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Optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy

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Abstract

The main purpose of this paper is to investigate the optimal retailer's replenishment decisions under two levels of trade credit policy within the economic production quantity (EPQ) framework. We assume that the supplier would offer the retailer a delay period and the retailer also adopts the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model under the replenishment rate is finite. Furthermore, we assume that the retailer's trade credit period offered by supplier M is not shorter than the customer's trade credit period offered by retailer N ($M \geqslant N$). Since the retailer cannot earn any interest in this situation, $M < N$.

Based upon the above arguments, this paper incorporates both Chung and Huang [K.J. Chung, Y.F. Huang, The optimal cycle time for EPQ inventory model under permissible delay in payments, International Journal of Production Economics 84 (2003) 307–318] and Huang [Y.F. Huang, Optimal retailer's ordering policies in the EOQ model under trade credit financing, Journal of the Operational Research Society 54 (2003) 1011–1015] under above conditions. In addition, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal replenishment decisions. Then three theorems are developed to efficiently determine the optimal replenishment decisions for the retailer. We deduce some previously published results of other authors as special cases. Finally, numerical examples are given to illustrate the theorems obtained in this paper. Then, as well as, we obtain a lot of managerial insights from numerical examples.
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Keywords: EPQ; Inventory; Two levels of trade credit; Permissible delay in payments

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1. Introduction

The traditional economic order quantity (EOQ) model is widely used by practitioners as a decision-making tool for the control of inventory. The EOQ model assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as the items are received. However, this may not be true. In practice, the supplier will offer the retailer a delay period, which is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. In a real world, the supplier often makes use of this policy to promote his commodities.

Goyal [16] established a single-item inventory model under permissible delay in payments. Chand and Ward [3] analyzed Goyal's problem [16] under assumptions of the classical economic order quantity model, and obtained different results. Chung [10,11] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Shah [28], Aggarwal and Jaggi [1] considered the inventory model with exponential deterioration rate under the condition of permissible delay in payments. Chu et al. [9] and Chung et al. [13] also extended Goyal's model [16] to the case of deterioration. Liao et al. [24] and Sarker et al. [26] investigated this topic with inflation. Jamal et al. [21] and Chang and Dye [6] extended this issue with allowable shortage. Chung [12] developed an alternative approach to modify Shah's [28] solution. Chang et al. [7] extended this issue with linear trend demand. Chen and Chuang [8] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow. Kim et al. [23] developed an optimal credit policy to increase wholesaler's profits with price-dependent demand functions. Hwang and Shinn [20] modeled an inventory system for retailer's pricing and lot sizing policy for exponential deteriorating products under the condition of permissible delay in payment. Jamal et al. [22] and Sarker et al. [27] addressed the optimal payment time under permissible delay in payment with deterioration. Shawky and Abou-El-Ata [29] investigated the production lot-size model with both restrictions on the average inventory level and trade-credit policy using geometric programming and Lagrange approaches. Teng [31] assumed that the selling price not equal to the purchasing price to modify the inventory model under permissible delay in payments. Shinn and Hwang [30] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. Arcelus et al. [2] modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. Chung and Huang [14] extended this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang and Chung [19] extended Goyal's model [16] to cash discount policy for early payment. Salameh et al. [25] extended this issue to continuous review inventory model. Chang et al. [5] and Chung and Liao [15] deal with the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity. Chang [4] extended this issue to inflation and finite time horizon. Huang [18] investigated that the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under supplier's trade credit policy.

All above models assumed that the supplier would offer the retailer a delay period and the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period. They implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. That is, they assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to his/her customer in previously published results. In most business transactions, this assumption is debatable. We define this situation as one level of trade credit. In this paper, we adopt the viewpoint of Huang [17] to modify this assumption to assume that the retailer will adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. We define this situation as two levels of trade credit. Furthermore, we also adopt Huang's assumption [17] that the retailer's trade credit period offered by supplier M is not shorter than the customer's trade credit period offered by retailer N ($M \geqslant N$). Since the retailer cannot earn any interest in this situation, $M < N$.

Another unrealistic assumption in the EOQ model is the infinite replenishment rate. So, we relax this assumption to finite replenishment rate. That is, the well-known economic production quantity (EPQ) framework. This viewpoint can be found in Chung and Huang [14]. Under these conditions, this paper incorporates both Chung and Huang [14] and Huang [17] under above conditions. Then we model the retailer's inventory system to investigate the optimal retailer's replenishment decisions under two levels of trade credit policy within the EPQ framework. Three theorems are developed to efficiently determine the optimal replenishment decisions for the retailer. We deduce some previously published results of other authors as special cases. Finally, numerical examples are given to illustrate these theorems obtained in this paper. In addition, we obtain a lot of managerial insights from numerical examples.

2. Model formulation and the convexity

The following notation and assumptions will be used throughout, most notation and assumptions adopted are the same as those in Chung and Huang [14] and Huang [17]:

Notation:

D	demand rate per year
P	replenishment rate per year, $P \geqslant D$
A	ordering cost per order
ρ	$1 - \frac{D}{P} \geqslant 0$
c	unit purchasing price
s	unit selling price, $s \geqslant c$
h	unit stock holding cost per item per year excluding interest charges
I_e	interest earned per \$ per year
I_k	interest charged per \$ in stocks per year by the supplier
M	retailer's trade credit period offered by supplier in years
N	customer's trade credit period offered by retailer in years
T	cycle time in years
$\mathrm{TVC}(T)$	annual total relevant cost, which is a function of T
T^*	optimal cycle time of $\mathrm{TVC}(T)$

Assumptions:

(1) Demand rate, D, is known and constant.
(2) Replenishment rate, P, is known and constant.
(3) Shortages are not allowed.
(4) Time horizon is infinite.
(5) $I_k \geqslant I_e$, $M \geqslant N$.
(6) When $T \geqslant M$, the account is settled at $T = M$, the retailer pays off all units sold and keeps his/her profits, and the retailer starts paying for the interest charges on the items in stock with rate I_k. When $T \leqslant M$, the account is settled at $T = M$ and the retailer does not need to pay any interest charge.
(7) The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is,

the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

The annual total relevant cost consists of the following elements.

(1) Annual ordering cost $= \frac{A}{T}$.
(2) Annual stock holding cost (excluding interest charges) (as shown in Fig. 1)

$$= \frac{hT(P-D)\frac{DT}{P}}{2T} = \frac{DTh}{2}\left(1-\frac{D}{P}\right) = \frac{DTh\rho}{2}.$$

(3) According to assumption (6), there are four cases to occur in interest charged for the items kept in stock per year.

Case 1: $M \leqslant \frac{PM}{D} \leqslant T$, as shown in Fig. 1.

$$\text{Annual interest payable} = cI_k\left[\frac{DT^2\rho}{2} - \frac{(P-D)M^2}{2}\right]\Big/T = cI_k\rho\left(\frac{DT^2}{2} - \frac{PM^2}{2}\right)\Big/T.$$

Case 2: $M \leqslant T \leqslant \frac{PM}{D}$, as shown in Fig. 2.

$$\text{Annual interest payable} = cI_k\left[\frac{D(T-M)^2}{2}\right]\Big/T.$$

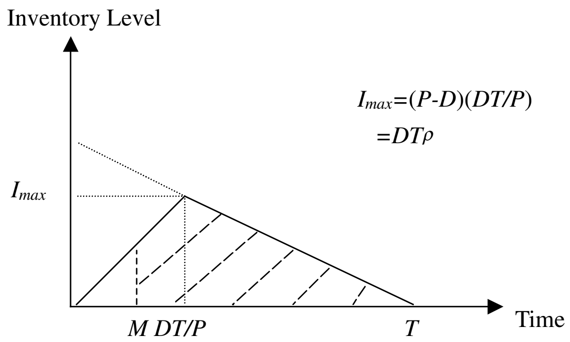

Fig. 1. The total amount of interest payable when $PM/D \leqslant T$.

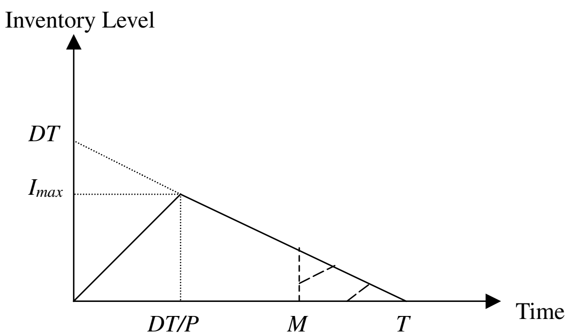

Fig. 2. The total amount of interest payable when $M \leqslant T \leqslant PM/D$.

Case 3: $N \leqslant T \leqslant M$.
In this case, annual interest payable = 0.
Case 4: $0 < T \leqslant N$.
Similar as Case 3, annual interest payable =0.
(4) According to assumption (7), there are four cases to occur in interest earned per year.
Case 1: $M \leqslant \frac{PM}{D} \leqslant T$, as shown in Fig. 3.

$$\text{Annual interest earned} = sI_e\left[\frac{(DN + DM)(M - N)}{2}\right] \Big/ T = sI_eD(M^2 - N^2)/2T.$$

Case 2: $M \leqslant T \leqslant \frac{PM}{D}$.
Similar as Case 1, annual interest earned $= sI_eD(M^2 - N^2)/2T$.
Case 3: $N \leqslant T \leqslant M$, as shown in Fig. 4.

$$\text{Annual interest earned} = sI_e\left[\frac{(DN + DT)(T - N)}{2} + DT(M - T)\right] \Big/ T = sI_eD(2MT - N^2 - T^2)/2T.$$

Case 4: $T \leqslant N$, as shown in Fig. 5.
Annual interest earned $= sI_eDT(M - N)/T$.
From the above arguments, the annual total relevant cost for the retailer can be expressed as TVC(T) = ordering cost + stock-holding cost + interest payable − interest earned.

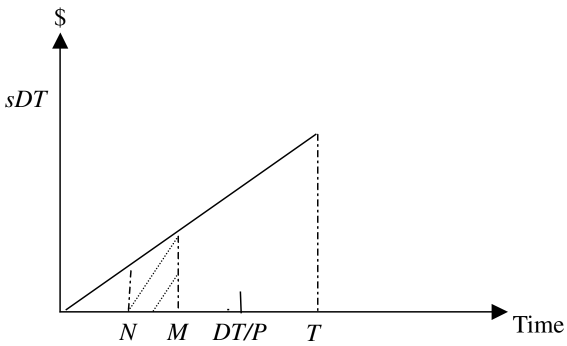

Fig. 3. The total amount of interest earned when $M \leqslant PM/D \leqslant T$.

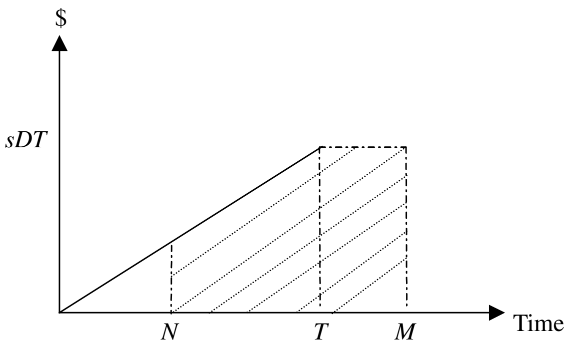

Fig. 4. The total amount of interest earned when $N \leqslant T \leqslant M$.

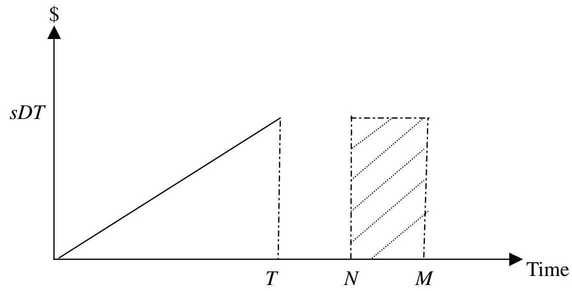

Fig. 5. The total amount of interest earned when $T \leqslant N$.

$$\mathrm{TVC}(T) = \begin{cases} \mathrm{TVC}_1(T) & \text{if } T \geqslant \frac{PM}{D}, & \text{(a)} \\ \mathrm{TVC}_2(T) & \text{if } M \leqslant T \leqslant \frac{PM}{D}, & \text{(b)} \\ \mathrm{TVC}_3(T) & \text{if } N \leqslant T \leqslant M, & \text{(c)} \\ \mathrm{TVC}_4(T) & \text{if } 0 < T \leqslant N, & \text{(d)} \end{cases} \tag{1}$$

where

$$\mathrm{TVC}_1(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho(DT^2 - PM^2)/2T - sI_eD(M^2 - N^2)/2T, \tag{2}$$

$$\mathrm{TVC}_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_kD(T - M)^2/2T - sI_eD(M^2 - N^2)/2T, \tag{3}$$

$$\mathrm{TVC}_3(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_eD(2MT - N^2 - T^2)/2T \tag{4}$$

and

$$\mathrm{TVC}_4(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_eD(M - N). \tag{5}$$

Since $\mathrm{TVC}_1(\frac{PM}{D}) = \mathrm{TVC}_2(\frac{PM}{D})$, $\mathrm{TVC}_2(M) = \mathrm{TVC}_3(M)$ and $\mathrm{TVC}_3(N) = \mathrm{TVC}_4(N)$, $\mathrm{TVC}(T)$ is continuous and well-defined. All $\mathrm{TVC}_1(T)$, $\mathrm{TVC}_2(T)$, $\mathrm{TVC}_3(T)$, $\mathrm{TVC}_4(T)$ and $\mathrm{TVC}(T)$ are defined on $T > 0$. Eqs. (2)–(5) yield

$$\mathrm{TVC}_1'(T) = -\left[\frac{2A - cM^2I_k(P - D) - sDI_e(M^2 - N^2)}{2T^2}\right] + D\rho\left(\frac{h + cI_k}{2}\right), \tag{6}$$

$$\mathrm{TVC}_1''(T) = \frac{2A - cM^2I_k(P - D) - sDI_e(M^2 - N^2)}{T^3}, \tag{7}$$

$$\mathrm{TVC}_2'(T) = -\left[\frac{2A + cDM^2I_k - sDI_e(M^2 - N^2)}{2T^2}\right] + D\left(\frac{h\rho + cI_k}{2}\right), \tag{8}$$

$$\mathrm{TVC}_2''(T) = \frac{2A + cDM^2I_k - sDI_e(M^2 - N^2)}{T^3}, \tag{9}$$

$$\mathrm{TVC}_3'(T) = -\left[\frac{2A + sDN^2I_e}{2T^2}\right] + D\left(\frac{h\rho + sI_e}{2}\right), \tag{10}$$

$$\mathrm{TVC}_3''(T) = \frac{2A + sDN^2I_e}{T^3} > \quad 0, \tag{11}$$

$$\mathrm{TVC}_4'(T) = \frac{-A}{T^2} + \frac{Dh\rho}{2} \tag{12}$$

and

$$\mathrm{TVC}_4''(T) = \frac{2A}{T^3} > 0. \tag{13}$$

Eqs. (11) and (13) imply that $\mathrm{TVC}_3(T)$ and $\mathrm{TVC}_4(T)$ are convex on $T > 0$. However, $\mathrm{TVC}_1(T)$ is convex on $T > 0$ if $2A - cM^2I_k(P-D) - sDI_e(M^2 - N^2) > 0$ and $\mathrm{TVC}_2(T)$ is convex on $T > 0$ if $2A + cDM^2I_k - sDI_e(M^2 - N^2) > 0$. Furthermore, we have $\mathrm{TVC}_1'(\frac{PM}{D}) = \mathrm{TVC}_2'(\frac{PM}{D})$, $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M)$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N)$. Now, we let $\alpha = 2A - cM^2I_k(P-D) - sDI_e(M^2 - N^2)$, $\beta = 2A + cDM^2I_k - sDI_e(M^2 - N^2)$, and easily find $\beta > \alpha$. Therefore, Eqs. (1a–d) imply that $\mathrm{TVC}(T)$ is convex on $T > 0$ if $\alpha > 0$. Then we can obtain following results.

Theorem 1

(A) *If* $\beta \leqslant 0$, *then* $\mathrm{TVC}(T)$ *is convex on* $(0, M]$ *and concave on* $[M, \infty)$.
(B) *If* $\alpha \leqslant 0$ *and* $\beta > 0$, *then* $\mathrm{TVC}(T)$ *is convex on* $(0, PM/D]$ *and concave on* $[PM/D, \infty)$.
(C) *If* $\alpha > 0$, *then* $\mathrm{TVC}(T)$ *is convex on* $(0, \infty)$.

Let $\mathrm{TVC}_i'(T_i^*) = 0$ for all $i = 1, 2, 3, 4$. We can obtain

$$T_1^* = \sqrt{\frac{2A - cM^2I_k(P-D) - sDI_e(M^2 - N^2)}{D\rho(h + cI_k)}} \quad \text{if } \alpha > 0, \tag{14}$$

$$T_2^* = \sqrt{\frac{2A + cDM^2I_k - sDI_e(M^2 - N^2)}{D(h\rho + cI_k)}} \quad \text{if } \beta > 0, \tag{15}$$

$$T_3^* = \sqrt{\frac{2A + sDN^2I_e}{D(h\rho + sI_e)}} \tag{16}$$

and

$$T_4^* = \sqrt{\frac{2A}{Dh\rho}}. \tag{17}$$

Eqs. (6), (8), (10) and (12) yield that

$$\mathrm{TVC}_1'\left(\frac{PM}{D}\right) = \mathrm{TVC}_2'\left(\frac{PM}{D}\right) = \frac{-2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2)] + sDI_e(M^2 - N^2)}{2(\frac{PM}{D})^2}, \tag{18}$$

$$\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) = \frac{-2A + DM^2h\rho + sDI_e(M^2 - N^2)}{2M^2} \tag{19}$$

and

$$\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) = \frac{-2A + DN^2h\rho}{2N^2}. \tag{20}$$

Furthermore, we let

$$\Delta_1 = -2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2)] + sDI_e(M^2 - N^2), \tag{21}$$

$$\Delta_2 = -2A + DM^2h\rho + sDI_e(M^2 - N^2) \tag{22}$$

and

$$\Delta_3 = -2A + DN^2h\rho. \tag{23}$$

Then, we have $\Delta_1 \geqslant \Delta_2 \geqslant \Delta_3$.

3. Decision rules of the optimal cycle time T^*

In this section, we develop efficient decision rules to find the optimal cycle time for the retailer.

3.1. Suppose that $\beta \leqslant 0$

When $\beta \leqslant 0$, then $\alpha < 0$, and we can find $\mathrm{TVC}_1(T)$ is increasing on $[PM/D, \infty)$ from Eq. (6) and $\mathrm{TVC}_2(T)$ is increasing on $[M, PM/D\,]$ from Eq. (8). In addition, we can obtain $\Delta_1 \geqslant \Delta_2 > 0$ from Eqs. (21) and (22). By the convexity of $\mathrm{TVC}_i(T)$ ($i = 3$ and 4), we see

$$\mathrm{TVC}_i'(T)\begin{cases} < 0 & \text{if } T < T_i^*, \quad \text{(a)} \\ = 0 & \text{if } T = T_i^*, \quad \text{(b)} \\ > 0 & \text{if } T > T_i^*. \quad \text{(c)} \end{cases} \tag{24}$$

Then, we have the following result to determine the optimal cycle time T^*.

Theorem 2. *Suppose that* $\beta \leqslant 0$, *then*

(A) *If* $\Delta_3 \geqslant 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_4^*)$ *and* $T^* = T_4^*$.
(B) *If* $\Delta_3 < 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_3^*)$ *and* $T^* = T_3^*$.

Proof. See Appendix A. □

3.2. Suppose that $\alpha \leqslant 0$ and $\beta > 0$

When $\alpha \leqslant 0$ and $\beta > 0$, we can find $\mathrm{TVC}_1(T)$ is increasing on $[PM/D, \infty)$ from Eq. (6) and $\Delta_1 > 0$ from Eq. (21). By the convexity of $\mathrm{TVC}_i(T)$ ($i = 2, 3, 4$), we see

$$\mathrm{TVC}_i'(T)\begin{cases} < 0 & \text{if } T < T_i^*, \quad \text{(a)} \\ = 0 & \text{if } T = T_i^*, \quad \text{(b)} \\ > 0 & \text{if } T > T_i^*. \quad \text{(c)} \end{cases} \tag{25}$$

Then, we have the following results to determine the optimal cycle time T^*.

Theorem 3. *Suppose that* $\alpha \leqslant 0$ *and* $\beta > 0$, *then*

(A) *If* $\Delta_3 \geqslant 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_4^*)$ *and* $T^* = T_4^*$.
(B) *If* $\Delta_2 \geqslant 0$ *and* $\Delta_3 < 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_3^*)$ *and* $T^* = T_3^*$.
(C) *If* $\Delta_2 < 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_2^*)$ *and* $T^* = T_2^*$.

Proof. See Appendix B. □

3.3. Suppose that $\alpha > 0$

When $\alpha > 0$, all T_i^* $(i = 1, 2, 3, 4)$ are well-defined. By the convexity of $\mathrm{TVC}_i(T)$ $(i = 1, 2, 3, 4)$, we see

$$\mathrm{TVC}_i'(T)\begin{cases} < 0 & \text{if } T < T_i^*, \quad \text{(a)} \\ = 0 & \text{if } T = T_i^*, \quad \text{(b)} \\ > 0 & \text{if } T > T_i^*. \quad \text{(c)} \end{cases} \tag{26}$$

Then, we have the following results to determine the optimal cycle time T^*.

Theorem 4. *Suppose that* $\alpha > 0$, *then*

(A) *If* $\Delta_3 \geqslant 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_4^*)$ *and* $T^* = T_4^*$.
(B) *If* $\Delta_2 \geqslant 0$ *and* $\Delta_3 < 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_3^*)$ *and* $T^* = T_3^*$.
(C) *If* $\Delta_1 > 0$ *and* $\Delta_2 < 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_2^*)$ *and* $T* = T_2^*$.
(D) *If* $\Delta_1 \leqslant 0$, *then* $\mathrm{TVC}(T^*) = \mathrm{TVC}(T_1^*)$ *and* $T^* = T_1^*$.

Proof. See Appendix C. □

4. Special cases

4.1. (I) Chung and Huang's model [14]

When $N = 0$ and $s = c$, let

$$\mathrm{TVC}_5(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho\left(\frac{DT^2}{2} - \frac{PM^2}{2}\right)\Big/T - cI_e\left(\frac{DM^2}{2}\right)\Big/T,$$

$$\mathrm{TVC}_6(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\left[\frac{D(T-M)^2}{2}\right]\Big/T - cI_e\left(\frac{DM^2}{2}\right)\Big/T,$$

$$\mathrm{TVC}_7(T) = \frac{A}{T} + \frac{DTh\rho}{2} - cI_e\left[\frac{DT^2}{2} + DT(M-T)\right]\Big/T,$$

$$T_5^* = \sqrt{\frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{D\rho(h + cI_k)}},$$

$$T_6^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)}}$$

and

$$T_7^* = \sqrt{\frac{2A}{D(h\rho + cI_e)}}.$$

Then $\mathrm{TVC}_i'(T_i^*) = 0$ for $i = 5, 6, 7$. Eq. (1a–d) will be reduced as follows:

$$\mathrm{TVC}(T) = \begin{cases} \mathrm{TVC}_5(T) & \text{if } T \geqslant \frac{PM}{D}, & \text{(a)} \\ \mathrm{TVC}_6(T) & \text{if } M \leqslant T \leqslant \frac{PM}{D}, & \text{(b)} \\ \mathrm{TVC}_7(T) & \text{if } 0 < T \leqslant M. & \text{(c)} \end{cases} \tag{27}$$

Eqs. (27a–c) will be consistent with Eq. (6a–c) in Chung and Huang [14], respectively. In addition, Theorems 3 and 4 in this paper will be modified as Theorems 2 and 3 in Chung and Huang [14]. Hence, Chung and Huang [14] will be a special case of this paper.

4.2. (II) Huang's model [17]

When $P \to \infty$ and $s = c$, let

$$\mathrm{TVC}_8(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T-M)^2/2T - cI_e D(M^2 - N^2)/2T,$$

$$\mathrm{TVC}_9(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e D(2MT - N^2 - T^2)/2T,$$

$$\mathrm{TVC}_{10}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e D(M - N),$$

$$T_8^* = \sqrt{\frac{2A + cD[M^2(I_k - I_e) + N^2 I_e]}{D(h + cI_k)}},$$

$$T_9^* = \sqrt{\frac{2A + cDN^2 I_e}{D(h + cI_e)}}$$

and

$$T_{10}^* = \sqrt{\frac{2A}{Dh}}.$$

Then $\mathrm{TVC}_i'(T_i^*) = 0$ for $i = 8, 9, 10$. Eqs. (1a–d) will be reduced as follows:

$$\mathrm{TVC}(T) = \begin{cases} \mathrm{TVC}_8(T) & \text{if } T \geqslant M, & \text{(a)} \\ \mathrm{TVC}_9(T) & \text{if } N \leqslant T \leqslant M, & \text{(b)} \\ \mathrm{TVC}_{10}(T) & \text{if } 0 < T \leqslant N. & \text{(c)} \end{cases} \tag{28}$$

Eqs. (28a–c) will be consistent with Eqs. (1a–c) in Huang [17], respectively. In addition, Theorem 4 in this paper will be modified as Theorem 1 in Huang [17]. Hence, Huang [17] will be a special case of this paper.

4.3. (III) Goyal's model [16]

When $P \to \infty$, $N = 0$ and $s = c$, let

$$\mathrm{TVC}_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k \left[\frac{D(T-M)^2}{2}\right] \Big/ T - cI_e \left(\frac{DM^2}{2}\right) \Big/ T,$$

$$\mathrm{TVC}_{12}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e \left[\frac{DT^2}{2} + DT(M - T)\right] \Big/ T,$$

$$T_{11}^{*}=\sqrt{\frac{2A+DM^{2}c(I_{k}-I_{e})}{D(h+cI_{k})}}$$

and

$$T_{12}^{*}=\sqrt{\frac{2A}{D(h+cI_{e})}}.$$

Then $\mathrm{TVC}_i'(T_i^*)=0$ for $i=11, 12$. Eq. (1a–d) will be reduced as follows:

$$\mathrm{TVC}(T)=\begin{cases}\mathrm{TVC}_{11}(T) & \text{if } M\leqslant T, \qquad \text{(a)}\\ \mathrm{TVC}_{12}(T) & \text{if } 0<T\leqslant M. \qquad \text{(b)}\end{cases} \tag{29}$$

Eq. (29a,b) will be consistent with Eqs. (1) and (4) in Goyal [16], respectively. Hence, Goyal [16] will be a special case of this paper. Theorem 4 in this paper can be modified as Theorem 1 in Chung [10]. So Theorem 1 in Chung [10] is a special case of Theorem 4 of this paper.

5. Numerical examples

To illustrate all results obtained in this paper, let us apply the proposed method to efficiently solve the following numerical examples.

From above Tables 1 and 2, we can observe the optimal cycle time with various parameters of P, N and s, respectively. The following inferences can be made based in Tables 1 and 2.

(1) When replenishment rate P is increasing, the optimal cycle time for the retailer will be decreasing. The retailer will order less quantity since the replenishment rate is faster enough. These results are easily understood and can be found in Tables 1 and 2.

Table 1
The optimal cycle time with various values of P and N

Example 1: Let A = \$150/order, D = 2500 units/year, c = \$50/unit, s = \$75/unit, h = \$15/unit/year, I_k = \$0.15/\$/year, I_e = \$0.1/\$/year, M = 0.1year

N (year)	P = 3000 units/year						P = 4000 units/year						P = 5000 units/year					
	α	β	Δ_1	Δ_2	Δ_3	T^*	α	β	Δ_1	Δ_2	Δ_3	T^*	α	β	Δ_1	Δ_2	Δ_3	T^*
0.02	>0	>0	>0	<0	<0	$T_2^*=0.1109$	>0	>0	>0	>0	<0	$T_3^*=0.0968$	<0	>0	>0	>0	<0	$T_3^*=0.0906$
0.05	>0	>0	>0	<0	<0	$T_2^*=0.1178$	>0	>0	>0	<0	<0	$T_2^*=0.1028$	<0	>0	>0	>0	<0	$T_3^*=0.0962$
0.08	>0	>0	<0	<0	<0	$T_1^*=0.1442$	>0	>0	>0	<0	<0	$T_2^*=0.1131$	>0	>0	>0	<0	<0	$T_2^*=0.1058$

Table 2
The optimal cycle time with various values of P and s

Example 2: Let A = \$150/order, D = 2500 units/year, c = \$50/unit, h = \$15/unit/year, I_k = \$0.15/\$/year, I_e = \$0.1/\$/year, M = 0.1 year, N = 0.05 year

s (\$/unit)	P = 3000 units/year						P = 4000 units/year						P = 5000 units/year					
	α	β	Δ_1	Δ_2	Δ_3	T^*	α	β	Δ_1	Δ_2	Δ_3	T^*	α	β	Δ_1	Δ_2	Δ_3	T^*
50	>0	>0	<0	<0	<0	$T_1^*=0.1342$	>0	>0	>0	<0	<0	$T_2^*=0.1095$	>0	>0	>0	<0	<0	$T_2^*=0.1025$
75	>0	>0	>0	<0	<0	$T_2^*=0.1178$	>0	>0	>0	<0	<0	$T_2^*=0.1028$	<0	>0	>0	>0	<0	$T_3^*=0.0962$
100	>0	>0	>0	<0	<0	$T_2^*=0.1095$	>0	>0	>0	>0	<0	$T_3^*=0.0963$	<0	>0	>0	>0	<0	$T_3^*=0.091$

(2) When the customer's trade credit period offered by retailer N is increasing, the optimal cycle time for the retailer will be increasing. It implies that the retailer will order more quantity to get more interest earned offered by the supplier to compensate the loss of interest earned from longer trade credit period offered to his/her customer. Table 1 shows this computed result.
(3) In Table 2, we can find that the optimal cycle time for the retailer will be decreasing when the unit selling price s is increasing. This result implies that the retailer will order less quantity to take the benefits of the trade credit more frequently.

6. Summary and conclusions

This paper incorporates both Chung and Huang [14] and Huang [17] to investigate the optimal retailer's replenishment decisions under two levels of trade credit policy within the economic production quantity (EPQ) framework to reflect the realistic business situations. Theorems 2–4 help the retailer in accurately and quickly determining the optimal replenishment decisions under minimizing the annual total relevant cost. When the customer's trade credit period offered by the retailer equals to zero and the unit purchasing price is equal to the unit selling price, the inventory model discussed in this paper is reduced to Chung and Huang [14]. When the replenishment rate is infinite and the unit purchasing price is equal to the unit selling price, the inventory model discussed in this paper is reduced to Huang [17]. When the customer's trade credit period offered by the retailer equals to zero, the replenishment rate is infinite and the unit purchasing price is equal to the unit selling price, the inventory model discussed in this paper is reduced to Goyal [16]. Finally, numerical examples are used to illustrate all results obtained in this paper. In addition, we obtain a lot of managerial insights from numerical examples.

A future study will further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, allowable shortages, multi-supplier, multi-retailer, multi-customer etc.

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Appendix A. Proof of Theorem 2

(A) If $\Delta_3 \geqslant 0$, then $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) \geqslant 0$. Eqs. (24a–c) imply that
  (i) $\mathrm{TVC}_3(T)$ is increasing on $[N, \infty)$.
  (ii) $\mathrm{TVC}_4(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, N]$.
  Combining (i), (ii) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, \infty)$. Consequently, $T^* = T_4^*$.

(B) If $\Delta_3 < 0$, then $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eq (24a–c) imply that
  (i) $\mathrm{TVC}_3(T)$ is decreasing on $[N, T_3^*]$ and increasing on $[T_3^*, \infty)$.
  (ii) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
  Combining (i), (ii) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, \infty)$. Consequently, $T^* = T_3^*$.

Incorporating the above arguments, we have completed the proof of Theorem 2.

Appendix B. Proof of Theorem 3

(A) If $\Delta_3 \geqslant 0$ then $\Delta_2 \geqslant 0$, therefore $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) \geqslant 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) \geqslant 0$. Eqs. (25a–c) imply that
  (i) $\mathrm{TVC}_2(T)$ is increasing on $[M, \infty)$.
  (ii) $\mathrm{TVC}_3(T)$ is increasing on $[N, M]$.
  (iii) $\mathrm{TVC}_4(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, N]$.
  Combining (i)–(iii) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, \infty)$. Consequently, $T^* = T_4^*$.

(B) If $\Delta_2 \geqslant 0$ and $\Delta_3 < 0$, then $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) \geqslant 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eqs. (25a–c) imply that
  (i) $\mathrm{TVC}_2(T)$ is increasing on $[M, \infty)$.
  (ii) $\mathrm{TVC}_3(T)$ is decreasing on $[N, T_3^*]$ and increasing on $[T_3^*, M]$.
  (iii) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
  Combining (i)–(iii) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, \infty)$. Consequently, $T^* = T_3^*$.

(C) If $\Delta_2 < 0$ then $\Delta_3 < 0$, therefore $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) < 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eqs. (25a–c) imply that
  (i) $\mathrm{TVC}_2(T)$ is decreasing on $[M, T_2^*]$ and increasing on $[T_2^*, \infty)$.
  (ii) $\mathrm{TVC}_3(T)$ is decreasing on $[N, M]$.
  (iii) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
  Combining (i)–(iii) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_2^*]$ and increasing on $[T_2^*, \infty)$. Consequently, $T^* = T_2^*$.

Incorporating the above arguments, we have completed the proof of Theorem 3.

Appendix C. Proof of Theorem 4

(A) If $\Delta_3 \geqslant 0$ then $\Delta_1 > 0$, $\Delta_2 \geqslant 0$, therefore $\mathrm{TVC}_1'(\frac{PM}{D}) = \mathrm{TVC}_2'(\frac{PM}{D}) > 0$, $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) \geqslant 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) \geqslant 0$. Eqs. (26a–c) imply that
  (i) $\mathrm{TVC}_1(T)$ is increasing on $[\frac{PM}{D}, \infty)$.
  (ii) $\mathrm{TVC}_2(T)$ is increasing on $[M, \frac{PM}{D}]$.
  (iii) $\mathrm{TVC}_3(T)$ is increasing on $[N, M]$.
  (iv) $\mathrm{TVC}_4(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, N]$.
  Combining (i)–(iv) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_4^*]$ and increasing on $[T_4^*, \infty)$. Consequently, $T^* = T_4^*$.

(B) If $\Delta_2 \geqslant 0$ and $\Delta_3 < 0$ then $\Delta_1 > 0$, therefore $\mathrm{TVC}_1'(\frac{PM}{D}) = \mathrm{TVC}_2'(\frac{PM}{D}) > 0$, $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) \geqslant 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eqs. (26a–c) imply that
  (i) $\mathrm{TVC}_1(T)$ is increasing on $[\frac{PM}{D}, \infty)$.
  (ii) $\mathrm{TVC}_2(T)$ is increasing on $[M, \frac{PM}{D}]$.
  (iii) $\mathrm{TVC}_3(T)$ is decreasing on $[N, T_3^*]$ and increasing on $[T_3^*, M]$.
  (iv) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
  Combining (i)–(iv) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, \infty)$. Consequently, $T* = T_3^*$.

(C) If $\Delta_1 > 0$ and $\Delta_2 < 0$ then $\Delta_3 < 0$, therefore $\mathrm{TVC}_1'(\frac{PM}{D}) = \mathrm{TVC}_2'(\frac{PM}{D}) > 0$, $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) < 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eqs. (26a–c) imply that

(i) $\mathrm{TVC}_1(T)$ is increasing on $[\frac{PM}{D}, \infty)$.
(ii) $\mathrm{TVC}_2(T)$ is decreasing on $[M, T_2^*]$ and increasing on $[T_2^*, \frac{PM}{D}]$.
(iii) $\mathrm{TVC}_3(T)$ is decreasing on $[N, M]$.
(iv) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
Combining (i)–(iv) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_2^*]$ and increasing on $[T_2^*, \infty)$. Consequently, $T^* = T_2^*$.

(D) If $\Delta_1 \leqslant 0$ then $\Delta_2 < 0$ and $\Delta_3 < 0$, therefore $\mathrm{TVC}_1'(\frac{PM}{D}) = \mathrm{TVC}_2'(\frac{PM}{D}) \leqslant 0$, $\mathrm{TVC}_2'(M) = \mathrm{TVC}_3'(M) < 0$ and $\mathrm{TVC}_3'(N) = \mathrm{TVC}_4'(N) < 0$. Eqs. (26a–c) imply that
(i) $\mathrm{TVC}_1(T)$ is decreasing on $[\frac{PM}{D}, T_1^*]$ and increasing on $[T_1^*, \infty)$.
(ii) $\mathrm{TVC}_2(T)$ is decreasing on $[M, \frac{PM}{D}]$.
(iii) $\mathrm{TVC}_3(T)$ is decreasing on $[N, M]$.
(iv) $\mathrm{TVC}_4(T)$ is decreasing on $(0, N]$.
Combining (i)–(iv) and Eqs. (1a–d), we have that $\mathrm{TVC}(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, \infty)$. Consequently, $T^* = T_1^*$.

Incorporating the above arguments, we have completed the proof of Theorem 4.

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