

An EOQ model under retailer partial trade credit policy in supply chain

Yung-Fu Huang^a, Kuang-Hua Hsu^{b,*}

^a*Department of Marketing and Logistics Management, Chaoyang University of Technology, Taichung, Taiwan, ROC*

^b*Department of Finance, Chaoyang University of Technology, Taichung, Taiwan, ROC*

Received 13 January 2006; accepted 25 May 2007

Available online 10 July 2007

Abstract

The main purpose of this paper is to investigate the retailer's inventory policy under two levels of trade credit to reflect the supply chain management situation. In this paper, we assume that the retailer has the powerful decision-making right. So, we extend the assumption that the retailer can obtain the full trade credit offered by the supplier and the retailer just offers the partial trade credit to his/her customer. Then, we investigate the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory policy under the supply chain management. Two easy-to-use theorems are developed to efficiently determine the optimal inventory policy for the retailer. We deduce some previously published results of other researchers as special cases. Finally, numerical examples are given to illustrate the theorems and obtain a lot of managerial phenomena.

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Keywords: EOQ; Inventory; Partial trade credit; Supply chain

1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as they are received. However, this may not be true. In practice, the supplier will offer the retailer a delay period, that is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn

interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. In the real world, the supplier often makes use of this policy to promote his/her commodities.

Goyal (1985) established a single-item inventory model under trade credit. Chung (1998) developed an alternative approach to determine the EOQ under the condition of trade credit. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of trade credit. Chang et al. (2002) extended this issue to the varying rate of deterioration. Liao et al. (2000) and Sarker et al. (2000a) investigated this topic with inflation. Jamal et al. (1997) and Chang and Dye (2001) extended this issue with allowable shortage. Chang et al. (2001) extended this issue

*Corresponding author. No. 168, Jifong E. Road, Wufong Township, Taichung County 41349, Taiwan, ROC.
Tel.: +886 4 247 39 477; fax: +886 4 247 29 772.

E-mail addresses: huf@cyut.edu.tw (Y.-F. Huang),
khhsu@cyut.edu.tw (K.-H. Hsu).

with linear trend demand. [Chen and Chuang \(1999\)](#) investigated a light buyer's inventory policy under trade credit by the concept of discounted cash flow. [Hwang and Shinn \(1997\)](#) modeled an inventory system for a retailer's pricing and lot-sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. [Jamal et al. \(2000\)](#) and [Sarker et al. \(2000b\)](#) addressed the optimal payment time under permissible delay in payment with deterioration. [Teng \(2002\)](#) assumed that the selling price is not equal to the purchasing price to modify [Goyal's model \(1985\)](#). [Chung et al. \(2002\)](#) discussed this issue under the selling price not equal to the purchasing price and different payment rule. [Shinn and Hwang \(2003\)](#) determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. [Chung and Huang \(2003\)](#) extended this problem within the economic production quantity (EPQ) framework and developed an efficient procedure to determine the retailer's optimal ordering policy. [Huang and Chung \(2003\)](#) extended [Goyal's model \(1985\)](#) to cash discount policy for early payment. [Salameh et al. \(2003\)](#) extended this issue to the continuous review inventory model. [Chang et al. \(2003\)](#) and [Chung and Liao \(2004\)](#) dealt with the problem of determining the EOQ for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity. [Chang \(2004\)](#) extended this issue to inflation and finite time horizon. [Huang \(2004\)](#) investigated that the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under a supplier's trade credit policy. There are several interesting and relevant papers related to trade credit such as [Chung et al. \(2005\)](#), [Chung and Liao \(2006\)](#), and [Huang \(2007\)](#) and their references.

All the above articles assumed that the supplier would offer the retailer a delay period and the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period. They implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. That is, they assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to his/her customer in previously published results. That is one level of trade credit. In most business

transactions, this assumption is unrealistic. Recently, [Huang \(2003\)](#) modified this assumption to assume that the retailer will adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. That is two levels of trade credit. This new viewpoint is more matched to real-life situations in the supply chain model. Therefore, we want to extend [Huang's model \(2003\)](#) to investigate the situation under which the retailer has the powerful decision-making right. That is, we want to assume that the retailer can obtain the full trade credit offered by the supplier and the retailer just offers the partial trade credit to his/her customer. In practice, this circumstance is very realistic. For example, the Toyota Company can require his supplier to offer the full trade credit to him and just offer partial trade credit to his dealership. That is, the Toyota Company can delay the full amount of purchasing cost until the end of the delay period offered by his supplier. But the Toyota Company only offers partial delay payment to his dealership on the permissible credit period and the rest of the total amount is payable at the time the dealership places a replenishment order. In addition, we want to relax three assumptions in [Huang's model \(2003\)](#) that unit purchasing price equals unit selling price, $c = s$, interest charge rate is larger than interest earned rate, $I_k \geq I_e$, and the retailer's trade credit period offered by the supplier is longer than the customer's trade credit period offered by the retailer, $M \geq N$. Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal ordering policies.

2. Model formulation and convexity

The following notation and assumptions will be used throughout:

Notation

D	demand rate per year
A	ordering cost per order
c	unit purchasing price
s	unit selling price, $s \geq c$
h	unit stock-holding cost per year excluding interest charges
α	customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer, $0 \leq \alpha \leq 1$
I_e	interest earned per \$ per year

- I_k interest charged per \$ in stocks per year by the supplier
- M retailer's trade credit period offered by the supplier in years
- N customer's trade credit period offered by the retailer in years
- T cycle time in years
- $TRC(T)$ annual total relevant cost, which is a function of T
- T^* optimal cycle time of $TRC(T)$
- Q^* optimal order quantity = DT^*

Assumptions

- (1) Demand rate, D , is known and constant.
- (2) Shortages are not allowed.
- (3) Time horizon is infinite.
- (4) Replenishments are instantaneous.
- (5) The supplier offers the full trade credit to the retailer. When $T \geq M$, the account is settled at $T = M$, the retailer pays off all units sold and keeps his/her profits, and starts paying for the interest charges on the items in stock with rate I_k . When $T \leq M$, the account is settled at $T = M$ and the retailer does not need to pay any interest charge.
- (6) The retailer just offers the partial trade credit to his/her customer. Hence, his/her customer must make a partial payment to the retailer when the item is sold. Then his/her customer must pay off the remaining balance at the end of the trade credit period offered by the retailer. That is, the retailer can accumulate interest from his/her customer payment with rate I_e .

The annual total relevant cost consists of the following elements. Two situations may arise: (I) $M \geq N$ and (II) $M < N$.

Case I: Suppose that $M \geq N$.

- (1) Annual ordering cost = A/T .
- (2) Annual stock holding cost (excluding interest charges) = $DTh/2$.
- (3) According to assumption (5), there are three cases that occur in costs of interest charges for the items kept in stock per year.
 - (i) $M \leq T$.
Annual interest payable = $cI_k D(T-M)^2/2T$.
 - (ii) $N \leq T \leq M$.
In this case, annual interest payable = 0.
 - (iii) $T \leq N$.

In this case, annual interest payable = 0.
(4) According to assumption (6), there are three cases that occur in interest earned per year.

- (i) $M \leq T$, as shown in Fig. 1.
Annual interest earned

$$= sI_e \left[\frac{\alpha DN^2}{2} + \frac{(DN + DM)(M - N)}{2} \right] / T$$

$$= sI_e D [M^2 - (1 - \alpha)N^2] / 2T.$$

- (ii) $N \leq T \leq M$, as shown in Fig. 2.
Annual interest earned

$$= sI_e \left[\frac{\alpha DN^2}{2} + \frac{(DN + DT)(T - N)}{2} + DT(M - T) \right] / T$$

$$= sI_e D [2MT - (1 - \alpha)N^2 - T^2] / 2T.$$

- (iii) $T \leq N$, as shown in Fig. 3.
Annual interest earned

$$= sI_e \left[\frac{\alpha DT^2}{2} + \alpha DT(N - T) + DT(M - N) \right] / T$$

$$= sI_e D \left[M - (1 - \alpha)N - \frac{\alpha T}{2} \right].$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

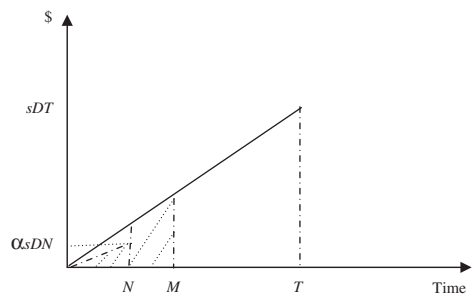


Fig. 1. Total amount of interest earned when $M \leq T$.

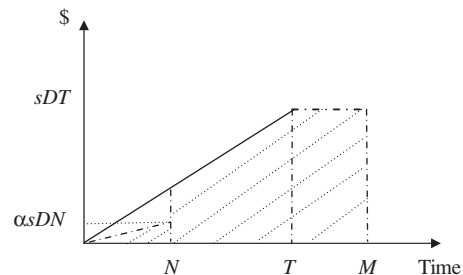


Fig. 2. Total amount of interest earned when $N \leq T \leq M$.

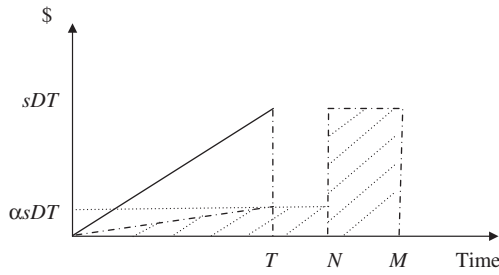


Fig. 3. Total amount of interest earned when $T \leq N$.

$TRC(T)$ = ordering cost + stock-holding cost + interest payable – interest earned.

$$TRC(T) = \begin{cases} TRC_1(T) & \text{if } T \geq M, \\ TRC_2(T) & \text{if } N \leq T \leq M, \text{ and} \\ TRC_3(T) & \text{if } 0 < T \leq N, \end{cases} \quad (1a-c)$$

where

$$TRC_1(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T - M)^2 / 2T - sI_e D[M^2 - (1 - \alpha)N^2] / 2T, \quad (2)$$

$$TRC_2(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e D[2MT - (1 - \alpha)N^2 - T^2] / 2T, \quad (3)$$

and

$$TRC_3(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e D \left[M - (1 - \alpha)N - \frac{\alpha T}{2} \right]. \quad (4)$$

Since $TRC_1(M) = TRC_2(M)$ and $TRC_2(N) = TRC_3(N)$, $TRC(T)$ is continuous and well-defined. All $TRC_1(T)$, $TRC_2(T)$, $TRC_3(T)$, and $TRC(T)$ are defined on $T > 0$. Eqs. (2)–(4) yield

$$TRC'_1(T) = - \left[\frac{2A + cDM^2 I_k - sDI_e(M^2 - (1 - \alpha)N^2)}{2T^2} \right] + D \left(\frac{h + cI_k}{2} \right), \quad (5)$$

$$TRC''_1(T) = \frac{2A + cDM^2 I_k - sDI_e(M^2 - (1 - \alpha)N^2)}{T^3}, \quad (6)$$

$$TRC'_2(T) = - \left[\frac{2A + sD(1 - \alpha)N^2 I_e}{2T^2} \right] + D \left(\frac{h + sI_e}{2} \right), \quad (7)$$

$$TRC''_2(T) = \frac{2A + sD(1 - \alpha)N^2 I_e}{T^3} > 0, \quad (8)$$

$$TRC'_3(T) = \frac{-A}{T^2} + D \left(\frac{h + s\alpha I_e}{2} \right), \quad (9)$$

and

$$TRC''_3(T) = \frac{2A}{T^3} > 0. \quad (10)$$

Eqs. (8) and (10) imply that $TRC_2(T)$ and $TRC_3(T)$ are convex on $T > 0$ and Eq. (6) implies that $TRC_1(T)$ is convex on $T > 0$ when $2A + cDM^2 I_k - sDI_e(M^2 - (1 - \alpha)N^2) > 0$. Furthermore, we have $TRC'_1(M) = TRC'_2(M)$ and $TRC'_2(N) = TRC'_3(N)$. Therefore, Eqs. (1a–c) imply that $TRC(T)$ is convex on $T > 0$ when $2A + cDM^2 I_k - sDI_e(M^2 - (1 - \alpha)N^2) > 0$.

Case II: Suppose that $M < N$.

- (1) Annual ordering cost = A/T .
- (2) Annual stock holding cost (excluding interest charges) = $DTh/2$.
- (3) According to assumption (5), there are two cases that occur in costs of interest charges for the items kept in stock per year.
 - (i) $M \leq T$.
Annual interest payable = $cI_k D(T - M)^2 / 2T$.
 - (ii) $M \geq T$.
In this case, annual interest payable = 0.
- (4) According to assumption (6), there are two cases that occur in interest earned per year.
 - (i) $M \leq T$, as shown in Fig. 4.
Annual interest earned = $sI_e D\alpha M^2 / 2T$.
 - (ii) $M \geq T$, as shown in Fig. 5.
Annual interest earned.

$$= sI_e \left[\frac{\alpha DT^2}{2} + \alpha DT(M - T) \right] / T$$

$$= sI_e D \left[\alpha M - \frac{\alpha T}{2} \right].$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

$TRC(T)$ = ordering cost + stock-holding cost + interest payable – interest earned.

$$TRC(T) = \begin{cases} TRC_4(T) & \text{if } T \geq M, \\ TRC_5(T) & \text{if } 0 < T \leq M, \end{cases} \quad (11a-b)$$

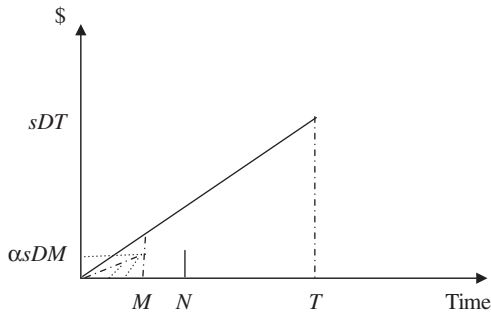


Fig. 4. Total amount of interest earned when $M \leq T$.

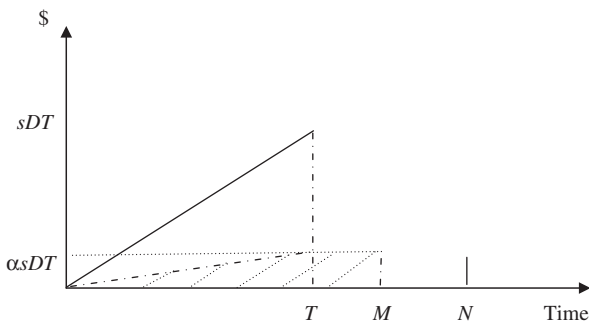


Fig. 5. Total amount of interest earned when $T \leq M$.

where

$$TRC_4(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T - M)^2 / 2T - sI_e D \alpha M^2 / 2T \quad (12)$$

and

$$TRC_5(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e D \left[\alpha M - \frac{\alpha T}{2} \right]. \quad (13)$$

Since $TRC_4(M) = TRC_5(M)$, $TRC(T)$ is continuous and well-defined. All $TRC_4(T)$, $TRC_5(T)$, and $TRC(T)$ are defined on $T > 0$. Eqs. (12) and (13) yield

$$TRC'_4(T) = - \left[\frac{2A + DM^2(cI_k - s\alpha I_e)}{2T^2} \right] + D \left(\frac{h + cI_k}{2} \right), \quad (14)$$

$$TRC''_4(T) = \frac{2A + DM^2(cI_k - s\alpha I_e)}{T^3}, \quad (15)$$

$$TRC'_5(T) = \frac{-A}{T^2} + D \left(\frac{h + s\alpha I_e}{2} \right), \quad (16)$$

and

$$TRC''_5(T) = \frac{2A}{T^3} > 0. \quad (17)$$

Eq. (17) implies that $TRC_5(T)$ is convex on $T > 0$ and Eq. (15) implies that $TRC_4(T)$ is convex on $T > 0$ when $2A + DM^2(cI_k - s\alpha I_e) > 0$. Furthermore, we have $TRC'_4(M) = TRC'_5(M)$. Therefore, Eqs. (11a,b) imply that $TRC(T)$ is convex on $T > 0$ when $2A + DM^2(cI_k - s\alpha I_e) > 0$.

3. Determination of the optimal cycle time T^*

Case I: Suppose that $M \geq N$.

Let $TRC'_i(T_i^*) = 0$ for all $i = 1, 2, 3$. We can obtain

$$T_1^* = \sqrt{\frac{2A + cDM^2 I_k - sDI_e [M^2 - (1 - \alpha)N^2]}{D(h + cI_k)}} \quad (18)$$

if

$$2A + cDM^2 I_k - sDI_e [M^2 - (1 - \alpha)N^2] > 0, \quad (18)$$

$$T_2^* = \sqrt{\frac{2A + sD(1 - \alpha)N^2 I_e}{D(h + sI_e)}}, \quad (19)$$

and

$$T_3^* = \sqrt{\frac{2A}{D(h + s\alpha I_e)}}. \quad (20)$$

Eq. (18) gives the optimal value of T^* for the case when $T \geq M$ so that $T_1^* \geq M$. We substitute Eq. (18) into $T_1^* \geq M$; then we obtain that

$T_1^* \geq M$ if and only if

$$-2A + DM^2 h + sDI_e [M^2 - (1 - \alpha)N^2] \leq 0.$$

Similarly, Eq. (19) gives the optimal value of T^* for the case when $N \leq T \leq M$ so that $N \leq T_2^* \leq M$. We substitute Eq. (19) into $N \leq T_2^* \leq M$; then we obtain that

$T_2^* \leq M$ if and only if

$$-2A + DM^2 h + sDI_e [M^2 - (1 - \alpha)N^2] \geq 0$$

and

$$N \leq T_2^* \text{ if and only if } -2A + DN^2(h + s\alpha I_e) \leq 0.$$

Finally, Eq. (20) gives the optimal value of T^* for the case when $T \leq N$ so that $T_3^* \leq N$. We substitute Eq. (20) into $T_3^* \leq N$; then we obtain that

$$T_3^* \leq N \text{ if and only if } -2A + DN^2(h + s\alpha I_e) \geq 0.$$

Furthermore, we let

$$\Delta_1 = -2A + DM^2h + sDI_e[M^2 - (1 - \alpha)N^2] \quad (21)$$

and

$$\Delta_2 = -2A + DN^2(h + s\alpha I_e). \quad (22)$$

Eqs. (21) and (22) imply that $\Delta_1 \geq \Delta_2$. From the above arguments, we obtain the following results:

Theorem 1.

- (A) If $\Delta_2 \geq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_3^*)$ and $T^* = T_3^*$.
- (B) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $\text{TRC}(T^*) = \text{TRC}(T_2^*)$ and $T^* = T_2^*$.
- (C) If $\Delta_1 \leq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_1^*)$ and $T^* = T_1^*$.

Case II: Suppose that $M < N$.

Let $\text{TRC}'_i(T_i^*) = 0$ for all $i = 4, 5$. We obtain

$$T_4^* = \sqrt{\frac{2A + DM^2(cI_k - s\alpha I_e)}{D(h + cI_k)}} \quad (23)$$

if $2A + DM^2(cI_k - s\alpha I_e) > 0$

and

$$T_5^* = \sqrt{\frac{2A}{D(h + s\alpha I_e)}}. \quad (24)$$

Eq. (23) gives the optimal value of T^* for the case when $T \geq M$ so that $T_4^* \geq M$. We substitute Eq. (23) into $T_4^* \geq M$; then we obtain that

$$T_4^* \geq M \text{ if and only if } -2A + DM^2(h + s\alpha I_e) \leq 0.$$

Similarly, Eq. (24) gives the optimal value of T^* for the case when $T \leq M$ so that $T_5^* \leq M$. We substitute Eq. (24) into $T_5^* \leq M$; then we obtain that $T_5^* \leq M$ if and only if $-2A + DM^2(h + s\alpha I_e) \geq 0$.

Furthermore, we let

$$\Delta_3 = -2A + DM^2(h + s\alpha I_e). \quad (25)$$

From the above arguments, we can obtain the following results.

Theorem 2.

- (A) If $\Delta_3 \geq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_5^*)$ and $T^* = T_5^*$.
- (B) If $\Delta_3 < 0$, then $\text{TRC}(T^*) = \text{TRC}(T_4^*)$ and $T^* = T_4^*$.

4. Special cases

4.1. Huang's model

When $M \geq N$, $s = c$, and $\alpha = 0$ (it means that the retailer also offers the full trade credit to his/her customer), let

$$\text{TRC}_6(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T - M)^2/2T - cI_e D(M^2 - N^2)/2T, \quad (26)$$

$$\text{TRC}_7(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e D(2MT - N^2 - T^2)/2T, \quad (27)$$

$$\text{TRC}_8(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e D(M - N), \quad (28)$$

$$T_6^* = \sqrt{\frac{2A + cD[M^2(I_k - I_e) + N^2 I_e]}{D(h + cI_k)}}, \quad (29)$$

$$T_7^* = \sqrt{\frac{2A + cDN^2 I_e}{D(h + cI_e)}}, \quad (30)$$

and

$$T_8^* = \sqrt{\frac{2A}{Dh}}. \quad (31)$$

Then $\text{TRC}'_i(T_i^*) = 0$ for $i = 6, 7, 8$.

Eqs. 1(a–c) will be modified as follows:

$$\text{TRC}(T) = \begin{cases} \text{TRC}_6(T) & \text{if } T \geq M, \\ \text{TRC}_7(T) & \text{if } N \leq T \leq M, \\ \text{TRC}_8(T) & \text{if } 0 < T \leq N, \end{cases} \quad (32a-c)$$

Eqs. 32(a–c) will be consistent with Eqs. 1(a–c), in Huang (2003), respectively. Eqs. (21) and (22) can be modified as $\Delta_1 = -2A + DM^2(h + cI_e) - cDN^2 I_e$ and $\Delta_2 = -2A + DN^2 h$, respectively. If we let $\bar{\Delta}_1 = -2A + DM^2(h + cI_e) - cDN^2 I_e$ and $\bar{\Delta}_2 = -2A + DN^2 h$, Theorem 1 can be modified as follows:

Theorem 3.

- (A) If $\bar{\Delta}_1 > 0$ and $\bar{\Delta}_2 \geq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_8^*)$ and $T^* = T_8^*$.
- (B) If $\bar{\Delta}_1 > 0$ and $\bar{\Delta}_2 < 0$ then $\text{TRC}(T^*) = \text{TRC}(T_7^*)$ and $T^* = T_7^*$.

(C) If $\bar{\Delta}_1 \leq 0$ and $\bar{\Delta}_2 < 0$, then $TRC(T^*) = TRC(T_6^*)$ and $T^* = T_6^*$.

Theorem 3 has been discussed in Theorem 1 of Huang (2003). Hence, Huang (2003) will be a special case of this paper.

4.2. Goyal's model

When $N = 0$, it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his/her customer. That is one level of trade credit. Therefore, when $s = c$, $\alpha = 0$, and $N = 0$, let

$$TRC_9(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k \left[\frac{D(T-M)^2}{2} \right] / T - cI_e \left(\frac{DM^2}{2} \right) / T, \tag{33}$$

$$TRC_{10}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e \left[\frac{DT^2}{2} + DT(M-T) \right] / T, \tag{34}$$

$$T_9^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h + cI_k)}}, \tag{35}$$

and

$$T_{10}^* = \sqrt{\frac{2A}{D(h + cI_e)}}. \tag{36}$$

Then $TRC'_i(T_i^*) = 0$ for $i = 9, 10$. Eqs. (1a–c) will be reduced as follows:

$$TRC(T) = \begin{cases} TRC_9(T) & \text{if } M \leq T, \\ TRC_{10}(T) & \text{if } 0 < T \leq M. \end{cases} \tag{37a–b}$$

Eqs. (37a,b) will be consistent with Eqs. (1) and (4) in Goyal (1985), respectively. Eq. (21) can be modified as $\Delta_1 = -2A + DM^2(h + cI_e)$. If we let

Table 1
Optimal solutions when $M \geq N$

α	N	s	Δ_1	Δ_2	Theorem	T^*	Q^*	$TRC(T^*)$
Let $A = \$80/\text{order}$, $D = 2000 \text{ units/year}$, $c = \$10/\text{unit}$, $h = \$7/\text{unit/year}$, $I_k = \$0.15/\text{year}$, $I_e = \$0.13/\text{year}$, and $M = 0.1 \text{ year}$.								
0.1	0.02	10	> 0	< 0	1-(B)	$T_2^* = 0.09846$	196.9	1374.48
		30	> 0	< 0	1-(B)	$T_2^* = 0.08642$	172.8	1103.94
		50	> 0	< 0	1-(B)	$T_2^* = 0.0781$	156.2	808.64
	0.05	10	> 0	< 0	1-(B)	$T_2^* = 0.09995$	199.9	1399.25
		30	> 0	< 0	1-(B)	$T_2^* = 0.09025$	180.5	1187.4
		50	> 0	< 0	1-(B)	$T_2^* = 0.08372$	167.4	960.48
	0.08	10	< 0	< 0	1-(A)	$T_1^* = 0.10261$	205.2	1444.3
		30	> 0	< 0	1-(B)	$T_2^* = 0.09696$	193.9	1333.63
		50	> 0	< 0	1-(B)	$T_2^* = 0.09327$	186.5	1218.29
0.5	0.02	10	> 0	< 0	1-(B)	$T_2^* = 0.09834$	196.7	1372.37
		30	> 0	< 0	1-(B)	$T_2^* = 0.08609$	172.2	1096.7
		50	> 0	< 0	1-(B)	$T_2^* = 0.0776$	155.2	795.28
	0.05	10	> 0	< 0	1-(B)	$T_2^* = 0.09917$	198.3	1386.19
		30	> 0	< 0	1-(B)	$T_2^* = 0.08824$	176.5	1143.68
		50	> 0	< 0	1-(B)	$T_2^* = 0.08079$	161.6	881.46
	0.08	10	< 0	< 0	1-(A)	$T_1^* = 0.10068$	201.4	1411.56
		30	> 0	< 0	1-(B)	$T_2^* = 0.09211$	184.2	1228.02
		50	> 0	< 0	1-(B)	$T_2^* = 0.08641$	172.8	1033.07
0.9	0.02	10	> 0	< 0	1-(B)	$T_2^* = 0.09821$	196.4	1370.25
		30	> 0	< 0	1-(B)	$T_2^* = 0.08575$	171.5	1089.44
		50	> 0	< 0	1-(B)	$T_2^* = 0.07711$	154.2	781.84
	0.05	10	> 0	< 0	1-(B)	$T_2^* = 0.09838$	196.8	1373.03
		30	> 0	< 0	1-(B)	$T_2^* = 0.08619$	172.4	1098.97
		50	> 0	< 0	1-(B)	$T_2^* = 0.07776$	155.5	799.46
	0.08	10	> 0	< 0	1-(B)	$T_2^* = 0.09869$	197.4	1378.18
		30	> 0	< 0	1-(B)	$T_2^* = 0.087$	174	1116.53
		50	> 0	> 0	1-(C)	$T_3^* = 0.0789$	157.8	831.81

$\Delta = -2A + DM^2(h + CI_c)$, Theorem 1 can be modified as follows:

Theorem 4.

- (A) If $\Delta > 0$, then $T^* = T_{10}^*$.
- (B) If $\Delta < 0$, then $T^* = T_9^*$.
- (C) If $\Delta = 0$, then $T^* = T_9^* = T_{10}^* = M$.

Theorem 4 has been discussed in Theorem 1 of Chung (1998). Hence, Goyal (1985) will be a special case of this paper.

5. Numerical examples

To illustrate the results developed in this paper, let us apply the proposed method to solve the following numerical examples. For convenience, the values of the parameters are selected randomly. The

optimal solutions for different parameters of α , N , and s when $M \geq N$ and $M < N$ are shown in Tables 1 and 2, respectively. The following inferences can be made based on Tables 1 and 2.

- (1) For fixed N and s , the larger the value of α , the smaller the value of the optimal cycle time and the lower the value of the annual total relevant cost.
- (2) For fixed α and s , the larger the value of N , the larger the value of the optimal cycle time and the higher the value of the annual total relevant cost when $M \geq N$; the optimal cycle time and the annual total relevant cost will be independent of N when $M < N$.
- (3) Finally, for fixed α and N , the larger the value of s , the smaller value of the optimal cycle time and the smaller the value of the annual total relevant cost.

Table 2
Optimal solutions when $M < N$

α	N	s	Δ_3	Theorem	T^*	Q^*	TRC(T^*)
Let $A = \$80/\text{order}$, $D = 5000 \text{ units/year}$, $c = \$10/\text{unit}$, $h = \$10/\text{unit/year}$, $I_k = \$0.1/\text{\$/year}$, $I_c = \$0.2/\text{\$/year}$, and $M = 0.05 \text{ year}$.							
0.1	0.06	10	<0	2-(B)	$T_4^* = 0.0556$	278	2807.77
		30	<0	2-(B)	$T_4^* = 0.05477$	273.9	2762.47
		50	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
	0.08	10	<0	2-(B)	$T_4^* = 0.0556$	278	2807.77
		30	<0	2-(B)	$T_4^* = 0.05477$	273.9	2762.47
		50	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
	0.1	10	<0	2-(B)	$T_4^* = 0.0556$	278	2807.77
		30	<0	2-(B)	$T_4^* = 0.05477$	273.9	2762.47
		50	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
0.5	0.06	10	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
		30	>0	2-(A)	$T_5^* = 0.04961$	248.1	2474.9
		50	>0	2-(A)	$T_5^* = 0.04619$	231	2214.1
	0.08	10	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
		30	>0	2-(A)	$T_5^* = 0.04961$	248.1	2474.9
		50	>0	2-(A)	$T_5^* = 0.04619$	231	2214.1
	0.1	10	<0	2-(B)	$T_4^* = 0.05394$	269.7	2716.48
		30	>0	2-(A)	$T_5^* = 0.04961$	248.1	2474.9
		50	>0	2-(A)	$T_5^* = 0.04619$	231	2214.1
0.9	0.06	10	<0	2-(B)	$T_4^* = 0.05222$	261.1	2622.28
		30	>0	2-(A)	$T_5^* = 0.04558$	229.9	2159.99
		50	>0	2-(A)	$T_5^* = 0.04104$	205.2	1648.72
	0.08	10	<0	2-(B)	$T_4^* = 0.05222$	261.1	2622.28
		30	>0	2-(A)	$T_5^* = 0.04558$	229.9	2159.99
		50	>0	2-(A)	$T_5^* = 0.04104$	205.2	1648.72
	0.1	10	<0	2-(B)	$T_4^* = 0.05222$	261.1	2622.28
		30	>0	2-(A)	$T_5^* = 0.04558$	229.9	2159.99
		50	>0	2-(A)	$T_5^* = 0.04104$	205.2	1648.72

6. Conclusions

This paper extends the assumption of the two levels of trade credit policy in the previously published result to investigate that the inventory problem of the retailer has powerful decision-making right. Theorems 1 and 2 help the retailer accurately and speedily to determine the optimal ordering policy after computing the numbers Δ_1 , Δ_2 , and Δ_3 . Huang's model (2003) and Goyal's model (1985) are the special cases of this extended model discussed in this paper. Finally, numerical examples are given to illustrate the results developed in this paper. There are some managerial phenomena as follows:

- (1) When the customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer is increasing, the retailer will order less quantity and increase order frequency. The retailer can accumulate more interest under higher order frequency and higher customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer.
- (2) When the customer's trade credit period offered by the retailer is increasing, the retailer will order more quantity to accumulate more interest to compensate the loss of interest earned when longer trade credit period is offered to his/her customer under the condition of $M \geq N$.
- (3) When the unit selling price is increasing, the retailer will order less quantity to take the benefits of the trade credit more frequently.

A future study will further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, allowable shortages, deteriorating items, or finite replenishment rate.

Acknowledgments

The authors would like to thank anonymous referees for their valuable and constructive comments and suggestions that have led to significant improvement on an earlier version of this paper. The NSC in Taiwan supported this research, and the project no. was NSC 94-2416-H-324-003.

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