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Research

Capability Measures for Processes with Multiple Characteristics

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Process capability indices, such as C_p, C_a, and C_{pk}, have been widely used in the manufacturing industry providing numerical measures on process precision, process accuracy, and process performance. Capability measures for processes with a single characteristic have been investigated extensively. However, capability measures for processes with multiple characteristics are comparatively neglected. In this paper, we consider a generalization of the yield index S_{pk} proposed by Boyles, for processes with multiple characteristics. We establish a relationship between the generalization and the process yield. We also develop a control chart based on the proposed generalization, which displays all the characteristic measures in one single chart. Using the chart, the engineers can effectively monitor and control the performance of all process characteristics simultaneously. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: capability zone; MCPCA chart; multiple characteristics; process capability index; process yield index

1. INTRODUCTION

Process capability indices, establishing the relationship between the actual process performance and the manufacturing specifications, have been the focus of recent research in quality assurance and capability analysis. Those capability indices quantifying process potential and process performance are essential to any successful quality improvement activities and quality program implementation. Some basic capability indices that have been widely used in the manufacturing industry include C_p, C_a, and C_{pk}, explicitly defined as follows[1–3]:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

$$C_a = 1 - \frac{|\mu - m|}{d}$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}$$

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where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, σ is the process standard deviation, $m = (\text{USL} + \text{LSL})/2$ is the mid-point of the specification interval, and $d = (\text{USL} - \text{LSL})/2$ is half the length of the specification interval.

The index C_p measures the overall process variation relative to the specification tolerance, therefore it only reflects process potential (or process precision). The index C_a measures the degree of process centering, which alerts the user if the process mean deviates from its target value. Therefore, the index C_a only reflects process accuracy. The index C_{pk} takes into account the magnitude of process variation as well as the degree of process centering, which measures process performance based on yield (proportion of conformities). For a normally distributed process with a fixed value of C_{pk}, the bounds on process yield, %Yield, are given by $2\Phi(3C_{pk}) - 1 \leq \%\text{Yield} < \Phi(3C_{pk})$, where $\Phi(\cdot)$ is the cumulative distribution function of $N(0, 1)$, the standard normal distribution. For example, if $C_{pk} = 1.00$, then it guarantees that the %Yield will be no less than 99.73%, or no greater than 2700 ppm (parts per million) of non-conformities. We note that the index C_{pk} only provides an approximate rather than an exact measure of the process yield. To obtain an exact measure, Boyles[4] considered a yield index, referred to as S_{pk}, for normally distributed processes. The index S_{pk} is defined as:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{\text{USL} - \mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu - \text{LSL}}{\sigma}\right)\right\}$$

where Φ^{-1} is the inverse function of Φ. For a process with $S_{pk} = c$, we can obtain $\%\text{Yield} = 2\Phi(3c) - 1$. Obviously, there is a one-to-one relationship between S_{pk} and the process yield. Thus, the yield index S_{pk} provides an exact measure of the process yield. For normally distributed processes, the number of non-conformities corresponding to a capable process with $S_{pk} = 1.00$ is 2700 ppm, a satisfactory process with $S_{pk} = 1.33$ is 63 ppm, an excellent process with $S_{pk} = 1.67$ is 0.6 ppm, and a super process with $S_{pk} = 2.00$ is 0.002 ppm.

2. *CAPABILITY MEASURE FOR MULTIPLE CHARACTERISTICS*

Capability measures for processes with a single characteristic have been investigated extensively, see e.g. Kane[1], Pearn *et al.*[2,3], Boyles[4], Chan *et al.*[5], Choi and Owen[6], Kotz and Johnson[7], Vännman[8], Deleryd and Vännman[9] and Pearn and Lin[10]. However, capability measures for processes with multiple characteristics is comparatively neglected. For processes with multiple characteristics, Bothe[11] considered a simple measure by taking the minimum measure of each single characteristic. For example, consider a ν-characteristic process with ν yield measures (percentage of conformities) $P_1, P_2, \ldots,$ and P_ν. The overall process yield is measured as $P = \min\{P_1, P_2, \ldots, P_\nu\}$. We note that this approach does not reflect the real situation accurately. Suppose the process has five characteristics ($\nu = 5$), with equal characteristic yield measures $P_1 = P_2 = P_3 = P_4 = P_5 = 99.73\%$. Using the approach considered by Bothe[11], the overall process yield is calculated as $P = \min\{P_1, P_2, P_3, P_4, P_5\} = 99.73\%$ (or 2700 ppm of non-conformities). Assuming that the five characteristics are mutually independent, then the actual overall process yield should be calculated as $P = P_1 \times P_2 \times \cdots \times P_5 = 98.66\%$ (or 134 273 ppm of non-conformities), which is significantly less than that calculated by Bothe[11].

To overcome the problem, we propose the following overall capability index, referred to as S_{pk}^{T}:

$$S_{pk}^{\text{T}} = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3S_{pkj}) - 1) + 1\right]\Big/2\right\}$$

where S_{pkj} denotes the S_{pk} value of the jth characteristic for $j = 1, 2, \ldots, \nu$, and ν is the number of characteristics. The new index, S_{pk}^{T}, may be viewed as a generalization of the single characteristic yield index, S_{pk}, considered by Boyles[4].

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Table I. Various S_{pk}^{T} values and the corresponding process yield

S_{pk}^{T}	Process yield
1.00	0.997 300 204
1.24	0.999 800 777
1.33	0.999 933 927
1.50	0.999 993 205
1.67	0.999 999 456
2.00	0.999 999 998

Given $S_{pk}^{\mathrm{T}} = c$, we have

$$\frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3S_{pkj})-1)+1\right]\bigg/2\right\} = c$$

Hence,

$$\prod_{j=1}^{\nu}[2\Phi(3S_{pkj})-1] = 2\Phi(3c)-1$$

A one-to-one correspondence relationship between the index S_{pk}^{T} and the overall process yield P can be established as:

$$P = \prod_{j=1}^{\nu} P_j = \prod_{j=1}^{\nu}[2\Phi(3S_{pkj})-1] = 2\Phi(3S_{pk}^{\mathrm{T}})-1.$$

Hence, the new index S_{pk}^{T} provides an exact measure of the overall process yield. For example, if $S_{pk}^{\mathrm{T}} = 1.00$, then the entire process yield would be exactly 99.73%. Table I displays various commonly used capability requirements and the corresponding overall process yield. For a process with ν characteristics, if the requirement for the overall process capability is $S_{pk}^{\mathrm{T}} \geq c_0$, a sufficient condition (which is minimal) for the requirement to each single characteristic can be obtained by the following. Let c' be the minimum S_{pk} value required for each single characteristic, then

$$\frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3S_{pkj})-1)+1\right]\bigg/2\right\} \geq \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3c')-1)+1\right]\bigg/2\right\}$$

Hence, if

$$\frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3c')-1)+1\right]\bigg/2\right\} \geq c_0$$

i.e.

$$c' \geq \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[\nu]{2\Phi(3c_0)-1}+1}{2}\right)$$

then we have

$$S_{pk}^{\mathrm{T}} = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{\nu}(2\Phi(3S_{pkj})-1)+1\right]\bigg/2\right\} \geq c_0$$

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Table II. Capability zones for multiple characteristics

Characteristic number v	S_{pkj} for single characteristic	
	Lower bound s_L	Upper bound s_U
1	1.000	1.333
2	1.068	1.387
3	1.107	1.417
4	1.133	1.439
5	1.153	1.455
6	1.170	1.468
7	1.183	1.479
8	1.195	1.489
9	1.205	1.497
10	1.214	1.505
11	1.222	1.511
12	1.230	1.518
13	1.236	1.523
14	1.243	1.528
15	1.248	1.533

Thus, if the requirement for each single characteristic

$$S_{pkj} \geq \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[v]{2\Phi(3c_0)-1}+1}{2}\right), \quad \text{for all } j = 1, 2, \ldots, v$$

is satisfied, then the overall capability requirement $S_{pk}^{\mathrm{T}} \geq c_0$ would be satisfied. For example, if c_0 is set to be 1.00 with $v = 5$, i.e. the overall process yield is set to be no less than 0.9973. The overall capability requirement $S_{pk}^{\mathrm{T}} \geq 1.00$ would be satisfied if each single characteristic yield is no less than $(0.997\,300\,204)^{1/5} = 0.999\,459\,50$ (equivalent to 540 ppm of non-conformity items), and the capability for all the five characteristics is

$$S_{pkj} \geq \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[5]{2\Phi(3)-1}+1}{2}\right) = 1.153, \quad \text{for } j = 1, 2, \ldots, 5$$

If the requirement of the overall process capability is $c_1 \leq S_{pk}^{\mathrm{T}} \leq c_2$ for a process with v characteristics. The requirement would be satisfied, if the capability of jth characteristic satisfies $s_L \leq S_{pkj} \leq s_U$ for all $j = 1, 2, \ldots, v$, where the lower bound s_L and the upper bound s_U on each S_{pkj} can be calculated, respectively, as

$$s_L = \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[v]{2\Phi(3c_1)-1}+1}{2}\right) \quad \text{and} \quad s_U = \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[v]{2\Phi(3c_2)-1}+1}{2}\right)$$

Table II displays the lower bound s_L and upper bound s_U on S_{pkj} if the requirement of the overall process capability is $1.000 \leq S_{pk}^{\mathrm{T}} \leq 1.333$ for $v = 1(1)15$ characteristics. For example, suppose the requirement of the overall process capability is $1.000 \leq S_{pk}^{\mathrm{T}} \leq 1.333$ for process with five characteristics ($v = 5$), we can obtain the lower bound $s_L = 1.153$ and the upper bound $s_U = 1.455$ on all the five S_{pkj} values.

3. S_{pk} MULTI-CHARACTERISTIC PROCESS CAPABILITY ANALYSIS CONTROL CHART

Based on the yield index S_{pk}, Boyles[4] developed a tool called the S_{pk} contour plot which is a contour plot of index S_{pk} as a function of the process parameters (μ, σ) for monitoring and controlling process performance.

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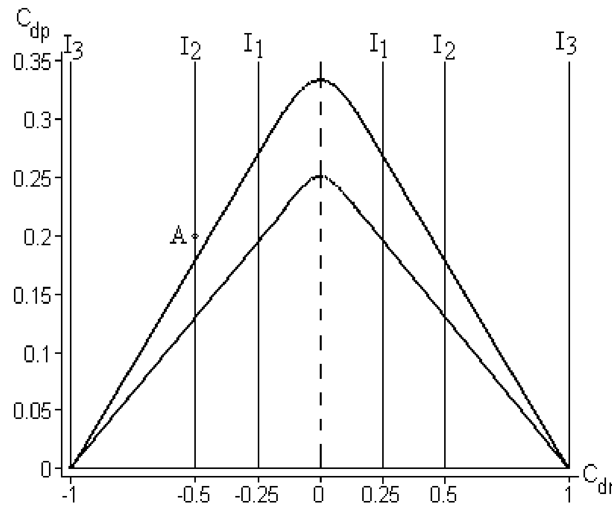

Figure 1. The S_{pk} MCPCA control chart with various departure control zones

In fact, the S_{pk} contour plot is a useful tool for evaluating multiple processes, as we can obtain the process yield and the process departure ratio by checking the location of the index value falling on the contour plot. For multiple processes with the same specification limits, the contour plot not only shows the process capability for multiple processes simultaneously, but also provides a quick reference to the parameters that should be targeted for process improvement. Note, however, that the S_{pk} contour plot is only applicable for multiple processes with the same specification limits on each single process, which may not be used on processes with multiple characteristics where the characteristic specifications are not the same.

To extend the applicability of the contour plot for processes with multiple characteristics, we apply the method developed by Deleryd and Vännman[9] who introduced a process capability plot, called the (δ, γ)-plot, which is an adjustment of Boyles' (μ, σ)-plot where $\delta = (\mu - T)/d$, $\gamma = \sigma/d$. We rewrite the definition of S_{pk} as below, which can be expressed as a function of $C_{dr} = (\mu - T)/d$ and $C_{dp} = \sigma/d$. Note that C_{dr} measures the departure ratio, and C_{dp} measures the variation relative to the specification tolerance. If $\mu < T$, we have $C_{dr} < 0$; if $\mu > T$, we have $C_{dr} > 0$; and if $\mu = T$, we have $C_{dr} = 0$ (the process is on target in this case). Obviously, if $\mu = \text{LSL}$, then $C_{dr} = -1$; if $\mu = \text{USL}$, then $C_{dr} = 1$. We will focus on the case where the specification interval is two-sided with the target value T at m, which is most common in practical situations. Henceforth we will assume that $T = m$.

$$
\begin{aligned}
S_{pk} &= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{\text{USL}-\mu}{\sigma}\right)+\frac{1}{2}\Phi\left(\frac{\mu-\text{LSL}}{\sigma}\right)\right\} \\
&= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{1-(\mu-m)/d}{\sigma/d}\right)+\frac{1}{2}\Phi\left(\frac{1+(\mu-m)/d}{\sigma/d}\right)\right\} \\
&= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{1-C_{dr}}{C_{dp}}\right)+\frac{1}{2}\Phi\left(\frac{1+C_{dr}}{C_{dp}}\right)\right\}
\end{aligned}
$$

Therefore, using C_{dr} as the x-axis and C_{dp} as the y-axis, we can plot the following point set forming the curve of S_{pk} (bold curves in Figure 1) on the (C_{dr}, C_{dp}) coordinates,

$$
\left\{(C_{dr}, C_{dp})\ \middle|\ \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1-C_{dr}}{C_{dp}}\right)+\frac{1}{2}\Phi\left(\frac{1+C_{dr}}{C_{dp}}\right)\right]=S_{pk}\right\}
$$

Note that the process capability plot is invariable irrespective of the value of the specification limits. Processes with multiple characteristics having different characteristic specification limits can thus be plotted

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Table III. Various control regions for process departure and improvement suggestions

Control region	C_{dr} value	Process improvement suggestion
I_1	± 0.25	Characteristic departure is tolerable
I_2	± 0.50	Characteristic departure is abnormal, need to investigate and improve
I_3	± 1.00	Characteristic departure is serious, need to recheck overall process

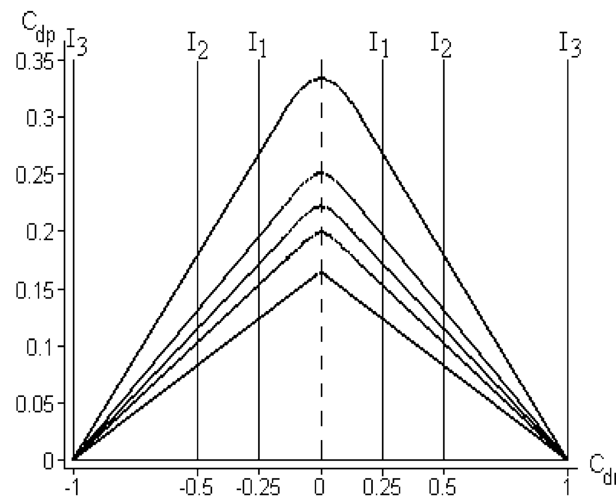

Figure 2. The MCPCA chart with contours of $S_{pk} = 1.0$, 1.33, 1.5, 1.67, 2.0 (from top to bottom)

simultaneously on a single chart. We shall call this control chart the multi-characteristic process capability analysis (MCPCA) chart. In Figure 1, the top curve is the capability plot for processes with $S_{pk} = 1.00$, and the bottom curve is the capability plot for processes with $S_{pk} = 1.33$.

As we pointed out earlier, S_{pk} is a yield-based index. However, according to today's modern quality improvement theories, reduction of variation from the target is as important as increasing the process yield (meeting the specifications). Therefore, three pairs of control limits I_1, I_2, and I_3 are drawn on the proposed MCPCA chart (see Figure 1) to monitor the variation from the target of each characteristic. Corresponding to the three pairs of control limits I_1, I_2, and I_3, the values of $C_{dr} = (\mu - T)/d$ are -1.0, -0.5, -0.25, 0.25, 0.5, and 1.0 from left to right, respectively.

Under the six-sigma quality improvement program formulated by Motorola (see Noguera and Nielsen[12]) assuming $d = 6\sigma$, the three pairs of control limits I_1, I_2, and I_3 correspond to $|\mu - T| = 1.5\sigma$, 3σ and 6σ. Research has shown that a typical process is likely to deviate from its natural centering condition by approximately 1.5σ at any given moment in time. Under the six-sigma quality improvement program, the process mean is allowed to shift as much as 1.5σ, i.e. all the pairs (C_{drj}, C_{dpj}) of the jth characteristic should not locate outside of the pair of control limits I_1. Note that six-sigma technically means having no more than 3.4 ppm of non-conformities by assuming that the specification limits are 6σ away from the target. Therefore, the three pairs of control limits I_1, I_2, and I_3 form various process accuracy (the degree of centering) control zones.

Various control regions (zones) for process departure and the improvement suggestions are summarized in Table III. The practitioners can judge the degree of centering of characteristic j by checking the location of the corresponding plotted point on the MCPCA chart. For example, the departure measure C_{dr} value is -0.5 for characteristic A in Figure 1. The departure ratio is considered to be significant which calls for an

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immediate check to find abnormal changes in the parameter settings. Using the MCPCA chart, the practitioners can effectively control and monitor both the variation and the departure from its target for each single process characteristic.

In the MCPCA chart (see Figure 2), we use the standardized measures on process departure and process variation. Therefore, the MCPCA chart can be used for processes with multiple characteristics where the individual characteristic specifications may not be identical. The MCPCA chart displays all the characteristic index values on one chart, and indicates the characteristic yield based on the S_{pk} contours. It can provide information instantly about the locations and spreads of all the studied characteristics by their (C_{dr}, C_{dp})-coordinate values. The MCPCA chart also displays the relative magnitudes of process variation and the process departure in terms of the standardized measures (C_{dr}, C_{dp}) which can be used to control and monitor all characteristics simultaneously. Therefore, the MCPCA chart provides in-time information so that the practitioners can understand the quality level of the product easily. In addition, the MCPCA chart also provides a clear direction on which parameter needs to be targeted for quality improvement.

4. AN APPLICATION

Development of the new index is novel and deals with a common and practical problem (yield measurement) occurring in the manufacturing industry. To the best of our knowledge, none of the existing methods published in the literature provides the same function as the new index S_{pk}^{T}. The new index is useful to the engineers/practitioners in measuring the yield, particularly for processes with multiple characteristics (a problem the industry must continue to face). We note that the exact sampling distribution of the estimated S_{pk}, which is a special case of the proposed new index with $\nu = 1$ (for single characteristic), has never been investigated. Statistical properties of the special case with $\nu = 1$ are in fact mathematically intractable. Lee *et al.*[13] obtained an approximate distribution of the estimated S_{pk} using the Taylor expansion. Statistical properties of the new index, S_{pk}^{T}, are expected to be considerably more difficult to deal with than that of S_{pk} (for a single characteristic).

Pearn and Kang[14] considered the sampling distribution of the estimated S_{pk}, and conducted a simulation to evaluate its accuracy. Their simulation results show that the estimated S_{pk} overestimates the true value of S_{pk}. However, with a sample size n greater than 150, the relative bias is less than 0.01, and the sampling error becomes negligible. For practical purposes, we suggest that a minimal value of $n = 150$ must be used in the applications to maintain a good accuracy.

For each single characteristic the C_{drj} and C_{dpj} values are calculated and the (C_{drj}, C_{dpj}) pair is plotted on the chart for $j = 1, 2, \ldots, \nu$. In real applications C_{drj} and C_{dpj}, representing the jth characteristic (with target value T_j), are replaced by their natural estimators:

$$\hat{C}_{drj} = \frac{(\bar{X}_j - T_j)}{d_j}, \quad \hat{C}_{dpj} = \frac{s_j}{d_j}$$

where d_j is the half length of the jth characteristic's specification interval, $\bar{X}_j = \sum_{i=1}^{n} x_{ij}/n$ and $s_j^2 = \sum_{i=1}^{n} (x_{ij} - \bar{X}_j)^2/(n-1)$ are the sample mean and the sample variance of the jth characteristic, respectively. Hence, the natural estimators $\hat{S}_{pkj}$ of index S_{pkj} corresponding to the jth characteristic are

$$\hat{S}_{pkj} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{1-\hat{C}_{drj}}{\hat{C}_{dpj}}\right) + \frac{1}{2}\Phi\left(\frac{1+\hat{C}_{drj}}{\hat{C}_{dpj}}\right)\right\}, \quad \text{for } j = 1, 2, \ldots, \nu$$

To illustrate the application of the MCPCA chart, we consider a real example taken from an electronic thermos manufacturer, located in Taiwan, adapting the six-sigma quality improvement program. One special type of thermos investigated has five target-the-best quality characteristics with unequal manufacturing specifications.

The quality requirement for the final product is $1.00 \leq S_{pk}^{\mathrm{T}} \leq 1.33$, i.e. the requirement for the process yield is no less than 99.73%. For $\nu = 5$, we can obtain the lower bound $s_L = 1.153$ and the upper bound $s_U = 1.455$ for

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Table IV. Calculations for the electronic thermos process capability

j	LSL	T	USL	$\bar{X}_j$	s_j	$(\hat{C}_{drj}, \hat{C}_{dp})$	$\hat{S}_{pkj}$
1	5.598	6.220	6.842	5.909	0.124	(−0.500, 0.199)	0.915
2	606.5	680.0	753.5	683.3	17.13	(−0.045, 0.233)	1.406
3	0.279	0.310	0.341	0.332	0.0076	(0.710, 0.245)	0.521
4	31.5	35.0	38.5	34.48	0.525	(−0.149, 0.150)	1.931
5	30	40	50	43.5	0.80	(0.35, 0.08)	2.737

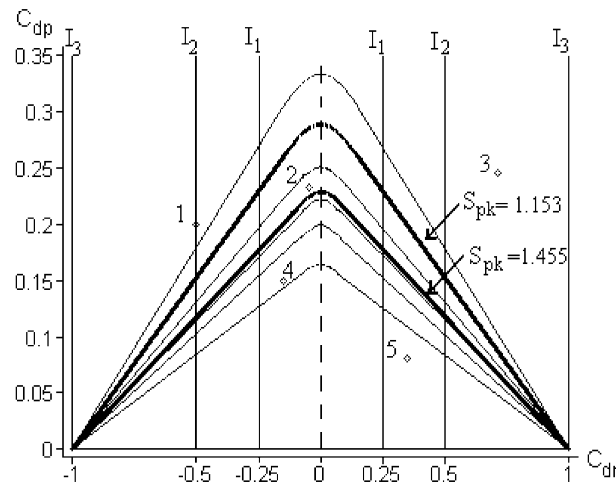

Figure 3. An application of the S_{pk} MCPCA chart

all five S_{pkj} values from Table II. Hence, a process is capable if $1.153 \leq S_{pkj} \leq 1.455$ and the pair (C_{drj}, C_{dpj}) of the jth characteristic must locate between the pair control limits I_1 for all $j = 1, 2, \ldots, 5$.

A sample of 150 or more items is taken from the factory to inspect. The specification, target value, sample mean, sample standard deviation, estimated C_{drj}, C_{dpj}, and S_{pkj} are summarized in Table IV. First, we plot two contours corresponding to $S_{pk} = 1.153$ and $S_{pk} = 1.455$ on the MCPCA chart, shown as two bold curves in Figure 3. At the same time, five pairs of the estimated C_{drj} and C_{dpj} values are plotted on the chart. This chart clearly shows the status of each in-process characteristic. Since all of these in-process characteristics are plotted in one chart, it is easy to determine their relative status. From the MCPCA chart, we can quickly conclude that the process is incapable because there are two plotted points out of the contour of $S_{pk} = s_L = 1.153$. We note that the estimated $S_{pk}^{\mathrm{T}} = 0.5135$. Based on the analysis of this chart, we can make the following conclusions and recommendations.

1. The plotted points corresponding to characteristics 1 and 3 are out of the contour of $S_{pk} = 1.153$ which show both $\hat{S}_{pk1}$ and $\hat{S}_{pk3}$ are less than the lower bound value $s_L = 1.153$ (the lower bound of quality requirement for all five S_{pkj}). Hence, the process is considered to be 'incapable' for characteristics 1 and 3. In fact, $\hat{S}_{pk1} = 0.915$ and $\hat{S}_{pk3} = 0.521$. Since both $\hat{S}_{pk1}$ and $\hat{S}_{pk3}$ are significantly less than 1.00 (the lower bound of quality requirement S_{pk}^{T} for the final product), then both characteristics 1 and 3 are candidates for high-priority quality improvement efforts. Furthermore, the plotted points 1 and 3 are out of the control limits I_1. Hence, both the variability and the deviation from target must be reduced to improve the process quality for characteristics 1 and 3.
2. The plotted point corresponding to characteristic 2 is between two contours corresponding to $S_{pk} = 1.153$ and $S_{pk} = 1.455$. At the same time, the plotted point 2 is located between the pair of the control limits I_1. This shows that the process is satisfactory for characteristic 2. Note that the corresponding $\hat{S}_{pk2} = 1.406$.

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3. The plotted points corresponding to characteristics 4 and 5 are inside the contour of $S_{pk} = 1.455$, which shows both $\hat{S}_{pk4}$ and $\hat{S}_{pk5}$ are greater than the upper bound value $s_U = 1.455$ (the upper bound of quality requirement for all five S_{pkj}). In fact, $\hat{S}_{pk4} = 1.931$ and $\hat{S}_{pk5} = 2.737$. Note that the plotted point 4 is located between the pair of the control limits I_1, but the plotted point 5 is out of the control limits I_1. A reduced sampling plan for characteristic 4 could be considered since the process is super. Under the six-sigma program, the quality improvement effort for characteristic 5 could be focused on the reduction of the process deviation from its target.

5. CONCLUSIONS

Process capability indices have been widely used in the manufacturing industry, providing numerical measures on process precision, process accuracy, and process performance. Capability measures for processes with a single characteristic has been investigated extensively. However, capability measures for processes with multiple characteristics are comparatively neglected. In this paper, we proposed a generalized capability measure, called S_{pk}^{T}, based on the yield index S_{pk} proposed by Boyles[4], for processes with multiple characteristics. We established a relationship between the new measure and the process yield. We also developed a control chart MCPCA based on the proposed new measure, which displays all the characteristic measures in one single chart. Using the control chart MCPCA, the engineers can effectively monitor and control the performance of all process characteristics simultaneously.

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REFERENCES

1. Kane VE. Process capability indices. *Journal of Quality Technology* 1986; **18**(1):41–52.
2. Pearn WL, Kotz S, Johnson NL. Distributional and inferential properties of process capability indices. *Journal of Quality Technology* 1992; **24**(4):216–231.
3. Pearn WL, Lin GH, Chen KS. Distributional and inferential properties of the process accuracy and process precision indices. *Communications in Statistics: Theory and Methods* 1998; **27**(4):985–1000.
4. Boyles RA. Process capability with asymmetric tolerances. *Communications in Statistics: Computation and Simulation* 1994; **23**(3):615–643.
5. Chan LK, Cheng SW, Spiring FA. A new measure of process capability C_{pm}. *Journal of Quality Technology* 1998; **20**(3):162–175.
6. Choi BC, Owen DB. A study of a new process capability index. *Communications in Statistics: Theory and Methods* 1990; **19**(4):1231–1245.
7. Kotz S, Johnson NL. *Process Capability Indices*. Chapman & Hall: London, 1993.
8. Vännman K. A unified approach to capability indices. *Statistica Sinica* 1995; **5**:805–820.
9. Deleryd M, Vännman K. Process capability plots—a quality improvement tool. *Quality and Reliability Engineering International* 1999; **15**:213–217.
10. Pearn WL, Lin GH. Estimating capability index C_{pk} for processes with asymmetric tolerances. *Communications in Statistics: Theory and Methods* 2000; **29**(11):2593–2604.
11. Bothe DR. A capability study for an entire product. *ASQC Quality Congress Transactions* 1992; **46**:172–178.
12. Noguera J, Nielsen T. Implement six sigma for interconnect technology. *ASQC Quality Congress Transactions* 1992; 538–544.

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13. Lee JC, Hung HN, Pearn WL, Kueng TL. On the distribution of the estimated process yield index S_{pk}. *Quality and Reliability Engineering International* 2002; **18**(2):111–116.
14. Pearn WL, Kang HY. A simulation program for testing process performance based on the yield. *Working Paper*, National Chiao Tung University, Taiwan, 2002.

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