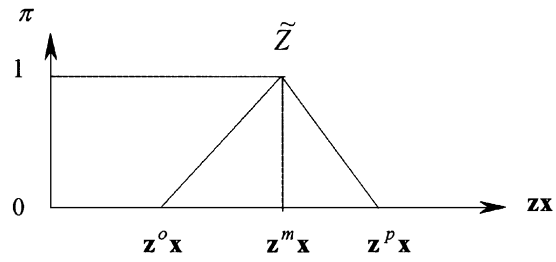

Fig. 3. The triangular possibility distribution of $\tilde{Z}$.

There are many MOLP approaches to solving the above problem, such as goal programming, utility theory, and so on. Since it is quite difficult for managers to determine the requisite objective goals or establish their utility functions, we suggest using Zimmermann's fuzzy programming method with the normalization process [21]. At first, the positive ideal solutions (*PIS*) and negative ideal solutions (*NIS*) of the three objective functions should be obtained. These are

$$\text{Min.}\quad z_1^{PIS} = \sum_{t=1}^{T}\sum_{p=1}^{P} \gamma_p^m * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} \omega_c^m * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J} \delta_j^m * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp} \right),$$

$$\text{Max.}\quad z_1^{NIS} = \sum_{t=1}^{T}\sum_{p=1}^{P} \gamma_p^m * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} \omega_c^m * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J} \delta_j^m * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp} \right),$$

$$\text{Max.}\quad z_2^{PIS} = \sum_{t=1}^{T}\sum_{p=1}^{P} (\gamma_p^m - \gamma_p^o) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} (\omega_c^m - \omega_c^o) * I_{tc}$$
$$+ \sum_{t=1}^{T}\sum_{j=1}^{J} (\delta_j^m - \delta_j^o) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp} \right),$$

$$\text{Min.}\quad z_2^{NIS} = \sum_{t=1}^{T}\sum_{p=1}^{P} (\gamma_p^m - \gamma_p^o) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} (\omega_c^m - \omega_c^o) * I_{tc}$$
$$+ \sum_{t=1}^{T}\sum_{j=1}^{J} (\delta_j^m - \delta_j^o) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp} \right),$$

$$\text{Min.}\quad z_3^{PIS} = \sum_{t=1}^{T}\sum_{p=1}^{P} (\gamma_p^p - \gamma_p^m) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} (\omega_c^p - \omega_c^m) * I_{tc}$$
$$+ \sum_{t=1}^{T}\sum_{j=1}^{J} (\delta_j^p - \delta_j^m) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp} \right),$$

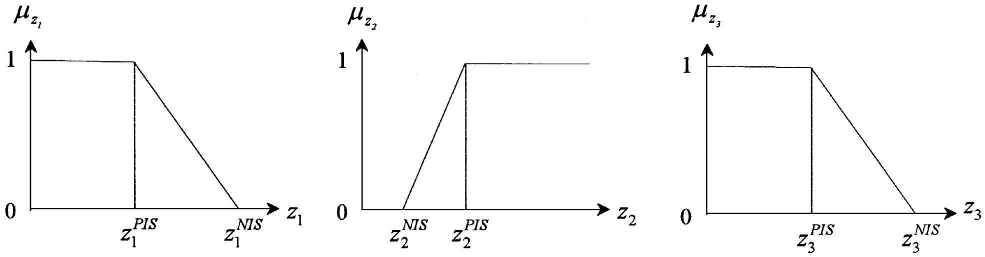

Fig. 4. The membership functions of the objectives z_1, z_2, and z_3.

$$\text{Max.} \quad z_3^{NIS} = \sum_{t=1}^{T}\sum_{p=1}^{P}(\gamma_p^p - \gamma_p^m) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C}(\omega_c^p - \omega_c^m) * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J}(\delta_j^p - \delta_j^m) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right).$$

The linear membership function of these objective functions shown in Fig. 4 can be computed as

$$\mu_{z_1} = \begin{cases} 1 & \text{if } z_1 < z_1^{PIS}, \\ \dfrac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & \text{if } z_1^{PIS} \leqslant z_1 \leqslant z_1^{NIS}, \\ 0 & \text{if } z_1 > z_1^{NIS}, \end{cases} \qquad \mu_{z_2} = \begin{cases} 1 & \text{if } z_2 > z_2^{PIS}, \\ \dfrac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}} & \text{if } z_2^{NIS} \leqslant z_2 \leqslant z_2^{PIS}, \\ 0 & \text{if } z_2 < z_2^{NIS}, \end{cases}$$

$$\mu_{z_3} = \begin{cases} 1 & \text{if } z_3 < z_3^{PIS}, \\ \dfrac{z_3^{NIS} - z_3}{z_3^{NIS} - z_3^{PIS}} & \text{if } z_3^{PIS} \leqslant z_3 \leqslant z_3^{NIS}, \\ 0 & \text{if } z_3 > z_3^{NIS}. \end{cases}$$

Finally, we apply Zimmermann's equivalent single-objective linear programming model (preference-based membership functions of the objective functions) to obtain the overall satisfaction compromise solution.

4. Numerical investigation

An example is given to illustrate the proposed PPP model. The presumed factory produces two types of products which are composed of six different materials. C_1 and C_2 are common to both products, C_3 and C_4 are unique to product P_1, as well as C_5 and C_6 which are unique to product P_2. All materials are purchased according to the strategies of safety stocks and forecast regulation. The final assembly of products is delayed until the firm orders have been received. The longest material acquisition lead time is two periods. The relationship between product–material and acquisition lead time for each material is shown on the right of Table 2.

Provided that the actual demand, FD_{tp0}, for each product per period has an identical normal distribution with mean $\mu_p = 10\,000$ and standard deviation $\sigma_p = 1000$. Moreover, assuming that the forecasting error rates for each product, defined as $(FD_{tpl_c} - FD_{tp0})/FD_{tp0}$, are the same and normally distributed with mean $\mu_{\text{error}} = 0$

Table 2
The required data in the demonstration

Product type	Demand distribution	Forecasting ability	Key machine type	Available capacity per unit	Material type					
	$N(\mu_p, \sigma_p)$	$N(\mu_{\text{error}}, \sigma_{\text{error}})$			C_1	C_2	C_3	C_4	C_5	C_6
P_1	(10 000, 1000)	(0, 0.1)	K_1	600	1	1	1	1	0	0
P_2	(10 000, 1000)	(0, 0.1)	K_2	700	1	1	0	0	1	1
		The acquisition lead time of each material			2	1	2	1	2	1

μ_p: mean demand per period for product p; σ_p: standard deviation of the demand per period for product p; μ_{error}: mean of the forecast error; σ_{error}: standard deviation of the forecast error.

and standard deviation $\sigma_{\text{error}} = 10\%$. Only two types of key machines, K_1 and K_2, are considered for the final product assembly. At each period, the capacity per machine per period is 600 and 700 units, respectively. All required data with respect to demand distribution, forecasting ability, capacity per key machine, acquisition lead time for each material and the product structure are presented in Table 2.

In accordance with past sales records and integrating the judgement of marketing experts, as well as the material control experiences of practitioners, managers approximate the possibility distributions for cost parameters. $(40, 30, 60)$ is supposed to be the imprecise unit stockout penalty costs for both products. It has a triangular possibility distribution with the most possible value $= 40$, the most optimistic value $= 30$ and the most pessimistic value $= 60$, respectively. In like manner, $(1, 0.3, 2)$ and $(1, 0.5, 2)$ are the imprecise unit inventory holding costs and unit idle capacity penalty costs, respectively. The objective function with imprecise cost coefficients is expressed by the following equation:

$$\text{Min.} \quad \sum_{t=1}^{T}\sum_{p=1}^{P}(40, 30, 60) * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C}(1, 0.3, 2) * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J}(1, 0.5, 2) * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right).$$

According to the approach described in Section 3.2, we transform the original ambiguous objective function into the following crisp multiple objective linear programming equation:

$$\text{Min.} \quad z_1 = \sum_{t=1}^{T}\sum_{p=1}^{P} 40 * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} 1 * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J} 1 * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right),$$

$$\text{Max.} \quad z_2 = \sum_{t=1}^{T}\sum_{p=1}^{P} 10 * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} 0.7 * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J} 0.5 * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right),$$

$$\text{Min.} \quad z_3 = \sum_{t=1}^{T}\sum_{p=1}^{P} 20 * SO_{tp} + \sum_{t=1}^{T}\sum_{c=1}^{C} 1 * I_{tc} + \sum_{t=1}^{T}\sum_{j=1}^{J} 1 * \left(CL_j * K_j - \sum_{p=1}^{P} AP_{tp}\right).$$

Before we start to compute the optimal decision set, it is necessary to generate the required input data. We supposed that the planning horizon is 12 months. For securing the robustness of our model, we randomly generated 20 sets of firm order quantities for each product using the presumed probability distribution. Each set contains 12 actual months of demands. Likewise, 20 sets of forecasting error rates for each product can be generated using the aforementioned normal distribution. Each set will include 24 forecasting error rates for different forecasting cycles, that is, $l_c = 1$ and 2, respectively. Based on our definition of the forecasting error rate described above, we may obtain 20 sets of demand forecasts for the next two periods, FD_{tp1} and FD_{tp2},

Table 3
Obtained results of the objective functions

The overall satisfaction level $\alpha^* = 0.8076$					
PIS for z_1	391 869	PIS for z_2	182 040	PIS for z_3	125 262
NIS for z_1	671 625	NIS for z_2	119 287	NIS for z_3	210 619
z_1^*	445 694	z_2^*	169 966	z_3^*	141 685

The optimal cost is imprecise and has a triangular possibility distribution of $(z_1^*, z_1^* - z_2^*, z_1^* + z_3^*)$, i.e. (445 694, 275 728, 587 379).

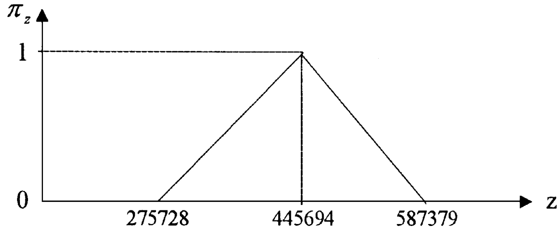

Fig. 5. Possibility distribution of the optimal value of the fuzzy objective function.

respectively. Therefore, all required forecasts and demands in each forecasting cycle on a rolling schedule basis can be placed in sequence into our model.

With these simulated input data, the objective function in our model is to minimize the sum of the 20 cost functions as shown in Eq. (1) with common decision variables: SS_c, α_{pl_c} and K_j. We applied LINDO 5.01 package on a PC-586 (Pentium-166) to solve the LP model. Following the previously described process, a trial run was completed. After 20 trial runs, the summary results, including the positive and the negative ideal solutions, as well as the compromise solutions of z_i $(i = 1, 2, 3)$, and the resulting overall satisfaction level, are shown in Table 3. Fig. 5 depicts the possibilistic distribution of the optimal value of the fuzzy objective function. Apparently, z_1^* is the most possible cost, $z_1^* - z_2^*$ (the most optimistic value) and $z_1^* + z_3^*$ (the most pessimistic value) are the least possible cost. The cost obtained range provides a useful reference for managers attempting to decide an operating budget in production planning. The optimal decision set of the demand forecast regulation factors, the material safety stock levels, and the numbers of key machines is shown in Table 4. The demand forecast regulation factors present a result in which the longer the forecasting cycle, the larger the regulating factor. This implies that forecasting error convolution effects exist in Eq. (8) of our PPP model.

In the end, it is worth noting that the longer the acquisition lead time (C_1, C_3, and C_5), the higher the safety stock level. Meanwhile, the higher the commonality (C_1, and C_2), the lower the safety stock level. Hence, we can conclude that shortening the length of acquisition lead time for materials and increasing material commonality may have representative effects for reducing the influence of demand uncertainty. The usual ways to achieve the preceding effects are taking into account the bill of material modifications and/or product/process redesign, which adopt product modularity to obtain the effects of risk-pooling.

5. Conclusions

Determining the appropriate safety stock levels for assembly materials and regulating the forecast demands are two strategies generally adopted by industries to moderate the effects of demand uncertainty in an ATO environment. Moreover, managers would better decide the numbers of key machines in advance for reducing

Table 4
The solutions for forecast regulation, material safety stock levels, and the numbers of key machines

Regulation values for demand forecasts:	Product type	
	P_1	P_2
Forecasts with lead time = 1	0.891	0.907
Forecasts with lead time = 2	0.863	0.886
Service level	99.62%[a]	99.74%
	Machine type	
Key machine required	K_1	K_2
Optimal quantities	17	13
Target capacity levels of key machines	10 200	9100
Capacity utilization levels	98.2%	93.5%

Safety stock levels for each material:

C_1	C_2	C_3	C_4	C_5	C_6
64.2%[b]	36.5%	70.1%	50.7%	69.2%	51.4%

[a] The service level is denoted the fill rate of the average demand per month.
[b] Safety stock levels are represented by the percentage of the average requirements per month of materials.

capital wastes. This paper provides an analytical model to determine these managerial decisions under the consideration of the ambiguous cost and the uncertain market demand. The proposed model integrates forecasting activities, material management, and production planning. Although solving possibilistic mathematical models remains a concern, this research gives evidence that the proposed model is competent in dealing with vague and imprecise data to solve the decision-making problems in ATO practices.

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