



## A solution to the unequal area facilities layout problem by genetic algorithm

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Received 3 March 2003; received in revised form 3 November 2003; accepted 28 June 2004

Available online 29 January 2005

### Abstract

The majority of the issued facilities layout problems (FLPs) minimize the material handling cost and ignore other factors, such as area utilization, department shape and site shape size. These factors, however, might influence greatly the objective function and should give consideration. The research range of this paper is focus on the unequal areas department facilities layout problem, and implement analysis of variance (ANOVA) of statistics to find out the best site size of layout by genetic algorithm. The proposed module takes the minimum total layout cost (TLC) into account. TLC is an objective function combining material flow factor cost (MFFC), shape ratio factor (SRF) and area utilization factor (AUF). In addition, a rule-based of expert system is implemented to create space-filling curve for connecting each unequal area department to be continuously placed without disjoint (partition). In this manner, there is no gap between each unequal area department. The experimental results show that the proposed approach is more feasible in dealing with the facilities layout problems in the real world.

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*Keywords:* Total layout cost; Genetic algorithms; Material flow factor cost; Shape ratio factor; Area utilization factor; Space-filling curve

### 1. Introduction

The facilities layout and material handling design affect the operating cost, profitability of the whole industry, and the material handling cost accounts for 20–50% of the total operating cost. An effective facilities layout and material handling design will reduce the operating cost of the industry by 10–30%

[1]. Achieving a minimal material handling cost becomes an ultimate goal for the facilities layout designers. Facilities layout problems (FLPs) could be classified into two kinds of problems, discrete layout problems (DLPs) and continual layout problems (CLPs). DLP divides the plant site into many rectangular blocks, each block has the same area and shape, and each block is assigned to a facility. If the facilities have unequal areas, they could occupy blocks and modeled into a cell. Quadratic assignment problem (QAP) is the most famous of discrete layout

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problem. Concerning the CLP, all the facilities may be placed anywhere within the planar site, and the facilities must not overlap each other. Some of those layout problems are presented in [2–4]. When the number of facilities layout departments is less than 15, these two kinds of problems are able to reach an optimal solution. However, when the number of facilities layout departments is more than 15, it has been validated to be a NP-complete problem. As the number of departments increasing, the computational time is exponentially increased by  $2^n$  [5]. Because the optimal solution is not easy to reaching, there are lot of heuristic approaches has been developed to get the near-optimal solution, such as simulation annealing [4,6,7], tabu searching [8,9], and genetic algorithms [10–12]. Generally speaking, genetic algorithm (GA) outperforms to other heuristic methods. GA is a simulation of the evolutionary competition and survival fitness in natural evolution. It is a parallel processing and multiple-points utilization algorithm in searching solution space. Therefore, it enhances the opportunity to achieve global optimal solution without falling into the local optimal solution. It has been widely implemented to solve combinatorial optimization problems and is considered as a robust approach by accompanying with artificial intelligence [13].

Regarding the discrete layout problems, several factors may influence the final result of layout. These factors are as follows: (1) the material flow factor cost (MFFC), (2) the area utilization factor (AUF) of whole layout, and (3) the shape ratio factor (SRF) of department. The first factor is concerning about material handling cost (MHC), minimal MHC is almost the general objective of the layout problem. The second factor, dependent on the plant site size, a large but inappropriate site size, apart from increase the land investment cost, also decreases the effective area utilization and increases the maintenance cost. About the third factor, the more regular individual department is (e.g. square), the lower cost of the department layout and arrangement is. The second and third factors mentioned above are as important as material flow cost. They are critical and significantly influential to final layout. Therefore concerning a facilities layout problem, the second and third factors should be considered into the model simultaneously as well.

In this paper, the objective function is the total layout cost (TLC) including MFFC, SRF, and AUF by

proposed approach. A rule-based approach of expert system is also proposed to create space-filling curve (SFC). SFC connects and places each unequal area department without disjoint (partition). By the way, we attempt to use ANOVA of statistics to find out the best site shape size layout by genetic algorithm. This paper is organized as follows: Section 2 presents the problem formulation. While the proposed solving approach based on genetic algorithm is described in Section 3. The implementation and experimental results of the proposed approach are summarized in Section 4. Finally, the concluding remarks are given in Section 5.

## 2. Problem formulation

Unequal area department facilities layout problem is a quadratic set covering problem (QSP). Traditional, QSP is based on flow factor and is constructed to find out minimal material flow cost. In this paper, we proposed a model to find out the minimal TLC.

### 2.1. The model base on material flow factor cost only

To measure the objective function value of unequal area department layout problem, most of researches consider only the material flow and directly minimize the total material flow cost between departments. The general model is shown as below:

$$\min \text{MFFC} = \sum \sum C_{ij} f_{ij} d_{ij} \quad (1)$$

where MFFC is the material flow factor cost between departments,  $C_{ij}$  the transportation cost for a unit material for a unit distance between departments  $i$  and  $j$ ,  $f_{ij}$  the material flow from departments  $i$  to  $j$  and  $d_{ij}$  is the rectilinear distance between center of departments  $i$  and  $j$ .

### 2.2. The model base on total layout cost (TLC)

An effective layout planning should include minimize material handling cost, reasonable geometric shape of department (site), efficient area utilization, flexibility arrangement, etc. Therefore, MFFC, SRF, and AUF are included in objective function in this study. Concerning facilities layout problem, the more regular shape of department (site),

the less cost of construction spent. The more area utilization, the less land investment is required. Thus, SRF and AUF have great impact to MFCC, they should be incorporated as well to precisely measure TLC.

In this paper, we define the individual department shape ratio is  $SRF_i$  and equal to  $P_i/(4\sqrt{A_i})$  [14]. The shape ratio of all departments is the geometric mean  $(\prod_{i=1}^N SR_i)^{1/N}$  (i.e., The ideal shape ratio of individual department is 1 for a square). Consolidated the shape ratio of all departments and the site shape ratio, hence the shape ratio factor of whole layout ( $SRF_{whole}$ ) is shown as following:

$$SRF_{whole} = \left( \prod_{i=1}^N SR_i \right)^{1/N} = \left( \prod_{i=1}^N \frac{P_i}{4\sqrt{A_i}} \right)^{1/N} \quad (2)$$

where  $SRF_{whole}$  is the shape ratio factor of overall layout,  $SR_i$  the shape ratio of department  $i$ ,  $N$  the number of department,  $P_i$  the perimeter of department  $i$  and  $A_i$  is the area required of department  $i$ .

We define the area utilization factor of whole layout ( $AUF_{whole}$ ) is a ratio of total areas required of all facilities to the smallest possible rectangle [15], which can envelop all the facilities. Hence, the area utilization factor of whole layout (e.g. 100% area utilization is the best layout) is shown in Eq. (3):

$$AUF_{whole} = \frac{\sum A_i}{\sum A_i + TBA} \quad (3)$$

where  $AUF_{whole}$  is the area utilization factor of whole layout,  $\sum A$  the total required areas of all departments and TBA is the total blank area of layout, Including the material flow factor cost (MFCC), the shape ratio factor of overall layout ( $SRF_{whole}$ ) and the area utilization factor of overall layout ( $AUF_{whole}$ ), the proposed model becomes:

zation factor of overall layout ( $AUF_{whole}$ ), the proposed model becomes:

$$\min TLC = MFCC \frac{SRF_{whole}}{AUF_{whole}} \quad (4)$$

$$\text{s.t. } \sum_{K=1}^N a_{ijk} \leq 1 \quad \text{for all } i \text{ and } j \quad (5)$$

$$\sum_{i=1}^W \sum_{j=1}^L a_{ijk} \leq A_k \quad \text{for all } k \quad (6)$$

$$\sum_{i=1}^W \sum_{j=1}^L \sum_{k=1}^N a_{ijk} \leq LW \quad (7)$$

where TLC is the total layout cost,  $a_{ijk} = 1$ , if department  $k$  is assigned to the location at the  $i$ th row and the  $j$ th column,  $a_{ijk} = 0$ , otherwise,  $A_k$  the area required of the department  $k$ ,  $L$  the maximum length (horizontal axis) of the plant site and  $W$  is the maximum width (vertical axis) of the plant site.

Therefore, the objective function is the total layout cost including MFCC, SRF and AUF. Constrain as formulated in Eq. (5) prohibits that more than one departments are assigned the same location (position). Constrain (6) represents that the locations assigned to each department is not allowed to be greater than the areas required of each department. Constrain (7) represents that the sum of areas required for all departments cannot be greater than the plant site. If ignoring SRF and AUF, the model only considers MFCC, constrain (4) becomes constrain (1).

An illustration is shown in Fig. 1. If areas required of departments 1, 2, 3, is 10, 5, 8, respectively, the material flow between departments 1 and 2, 1 and 3, 2

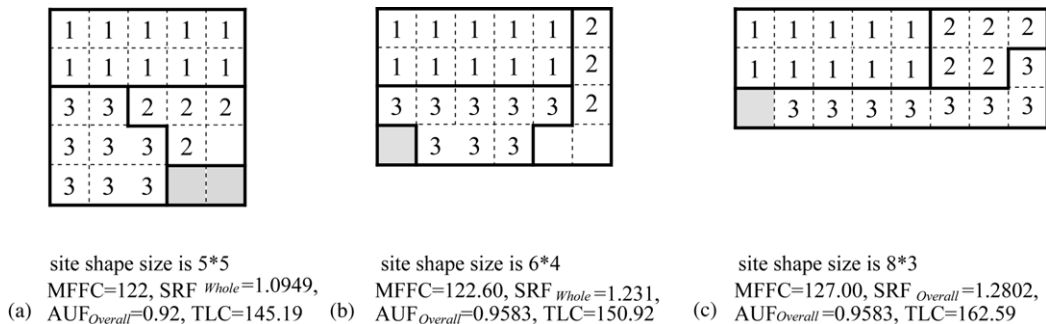


Fig. 1. Several layout options.

and 3, is 10, 8, 20, and the material handling cost is set equal to 1, all. In order to envelop the areas required for all departments, the site shape size of layout could be of several options as in Fig. 1(a)–(c). In Fig. 1(a),  $MFFC = 122$ ,  $SRF_1 = 1.1068$ ,  $SRF_2 = 1.1180$ ,  $SRF_3 = 1.0607$ ,  $SRF_{\text{whole}} = (SRF_1 \times SRF_2 \times SRF_3)^{1/3} = 1.094$ ,  $\sum A_i = 23$ ,  $TBA = 2$  (shadowy areas),  $AUF_{\text{whole}} = \frac{23}{23+2} = 0.92$ , the resultant total layout cost (TLC) based on Eq. (4) is equal to  $122 \times \frac{1.0949}{0.92}$  (145.1933). In Fig. 1(b), the MFFC and TLC is 122.6 and 150.92, respectively. In Fig. 1(c), the MFFC and TLC is 127 and 162.59, respectively. Considering only MFFC or TLC with respect to Fig. 1, the facilities layout designer may choose the cheaper one, which is obviously Fig. 1(a).

### 3. Methodology

This section mainly introduces the procedures of GA, layout representation as genetic code and GA operations and the procedures to generate space-filling curve is presented as well.

#### 3.1. The procedures of genetic algorithm (GA)

GA is a simulation of the evolutionary competition and survival fitness in natural evolution. It is a parallel processing, robust, and multiple-points utilization algorithm in searching solution space. Therefore, it enhances the opportunity to achieve global optimal solution without falling into the local optimal solution. It has become a popular search technology in recent years. The basic issue of GA proposed by Holland [16] is called simple genetic algorithm. The procedures of GA are shown in Fig. 2.

#### 3.2. Layout representation as genetic coding

In this study, the strings of genes encoding are represented as numeric value, which included five segments. The first segment shows department placement sequence. The second segment illustrates the required areas of each department. The third segment shows site size of length and width (such as:  $12 \times 12$  or  $14 \times 10$ ). The fourth segment shows sweeping direction (such as: 1 is horizontal, 2 is vertical). Finally, the fifth segment shows sweeping

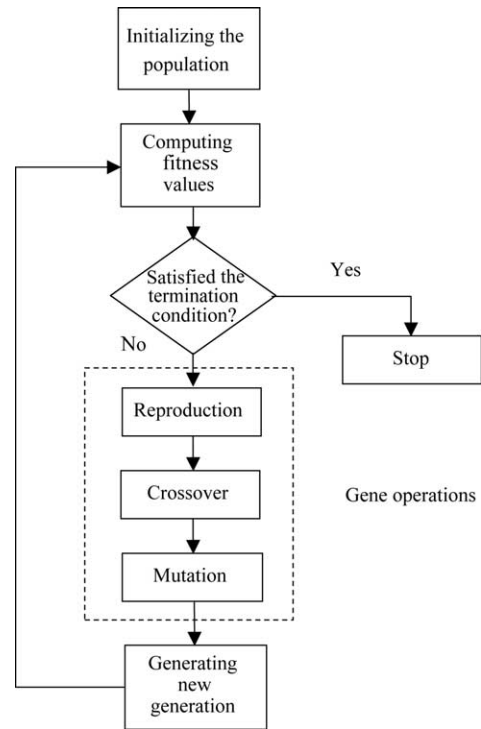


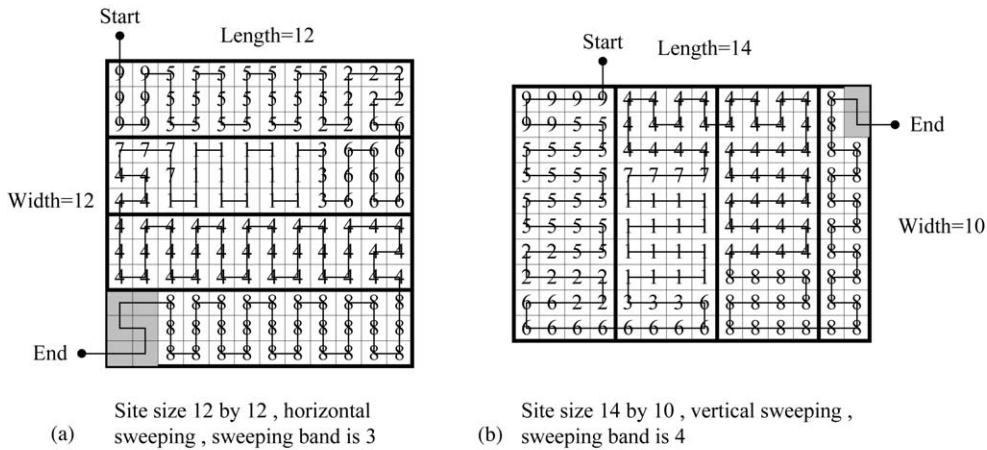
Fig. 2. The procedures of GA.

bands. The strings of genes, which comprise of these five segments, represent the whole floor layout.

An illustration is shown in Fig. 3. The first two segments about placement sequences, and areas in Fig. 3(a) and (b) are same as (9, 5, 2, 6, 3, 1, 7, 4, 8) and (6, 20, 8, 11, 3, 16, 4, 40, 30). But in Fig. 3(a), the site size is  $12 \times 12$  square, sweeping direction is horizontal (1), and sweeping band is 3. In Fig. 3(b), the site size is  $14 \times 10$  rectangular, sweeping direction is vertical (2), and sweeping band is 4. The curves of Fig. 3(a) and (b) from start to end are decoding procedure. These two floor layout plans of Fig. 3(a) and (b) could be resulting different MFFC and TLC, when the layout designer going to planning a firm should be consideration these alternatives.

##### 3.2.1. Fitness function

As far GA is concerned, it's better to have higher fitness values to provide more opportunities to be chosen in breeding new chromosomes. Objective function can be used as the fitness function to search



1 <sup>st</sup> segment	2 <sup>nd</sup> segment	3 <sup>rd</sup> segment	4 <sup>th</sup> segment	5 <sup>th</sup> segment
Sequence of departments	Required areas of departments	Site size of floor layout	Sweeping of direction	Sweeping band
9 5 2 6 3 1 7 4 8	6 20 8 11 3 16 4 40 30	12 12 <sup>(1)</sup>	1 <sup>(2)</sup>	3 <sup>(3)</sup>

<sup>(1)</sup> Site size of floor layout : Length by Width  
<sup>(2)</sup> Sweeping direction (D): either Horizontal (1) or Vertical (2)  
<sup>(3)</sup> Sweeping band (B) could be 1,2,3,...to n

Fig. 3. Layout representation as genetic coding.

for the maximum of the solution. On the contrast, the inverse of objective function can be used as the fitness function to search for the minimum of the solution. A fitness function including MFFC, SRF, and AUF in this paper is shown in Eq. (8):

$$\text{fitness} = \frac{1}{\text{TLC}} = \frac{1}{\text{MFFC}} \frac{\text{AUF}_{\text{overall}}}{\text{SRF}_{\text{overall}}} \quad (8)$$

### 3.2.2. Operations on genes

There are three genetic operations known as reproduction, crossover, and mutation. The purpose of reproduction is to breed chromosome with higher

fitness function value in replacing chromosomes with lower fitness function value in a population. If population has  $N$  chromosomes and its reproduction rate is  $P_r$ , there will be  $NP_r$  best chromosomes to be reproduced to replace  $NP_r$  worse chromosomes. The crossover operator operates chromosomes of the population and produces offspring. If there are  $N$  chromosomes of the population and the crossover rate is  $P_c$ , there will be  $NP_c$  chromosomes randomly chosen for crossover. Fig. 4 shows an example when two parents are randomly chosen for crossover. The crossover point at the third gene and generate two new offspring randomly.

		Crossover point															
Parent 1	1	2	5	6	1	3	4	10	8	6	5	20	3	8	7	2	2
Parent 2	2	1	3	6	4	2	5	5	20	6	3	8	10	9	6	1	3
Offspring 1	1	2	5	6	4	2	5	10	8	6	3	8	10	9	6	1	3
Offspring 2	2	1	3	6	1	3	4	5	20	6	5	20	3	8	7	2	2
Adjusted 1	1	2	5	6	4	1	3	10	8	6	3	5	20	9	6	1	3
Adjusted 2	2	1	3	6	2	5	4	5	20	6	8	10	3	8	7	2	2

Fig. 4. Crossover operator.

	Mutation point															
Parent	2	5	6	1	3	4	10	8	6	5	20	3	8	7	2	3
Offspring	2	5	4	1	3	6	10	8	3	5	20	6	8	7	2	3

Fig. 5. Mutation operator.

The mutation operator aims at increasing chromosome variability of population to enlarge new search directions. It enables a breakthrough in local optimal solution. If there are  $N$  chromosomes of the population and  $M$  genes in each chromosome, the mutation rate is set to  $P_m$ , there will be  $NMP_m$  genes of the population to be mutated randomly to generate new offspring as illustrated in Fig. 5.

### 3.3. Space-filling curve (SFC)

In facilities layout, SFC connects each position (location) and enables unequal areas of respective departments to be continuously placed without discontinuity (partition). Nevertheless, it requires many rules to verify the connection of all positions of a layout. Thus, the rule-based expert system becomes a tool to solve SFC in this study. Expert system had been validated to be effectively in the applications [17,18]. This study further applied IF-THEN rules of expert system to develop the procedures of SFC. These rules enable us to judge the next position using current row ( $I$ ) and column ( $J$ ), the sweeping direction ( $D$ ), sweeping band ( $B$ ), and the frequency of sweeping zone ( $F$ ). When each step is completed, the new position will be set as the current position until all positions (locations) in the whole layout are connected. The partial IF-THEN rule is described as below, the procedures of generating SFC, and the results of SFC are shown as in Figs. 6 and 7:

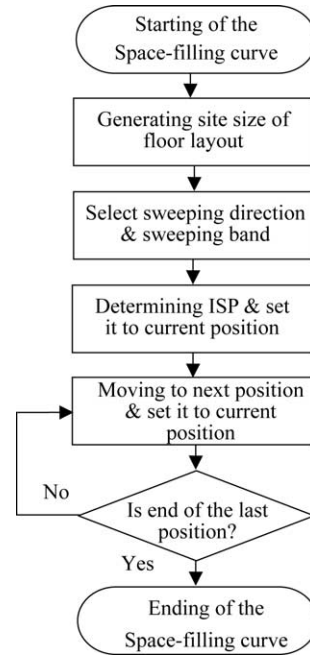


Fig. 6. The procedures to generate SFC.

width of floor layout ( $Y$ -axis);  $W$  the either even (E) or odd (O);  $D$  the sweeping direction,  $D$  is either horizon (H) or vertical (V);  $B$  the sweeping band,  $B = 1, 2, 3, \dots, n$ ;  $F$  the frequency of sweeping zone,  $F$  is either even (E: if horizon sweeping, from right to left; if vertical sweeping, from down to up) or odd (O: if horizon sweeping, from left to right; if vertical sweeping, from up to down);  $I$  the current position of the row,  $I = 1, 2, 3, \dots, W$ ;  $J$  the current position of the column,  $J = 1, 2, 3, \dots, L$ ;  $C$  the remainder of ( $I \text{ mode } B$ ),  $C = 0, 1, 2, 3, \dots, B - 1$ .

As illustrated in Fig. 7, assume that sweeping direction is horizontal sweeping, current position is at

IF  $\text{CP}_{wB}^{I,D}$  (Condition D & Condition F & Condition I & Condition J & Condition C)

THEN  $I = I + 1, F = F + 1$  Moving Downward to  $\text{NP}_{wB}^{I,D}(F+1, I+1, J, C)$  :

$$\text{CP}_{wB}^{I,D}(F+1, I+1, J, C) = \text{NP}_{wB}^{I,D}(F+1, I+1, J, C)$$

where ISP is the initial sweeping position; CP the current position; NP the next position;  $L$  the length of floor layout ( $X$ -axis),  $L$  is either even (E) or odd (O);  $W$  the

$F = 1, I = 3, J = 12, C = (I \text{ mode } B) = 0$ , then the next position will be  $I = I + 1 = 4, F = F + 1 = 2, J = 12, C = 1$ , the IF-THEN rule as described in below:

IF  $\begin{matrix} 12 & H \\ CP & (D \text{ is } H \ \& \ F \text{ is } 1 \ \& \ I \text{ is } 3 \ \& \ J \text{ is } 12 \ \& \ C \text{ is } 0) \\ 12 & 3 \end{matrix}$

THEN  $I = 3 + 1, F = 1 + 1$  Moving Downward to  $\begin{matrix} 12 & H \\ NP & (2,4,12,1) : \\ 12 & 3 \end{matrix}$

$\begin{matrix} 12 & H \\ CP & (2,4,12,1) = \\ 12 & 3 \end{matrix} = \begin{matrix} 12 & H \\ NP & (2,4,12,1) \\ 12 & 3 \end{matrix}$

### 3.4. Statistics test

One popular procedure used to deal with testing more than two population means is called the analysis of variance (ANOVA). The procedures of ANOVA to find the best site of plant layout are shown in Fig. 8.

## 4. Implementation and results

The proposed approach was programming in Visual Basic and executed by Pentium 3 PC. In order to validate the proposed algorithm, we take three cases to undertake experiments and comparison, case 1 take from Tompkins et al. [1] department number  $n = 8$  (site size  $18 \times 10$ ), case 2 from Islier [11] department number  $n = 12$  (site size  $19 \times 14$ ), and case 3 from Armour and Buffa [19] department number  $n = 20$  (site size

$30 \times 20$ ). To give consideration both the solution quality and calculating efficiency, the reproduction rate, crossover rate and mutation rate of the three cases are set same as 20%, 20% and 2%, whilst the population size are set to 100, 500 and 1000, respectively, and the numbers of generation are also 100, 500 and 1000. In addition, we design several different site sizes and using statistics method (ANOVA, The Scheffe’s multiple comparisons) to find the best site size (the minimum MFFC or TLC). Thus, in case 1  $n = 8$ , increases four site sizes which are  $14 \times 13, 16 \times 11, 20 \times 9$  and  $25 \times 7$ . In case 2  $n = 12$ , increases four site sizes which are  $17 \times 16, 21 \times 13, 25 \times 11$  and  $30 \times 9$ . In case 3  $n = 20$ , increases four site sizes which are  $25 \times 24, 40 \times 15, 50 \times 12$  and  $60 \times 10$ . Ten runs of each site size are executed by proposed algorithm and the results are listed in Table 1. Table 2 compared the results and optimal cost with others approaches. Tables 3, 5 and 7 are the results of ANOVA about three cases. Tables 4, 6 and 8 are the results of simultaneous confidence

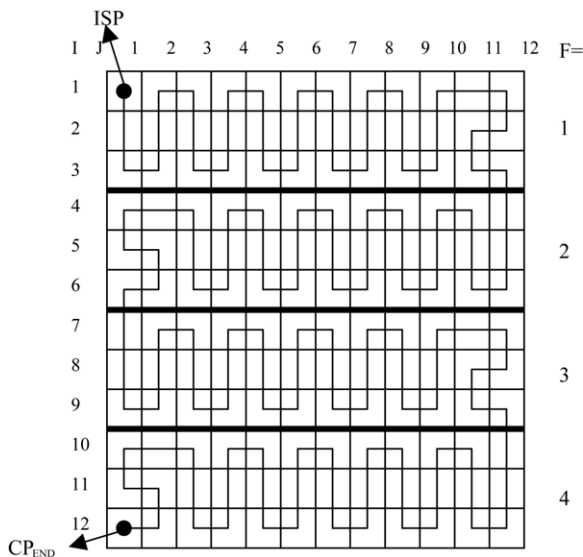


Fig. 7. SFC generating by horizontal sweeping,  $L = 12, W = 12, B = 3, ISP$  at  $F = 1, I = 1, J = 1$ .

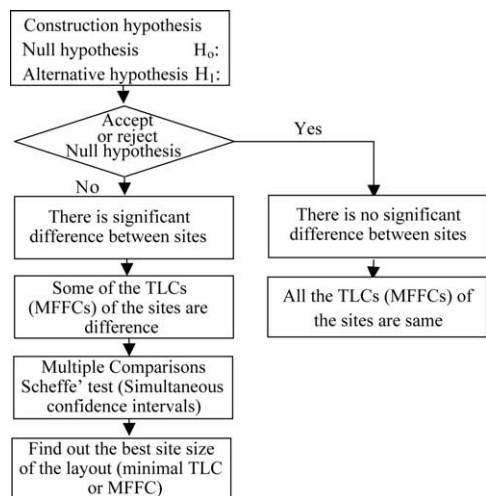


Fig. 8. The procedures of ANOVA to find the best site size.

Table 1  
Ten runs results of the three cases for several different site sizes

Run	Site size									
	14 × 13		16 × 11		18 × 10		20 × 9		25 × 7	
	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC
Case 1, $n = 8$										
1	50710	64636	45283.3	59644.2	42700	56030.8	42729.8	57837.4	54746.6	67701.2
2	50710	64636	45283.3	59644.2	44836.6	58294.6	42729.8	57837.4	53440	69791.8
3	50973.4	67790	45283.3	59644.2	44836.6	58294.6	46109.8	63614.4	54746.6	67701.2
4	49406.6	65380	45283.3	59644.2	42700	56030.8	42729.8	57837.4	53440	69791.8
5	49406.6	65380	45283.3	59644.2	44836.6	58294.6	42729.8	57837.4	54746.6	67701.2
6	49406.6	65380	45283.3	59644.2	44836.6	58294.6	42729.8	57837.4	53440	69791.8
7	49406.6	65380	45283.3	59644.2	42700	56030.8	42729.8	57837.4	54746.6	67701.2
8	50706.6	66684	45283.3	59644.2	42700	56030.8	42729.8	57837.4	53440	69791.8
9	50710	64636	45283.3	59644.2	42700	56030.8	42729.8	57837.4	53440	69791.8
10	49406.6	65380	45283.3	59644.2	42700	56030.8	42729.8	57837.4	53440	69791.8
Best	49406.6	64636	45283.3	59644.2	42700	56030.8	42729.8	57837.4	53440	67701.2
Average	50084.3	65528.2	45283.3	59644.2	43554.6	56936.3	43067.8	58415.1	53938.6	68955.5
Run	17 × 16		19 × 14		21 × 13		25 × 11		30 × 9	
	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC
	Case 2, $n = 12$									
1	38936.8	45949.2	40749.5	47362.6	38573.4	45757.7	40004.8	48338.2	42778.4	50790.2
2	39439.2	47227.2	38226.3	45505.4	37710.4	45403.2	40004.8	48338.2	43034.6	51727.1
3	40058.3	48786.3	38730.5	44396.7	37710.4	45403.2	40237.5	49014	42778.4	50790.3
4	37349.9	43686.2	38226.3	45505.4	37710.4	45403.2	40108.4	48811.8	43120.9	52906
5	37349.9	43686.2	38226.3	45505.4	38467.7	47255.2	40004.8	48338.2	42778.4	50790.2
6	37349.9	43686.2	38226.3	45505.4	38408.3	45232.7	40004.8	48338.2	43034.6	51727.1
7	37349.9	43686.2	38226.3	45505.4	37710.4	45403.2	40004.8	48338.2	43409.7	52740.1
8	37349.9	43686.2	38730.5	44396.7	38408.3	45232.7	40004.8	48338.2	42778.4	50790.3
9	39407.8	46609.1	38226.3	45505.4	39148.0	47036.5	39809.6	47665.1	43761.1	53654.3
10	39439.2	47227.7	38418.7	44891.9	37710.4	45403.2	40108.4	48811.8	42778.4	50790.3
Best	37349.9	45949.2	38226.3	44396.7	37710.4	45757.7	40004.8	48338.2	42778.4	50790.2
Average	38403.0	45423.0	38598.7	45408.0	38155.7	45753.0	40029.2	48433.1	43025.2	51670.5
Run	25 × 24		30 × 20		40 × 15		50 × 12		60 × 10	
	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC	MFFC	TLC
	Case 3, $n = 20$									
1	5759.3	6777.4	5926.6	6781.5	6030.2	6809.6	6254.7	7520.8	6597.7	7550.7
2	6072.5	7002.3	6418.9	7334.5	6068.7	6853	6301.6	7532.9	6487.9	7337.7
3	5912.8	6783.9	6127.5	7041.9	5988.6	6772.1	6448.2	7743.5	6638.3	7549.1
4	6044.2	6900.2	6065.9	6877.2	6223.8	6994	5997.9	7117.9	6689.5	7525.1
5	5842.2	6790.5	6056.7	6894.7	5946.7	6701.1	5827.3	7087.6	6878.2	7736.5
6	6097.9	7052.3	6020.1	6763.9	6002.6	6718.4	6091.4	7212.7	6576.0	7332.3
7	6015.0	6979.8	6049.6	6955.3	6039.0	6790.7	6356.1	7461.4	6471.6	7306.1
8	6068.3	7353.3	6259.4	7317.7	6090.0	6898.5	6223.4	7491	6816.1	7709.4
9	5787.4	6666.3	6017.1	6821.2	5918.5	6653.8	6058.1	7286.1	6771.8	7659.4
10	5806.7	6587.8	6121.5	6978.1	6105.1	6842.2	6234.9	7393.7	6562.6	7463.5
Best	5759.3	6587.8	5926.6	6763.9	5918.5	6653.8	5827.3	7087.6	6471.6	7306.1
Average	5940.63	6889.38	6106.33	6976.6	6041.32	6803.34	6179.36	7384.76	6648.97	7516.98



Table 2  
The results of the three cases and compared with other exiting algorithms

Department no.	Objective function	MFFC <sup>(1)</sup>	TLC <sup>(3)</sup> = MFFC $\frac{SRF_{whole}}{AUF_{whole}}$
Case 1, $n = 8$	Ref. [1], example 8.1, site size $18 \times 10$	56670 <sup>a(1)</sup>	68213.9 <sup>a(3)</sup>
	Proposed algorithm, site size $18 \times 10$	42700.0 <sup>(1)</sup> , 43554.6 <sup>(2)</sup>	56030.8 <sup>(3)</sup> , 56936.3 <sup>(4)</sup>
	Reduction cost (%)	24.7 <sup>(1)</sup>	17.9 <sup>(3)</sup>
Case 2, $n = 12$	Ref. [11], site size $19 \times 14$	39270.0 <sup>b(1)</sup>	44908.2 <sup>b(3)</sup>
	Proposed algorithm, site size $19 \times 14$	38226.3 <sup>(1)</sup> , 38598.7 <sup>(2)</sup>	44396.7 <sup>(3)</sup> , 45408.0 <sup>(4)</sup>
	Reduction cost (%)	2.7 <sup>(1)</sup>	1.1 <sup>(3)</sup>
Case 3, $n = 20$	Ref. [19], site size $30 \times 20$	7862.09 <sup>c(1)</sup>	8459.61 <sup>c(3)</sup>
	Ref. [20], MULTIPLE, site size $30 \times 20$	6857.9 <sup>d(1)</sup>	9916.5 <sup>d(3)</sup>
	Proposed algorithm, site size $30 \times 20$	5926.60 <sup>(1)</sup> , 6106.33 <sup>(2)</sup>	6781.5 <sup>(3)</sup> , 6976.60 <sup>(4)</sup>
	Reduction cost (%)	24.6 <sup>(1)</sup> , Armour and Buffa; 13.6 <sup>(1)</sup> , MULTIPLE	19.8 <sup>(3)</sup> , Armour and Buffa; 31.6 <sup>(1)</sup> , MULTIPLE

<sup>a(1)</sup> Abstracted from [1]; <sup>a(3)</sup> Tompkins  $n = 8$  is not available, recalculated with <sup>(3)</sup>  $SRF_{whole}=1.170$ ,  $AUF_{whole}=0.972$ ; <sup>b(1)</sup> abstracted from [11]; <sup>b(3)</sup> Islier  $n = 12$  is not available, recalculated with <sup>(3)</sup>  $SRF_{whole} = 1.139$ ,  $AUF_{whole} = 0.996$ ; <sup>c(1)</sup> abstracted from [19]; <sup>c(3)</sup> Armour and Buffa is not available, recalculated with <sup>(3)</sup>  $SRF_{whole}=1.076$ ,  $AUF_{whole}=1$ ; <sup>d(1)</sup> abstracted from [20]; <sup>d(3)</sup> MULTIPLE is not available, recalculated with <sup>(3)</sup>  $SRF_{whole} = 1.446$ ,  $AUF_{whole} = 1$ ; <sup>(1)</sup> the best objective function values of ten run, consider only the material flow factor cost (MFFC); <sup>(2)</sup> average objective function values of ten run, consider only the Material Flow Factor cost (MFFC); <sup>(3)</sup> the best objective function values of ten run about the total layout cost (TLC); <sup>(4)</sup> average objective function values of ten run about the total layout cost (TLC); NA: not available.

interval (Scheffe’s multiple comparisons) about these cases.

In Table 2, Tompkins  $n = 8$  site size  $18 \times 10$  has found out that the best MFFC is 56 670<sup>a(1)</sup>; should the shape ratio factor and area utilization factor in this paper be added in, the TLC would be 68 213.9<sup>a(3)</sup>. Aiming at site size  $18 \times 10$ , the best MFFC (TLC) are 42 700<sup>(1)</sup> (56 030.8<sup>(3)</sup>) done by the proposed algorithm, it decreases the cost by 24.7% (17.9%), the layout of which as shown in Fig. 9. In Table 2, Islier  $n = 12$  site size  $19 \times 14$  has found out that the best MFFC is 39 270<sup>b(1)</sup>; should the shape ratio factor and area utilization factor in this paper be added in, the TLC would be 44 908.2<sup>b(3)</sup>. Aiming at site shape size

$19 \times 14$ , the best MFFC and TLC are 38 226.3<sup>(1)</sup> and 44 396.7<sup>(3)</sup> done by the proposed algorithm, and they decrease the cost by 2.7% and 1.1% respectively to Islier, the layout of which as shown in Fig. 10. In Table 2, Armour and Buffa  $n = 20$  site size  $30 \times 20$  has found out that the best MFFC is 7862.09<sup>c(1)</sup>; should the shape ratio factor and area utilization factor in this paper be added in, the TLC by Armour and Buffa  $n = 20$  is 8459.61<sup>c(3)</sup>. Aiming at site size  $30 \times 20$ , the best MFFC and TLC are 5926.60<sup>(1)</sup> and 6781.5<sup>(3)</sup> done by the proposed algorithm, and they

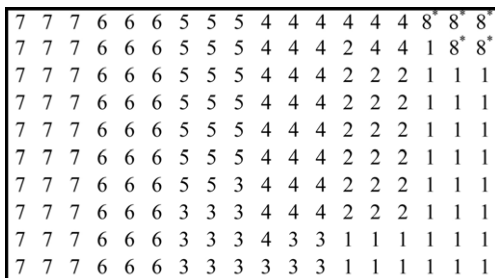


Fig. 9. Case 1 ( $n = 8$ ), the optimal layout site size  $18 \times 10$ , TLC = 56030.8,  $SRF_{whole} = 1.170$ ,  $AUF_{whole} = 0.972$ , MFFC = 42700.

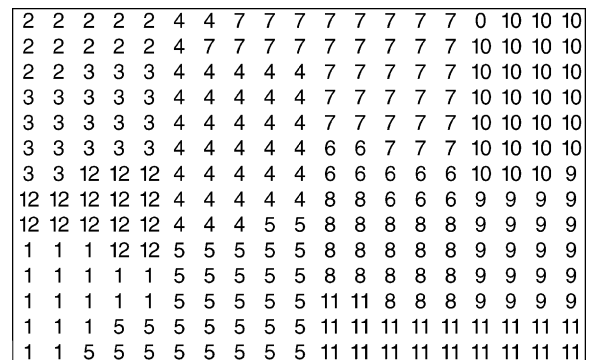


Fig. 10. Case 2 ( $n = 12$ ), site size  $19 \times 14$ , TLC = 44396.7,  $SRF_{whole} = 1.139$ ,  $AUF_{whole} = 0.996$ , MFFC = 38226.3.

Table 3  
ANOVA table by case 1

Source	Sum of square	d.f.	Mean square	<i>F</i>	<i>P</i> -value	Critical value
Between	8.81E+08	4	2.2E+08	330.5115	9.98E-33	2.5787
Error	29984036	45	666311.9			
Total	9.11E+08	49				

Table 4  
Scheffe's multiple comparison by case 1

<i>i</i>	<i>j</i>				
	14 × 13 (average = 50084.3)	16 × 11 (average = 45283.3)	18 × 10 (average = 43554.6)	20 × 9 (average = 43067.8)	25 × 7 (average = 53938.6)
14 × 13	–	4801* ± 1172.4	6529.7* ± 1172.4	7016.5* ± 1172.4	–3854.3* ± 1172.4
16 × 11		–	1728.7* ± 1172.4	2215.5* ± 1172.4	–8655.3* ± 1172.4
18 × 10			–	486.8 ± 1172.4	–10384* ± 1172.4
20 × 9				–	–10870.8* ± 1172.4
25 × 7					–

\* Significant difference between sites *i* and *j*.

Table 5  
ANOVA table by case 2

Source	Sum of square	d.f.	Mean square	<i>F</i>	<i>P</i> -value	Critical value
Between	1.64E+08	4	41072264	89.1203	8.64E–21	2.5787
Error	20738832	45	460862.9			
Total	1.85E+08	49				

decrease the cost by 24.6% and 19.8%, respectively, to Armour and Buffa, and superior to MULTIPLE in MFFC and TLC about 13.6% and 31.6%, the layout of which as shown in Fig. 11.

By the analysis of MFFC and TLC from these cases, the proposed approach in this study performs

well, is an effective layout scheme dealing with unequal area department problems and outperforms other approaches.

In Table 3, ANOVA analysis of case 1, the result is to reject null hypothesis, the means of MFFC between site sizes 14 × 13, 16 × 11, 18 × 10,

Table 6  
Scheffe's multiple comparison by case 2

<i>i</i>	<i>j</i>				
	17 × 16 (average = 38403.1)	19 × 14 (average = 38598.7)	21 × 13 (average = 38155.8)	25 × 11 (average = 43025.3)	30 × 9 (average = 40029.3)
17 × 16	–	–195.6 ± 975.1	247.3 ± 975.1	–1626.2* ± 975.1	–4622.2* ± 975.1
19 × 14		–	442.9 ± 975.1	–1430.6* ± 975.1	–4426.6* ± 975.1
21 × 13			–	–1873.5* ± 975.1	–4869.5* ± 975.1
25 × 11				–	–2996* ± 975.1
30 × 9					–

\* Significant difference between sites *i* and *j*.

Table 7  
ANOVA table by case 3

Source	Sum of square	d.f.	Mean square	F	P-value	Critical value
Between	3018355	4	754588.8	38.4207	5.67E-14	2.5787
Error	883807	45	19640.15			
Total	3902162	49				

Table 8  
Scheffe’s multiple comparison by case 3

<i>i</i>	<i>j</i>				
	25 × 24 (average = 5940.6)	30 × 20 (average = 6106.3)	40 × 15 (average = 6041.3)	50 × 12 (average = 6179.4)	60 × 10 (average = 6649.0)
25 × 24	–	–165.7 ± 201.3	–100.7 ± 201.3	–238.8* ± 201.3	–708.4* ± 201.3
30 × 20		–	65 ± 201.3	–73.1 ± 201.3	–542.7* ± 201.3
40 × 15			–	–138.1 ± 201.3	–607.7* ± 201.3
50 × 12				–	–469.6* ± 201.3
60 × 10					–

\* Significant difference between sites *i* and *j*.

20 × 9 and 25 × 7, are significant difference. Subsequently proceeding simultaneous confidence interval (Scheffe’s multiple comparisons) of case 1, the result is present as Table 4. In Table 4, the \* is significant difference between sites *i* and *j*, and there is no significant difference (possibly equal) between site sizes 18 × 10 and 20 × 9. Form these no significant difference sites, the minimal MFFC (TLC) is 42 700 (56 030.8), therefore, the best site

size of case 1 (*n* = 8) is 18 × 10, the optimal layout is shown as in Fig. 9.

In Table 5, ANOVA analysis of case 2, the result is to reject null hypothesis, the means of MFFC between site sizes 17 × 16, 19 × 14, 21 × 13, 25 × 11 and 30 × 9, are significant difference. Subsequently proceeding simultaneous confidence interval (Scheffe’s multiple comparisons) of case 2, the result is present as Table 6. In Table 6, the \* is significant difference between sites *i*

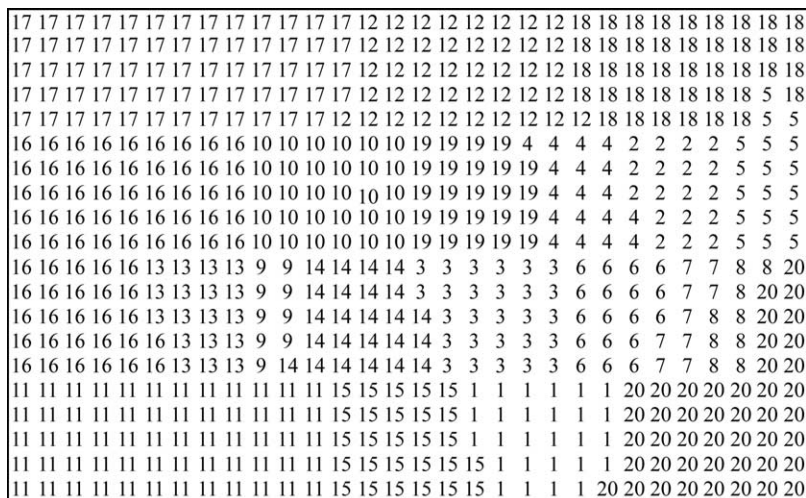


Fig. 11. Case 3 (*n* = 20), site size 30 × 20, MFFC = 5926.60, SRF<sub>whole</sub> = 1.144, AUF<sub>whole</sub> = 1, TLC = 6781.5.

12	12	12	3	3	3	3	4	4	4	4	4	4	4	4	5	5
12	12	12	3	3	3	3	4	4	4	4	4	4	4	4	5	5
12	12	12	3	3	3	3	4	4	4	4	4	4	4	4	5	5
12	12	12	3	3	3	3	4	4	4	4	4	4	4	4	5	5
12	12	12	3	3	3	3	4	4	4	4	4	4	4	4	5	5
1	1	1	1	2	2	7	7	7	7	7	7	7	7	7	5	5
1	1	1	1	2	2	7	7	7	7	7	7	7	7	7	5	5
1	1	1	1	2	2	7	7	7	7	7	7	7	7	5	5	5
10	1	1	1	2	2	2	7	7	7	7	7	7	7	5	5	5
10	1	1	1	2	2	2	7	7	7	7	7	7	5	5	5	5
10	10	10	10	9	9	9	9	6	6	8	8	8	8	11	11	11
10	10	10	10	9	9	9	9	6	6	8	8	8	8	11	11	11
10	10	10	10	9	9	9	9	6	6	8	8	8	8	11	11	11
10	10	10	10	9	9	9	9	6	6	8	8	8	8	11	11	11
0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11

Fig. 12. Case 2 ( $n=12$ ), the optimal layout site size  $17 \times 16$ , TLC = 43686.3,  $SRF_{\text{whole}} = 1.1395$ ,  $AUF_{\text{whole}} = 0.974$ , MFFC = 37349.9.

and  $j$ , and there are no significant difference (possibly equal) between site sizes  $17 \times 16$  and  $19 \times 14$ ,  $17 \times 16$  and  $21 \times 13$ ,  $19 \times 14$  and  $21 \times 13$ . Form these no significant difference sites, the minimal MFFC (TLC) is 37 349.9 (43 682.2), therefore, the

best site size of case 2 ( $n = 12$ ) is  $17 \times 16$ , the optimal layout is shown as Fig. 12.

In Table 7, ANOVA analysis of case 3, the result is to reject null hypothesis, the means of MFFC between site sizes  $25 \times 24$ ,  $30 \times 20$ ,  $40 \times 15$ ,  $50 \times 12$  and  $60 \times 10$ , are significant difference. Subsequently proceeding simultaneous confidence interval (Scheffe's multiple comparisons) of case 3, the result is present as Table 8. In Table 8, the \* is significant difference between sites  $i$  and  $j$ , and there are no significant difference (possibly equal) between site sizes  $25 \times 24$  and  $30 \times 20$ ,  $25 \times 24$  and  $40 \times 15$ ,  $30 \times 20$  and  $40 \times 15$ ,  $30 \times 20$  and  $50 \times 12$ ,  $40 \times 15$  and  $50 \times 12$ . Form these no significant difference sites, the minimal MFFC (TLC) is 5759.3 (6587.8), therefore, the best site size of case 3 ( $n = 20$ ) is  $25 \times 24$ , the optimal layout is shown as in Fig. 13.

By the experiment results of the last two cases, it comes out an interesting conclusion, that is, when the ratio of length to width of site size gets closer to 1 (square), its MFFC and TLC would be lower; when the ratio is larger (narrow rectangular), the MFFC and TLC become higher too. Regarding the layout problems, consequently, the viewpoint that has been

15	15	15	15	15	10	10	10	10	10	10	14	14	14	14	3	3	3	3	3	3	1	1	1	1	
15	15	15	15	15	10	10	10	10	10	10	14	14	14	14	3	3	3	3	3	3	3	1	1	1	1
15	15	15	15	15	10	10	10	10	10	10	14	14	14	14	3	3	3	3	3	3	1	1	1	1	
15	15	15	15	15	10	10	10	10	10	10	14	14	14	14	3	3	3	3	3	3	1	1	1	1	
15	15	15	15	15	10	10	10	10	10	10	14	14	14	14	3	3	3	3	3	3	1	1	1	1	
17	17	12	12	12	12	12	12	12	12	13	13	13	13	9	9	19	19	19	19	5	5	5	5	1	
17	17	12	12	12	12	12	12	12	12	13	13	13	13	9	9	19	19	19	19	5	5	5	5	1	
17	12	12	12	12	12	12	12	12	12	13	13	13	13	9	9	19	19	19	19	5	5	5	5	1	
17	12	12	12	12	12	12	12	12	12	13	13	13	13	9	9	19	19	19	19	5	5	5	5	1	
17	12	12	12	12	12	12	12	12	12	13	13	13	13	9	9	19	19	19	19	5	5	5	5	5	
17	17	17	17	17	17	17	17	17	17	2	2	2	2	2	4	4	4	4	7	7	8	8	20	20	
17	17	17	17	17	17	17	17	17	17	2	2	2	2	2	4	4	4	4	7	7	8	8	20	20	
17	17	17	17	17	17	17	17	17	17	2	2	2	2	2	4	4	4	4	7	7	8	8	20	20	
17	17	17	17	17	17	17	17	17	17	2	2	2	2	2	4	4	4	4	7	7	8	8	20	20	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	6	6	20	20	20	20	20	20	20	20	20	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	6	6	6	6	20	20	20	20	20	20	20	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	6	6	6	6	20	20	20	20	20	20	20	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	6	6	6	6	20	20	20	20	20	20	20	
11	16	16	16	16	16	16	16	16	16	16	16	16	16	6	6	6	6	20	20	20	20	20	20	20	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	18	18	18	18	18	18	18	18	18	18	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	18	18	18	18	18	18	18	18	18	18	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	18	18	18	18	18	18	18	18	18	18	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	18	18	18	18	18	18	18	18	18	18	

Fig. 13. Case 3 ( $n=20$ ), the optimal layout site size  $25 \times 24$ , TLC = 6777.4,  $SRF_{\text{whole}} = 1.177$ ,  $AUF_{\text{whole}} = 1$ , MFFC = 5759.3.

long believed by people, a square is the best site size, is proved in this article.

## 5. Conclusion

In terms of the facilities layout of unequal areas in discrete layout, different site size has a crucial effect on the material handling cost of the last layout, shape of the individual departments and site shape, utilization of the overall area. The genetic algorithm proposed in this article is to find out the minimum TLC. TLC is a multi-objection function combining MFFC, SRF and AUF all together; an effective layout scheme has to be on the minimum MFFC and SRF and the maximum AUF. Therefore, the site shape size of a best layout scheme is ought to be a scheme with the minimum TLC. Moreover, in the article, we also apply the IF-THEN rules of expert system to develop space-filling curve, which connects in order all the locations (positions) in the layout scheme to disallow two-segmented sub-department in the placement of the individual departments of unequal areas.

A crucial discovery has been found out through the experiment in the article, that is, the ratio of length and width of the site size in a best layout scheme has to be as close to 1:1 (square) as possible in order to get a minimum TLC (MFFC). The research module in the article makes a better effect than all the other algorithms. The proposed objection function, a multi-criterion function including material handling cost, the area utilization and the shape factor, meets the actual needs more in terms of practical application.

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