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An inventory model under two levels of trade credit and limited storage space derived without derivatives

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Abstract

This paper tries to incorporate both Huang's model [Y.F. Huang, Optimal retailer's ordering policies in the EOQ model under trade credit financing, J. Oper. Res. Soc. 54 (2003) 1011–1015] and Teng's model [J.T. Teng, On the economic order quantity under conditions of permissible delay in payments, J. Oper. Res. Soc. 53 (2002) 915–918] by considering the retailer's storage space limited to reflect the real-life situations. That is, we want to investigate the retailer's inventory policy under two levels of trade credit and limited storage space. Furthermore, we adopt Teng's viewpoint [J.T. Teng, On the economic order quantity under conditions of permissible delay in payments, J. Oper. Res. Soc. 53 (2002) 915–918] that the retailer's unit selling price and the purchasing price per unit are not necessarily equal. Then, an algebraic approach is provided and three easy-to-use theorems are developed to efficiently determine the optimal cycle time. Some previously published results of other researchers can be deduced as special cases. Finally, a numerical example is given to illustrate these theorems and managerial insights are drawn.

Keywords: Inventory; Two levels of trade credit; Derivatives; Limited storage space

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1. Introduction

Typically, inventory planning takes into account only data from the operations concerns. Therefore, the interdependencies among the operations, financing and marketing concerns are neglected. In most business transactions, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of his/her products. This credit term in financial management is denoted as "net 30". Teng [1] indicated that the trade credit produces two benefits to the supplier: (1) it should attract new customers who consider it to be a type of price reduction; and (2) it should cause a reduction in sales outstanding, since some established customers will pay more promptly in order to take advantage of trade credit more frequently. Over the years, a number of researchers have appeared in the literature that treat inventory problems with varying conditions under trade credit intended to link financing, marketing as well as operations concerns. Some of the prominent papers are discussed below.

Goyal [2] established a single-item inventory model under permissible delay in payments. Chung [3] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi [4] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Liao et al. [5] and Sarker et al. [6] investigated this topic in the presence of inflation. Jamal et al. [7] and Chang and Dye [8] extended this issue with allowable shortage. Chang et al. [9] extended this issue with linear trend demand. Hwang and Shinn [10] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal et al. [11] and Sarker et al. [12] addressed the optimal payment time under permissible delay in payment with deterioration. Teng [1] assumed that the selling price is not equal to the purchasing price to modify Goyal's model [2]. Chung and Huang [13] examined this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang and Chung [14] extended Goyal's model [2] to cash discount policy for early payment. Arcelus et al. [15] modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. Abad and Jaggi [16] developed a joint approach to determine for the seller the optimal unit price and the length of the credit period when end demand is price sensitive. Salameh et al. [17] extended this issue to continuous review inventory model. Shinn and Hwang [18] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. Chang et al. [19] and Chung and Liao [20] deal with the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity.

All previously published models discussed trade credit assumed that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to his/her customer. That is one level of trade credit. Recently, Huang [21] modified this assumption to assume that the retailer will adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. That is two levels of trade credit. This new viewpoint is more matched real-life situations in the supply chain model. For example, the Toyota Company can require his supplier offered the trade credit to him and he must also offer the trade credit to his dealership. Then, the Toyota Company can offer shorter delay period to his dealership than his supplier offered to him. In this transaction, the Toyota Company can obtain maximum advantages. But Huang [21] implicitly assumed that the unit selling price and the purchasing price per unit of the retailer are equal. However, as we know, the unit selling price for the retailer is usually significantly higher than the purchasing price per unit in order to obtain profit. Consequently, the viewpoint of Huang [21] can be extended.

Such trade credit policy is one kind of encouragement for the retailer to order large quantities because a delay of payments indirectly reduces inventory cost. Hence, the retailer may purchase more goods than that can be stored in his/her own warehouse (OW). These excess quantities are stored in a rented warehouse (RW). The proposed model is applicable for the business of small and medium sized retailers since their storage capacity are small and limited. Especially, Taiwan has traditionally relied on its small and medium sized firms to compete in international markets since the 1970s. Therefore, this proposed model is more applicable for the special industrial environment in Taiwan. In general, the inventory holding charges in RW are higher than those in OW. When the demand occurs, it first is replenished from the RW which storages those exceeding items. This is done to reduce the inventory costs. It is further assumed that the transportation costs between warehouses are negligible. Several researchers have studied in this area such as Benkherouf [22], Bhunia and Maiti [23], Goswami and Chaudhuri [24], Pakkala and Achary [25], Sarma [26] and Wu [27].

Therefore, this paper tries to incorporate both Huang's model [21] and Teng's model [1] by considering the retailer's storage space limited. That is, we want to investigate the retailer's inventory policy under two levels of trade credit and limited storage space. Furthermore, we adopt Teng's viewpoint [1] that the retailer's unit selling price and the purchasing price per unit are not necessarily equal. In addition, we try to use the more easily algebraic approach to find the optimal solution in this paper. In recent papers, Cárdenas-Barrón [28] and Grubbström and Erdem [29] showed that the formulae for the EOQ and EPQ with backlogging could be derived without differential calculus. Yang and Wee [30] developed algebraically the optimal replenishment policy of the integrated vendor–buyer inventory system without using differential calculus. Wu and Ouyang [31] modify Yang and Wee [30] to allow shortages using algebraic method.

Consequently, this paper deals with the retailer's inventory replenishment problem under two levels of trade credit and limited storage space derived without derivatives. In addition, we develop the easy-to-use procedures to efficiently find the optimal cycle time for the retailer under minimizing annual total relevant cost. Finally, a numerical example is given to illustrate these results and managerial insights are drawn.

2. Model formulation

In this section, we want to develop the retailer's inventory model under two levels of trade credit and limited storage space. For convenience, most notation and assumptions similar to Huang [21] will be used in this paper.

2.1. Notation

A	ordering cost per order
c	purchasing price per item
D	demand rate per year
h	OW stock-holding cost per item per year
I_e	interest earned per \$ per year
I_p	interest charged per \$ in stocks per year by the supplier
k	RW stock-holding cost per item per year
M	the retailer's trade credit period offered by supplier in years
N	the customer's trade credit period offered by retailer in years
s	selling price per item
t_w	the rented warehouse time in years, $t_w = \begin{cases} \dfrac{DT - W}{D} & \text{if } DT > W \left(T > \dfrac{W}{D}\right) \\ 0 & \text{if } DT \leqslant W \left(T \leqslant \dfrac{W}{D}\right) \end{cases}$
T	the cycle time in years
W	retailer's OW storage capacity
$\mathrm{TRC}(T)$	the annual total relevant cost, which is a function of T
T^*	the optimal cycle time of $\mathrm{TRC}(T)$

2.2. Assumptions

(1) Demand rate is known and constant.
(2) Shortages are not allowed.
(3) Time horizon is infinite.
(4) Replenishments are instantaneous.
(5) $s \geqslant c$, $k \geqslant h$, $I_p \geqslant I_e$ and $M \geqslant N$.
(6) If the order quantity is larger than retailer's OW storage capacity W, the retailer will rent the warehouse to storage these exceeding items. And the RW storage capacity is unlimited. When the demand occurs, it first is replenished from the RW which storages those exceeding items.
(7) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When $T \geqslant M$, the account is settled at $T = M$, the retailer pays off all units sold and keeps his/her profits, and starts paying for the higher interest charges on the items in stock with rate I_p. When $T \leqslant M$, the account is settled at $T = M$ and the retailer does not need to pay any interest charge.
(8) The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

2.3. The model

The total annual relevant cost consists of the following elements. Three situations may arise. (I) $M \geqslant N \geqslant W/D$, (II) $M \geqslant W/D \geqslant N$ and (III) $W/D \geqslant M \geqslant N$.

Case I: Suppose that $M \geqslant N \geqslant W/D$.

(1) Annual ordering cost $= \frac{A}{T}$.

(2) According to assumption (6), annual stock-holding cost (excluding interest charges) can be obtained as follows:

(i) $W/D < T$.

In this case, the order quantity is larger than retailer's OW storage capacity. So the retailer needs to rent the warehouse to storage the exceeding items. Hence

Annual stock-holding cost = annual stock-holding cost of rented warehouse + annual stock-holding cost of the storage capacity W

$$= \frac{kt_{\mathrm{w}}(DT - W)}{2T} + \frac{h\left[Wt_{\mathrm{w}} + \frac{W(T-t_{\mathrm{w}})}{2}\right]}{T} = \frac{k(DT - W)^2}{2DT} + \frac{hW(2DT - W)}{2DT}.$$

(ii) $T \leqslant W/D$.

In this case, the order quantity is not larger than retailer's storage capacity. So the retailer will not necessary to rent warehouse to storage items. Hence

Annual stock-holding cost $= \frac{DTh}{2}$.

(3) According to assumption (7), cost of interest charges for the items kept in stock per year can be obtained as follows:

(i) $M \leqslant T$, as shown in Fig. 1.

Cost of interest charges for the items kept in stock per year $= \frac{cI_{\mathrm{p}}D(T-M)^2}{2T}$.

(ii) $T \leqslant M$, as shown in Fig. 2.

In this case, no interest charges are paid for the items kept in stock.

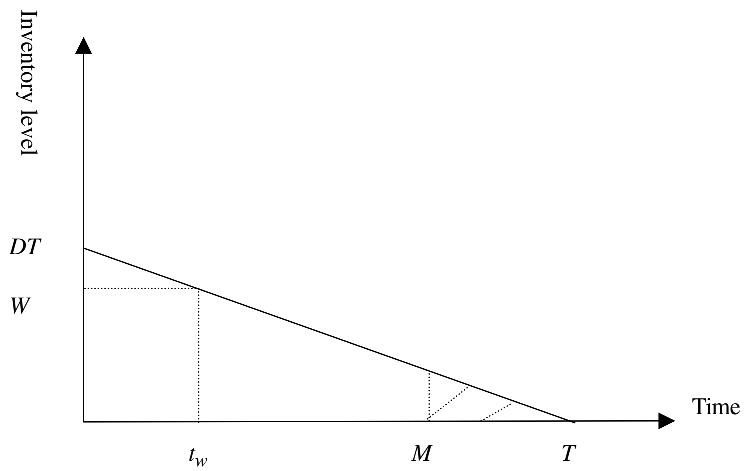

Fig. 1. The inventory level and the total accumulation of interest payable when $M \leqslant T$.

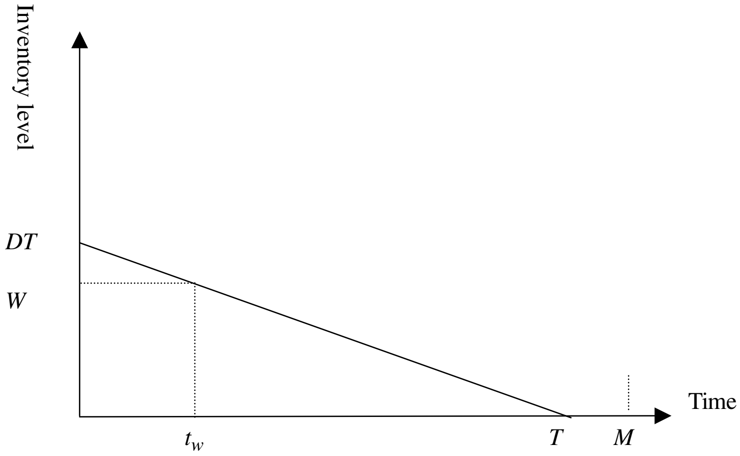

Fig. 2. The inventory level when $W/D < T \leqslant M$.

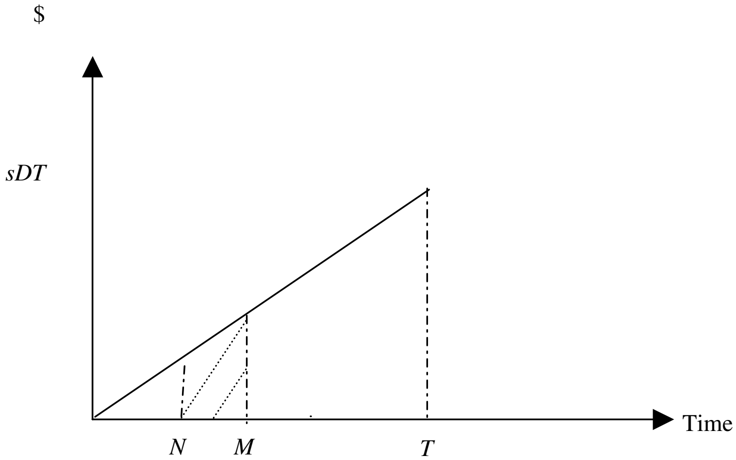

Fig. 3. The total accumulation of interest earned when $M \leqslant T$.

(4) According to assumption (8), interest earned per year can be obtained as follows:
  (i) $M \leqslant T$, as shown in Fig. 3.
  Annual interest earned $= sI_e\left[\frac{(DN+DM)(M-N)}{2}\right]/T = sI_eD(M^2 - N^2)/2T$.
  (ii) $N \leqslant T \leqslant M$, as shown in Fig. 4.
  Annual interest earned $= sI_e\left[\frac{(DN+DT)(T-N)}{2} + DT(M-T)\right]/T = sI_eD(2MT - N^2 - T^2)/2T$.
  (iii) $0 < T \leqslant N$, as shown in Fig. 5.
  Annual interest earned $= sI_eDT(M-N)/T$.

From the above arguments, the annual total relevant cost for the retailer can be expressed as TRC(T) = ordering cost + stock-holding cost + interest payable − interest earned.

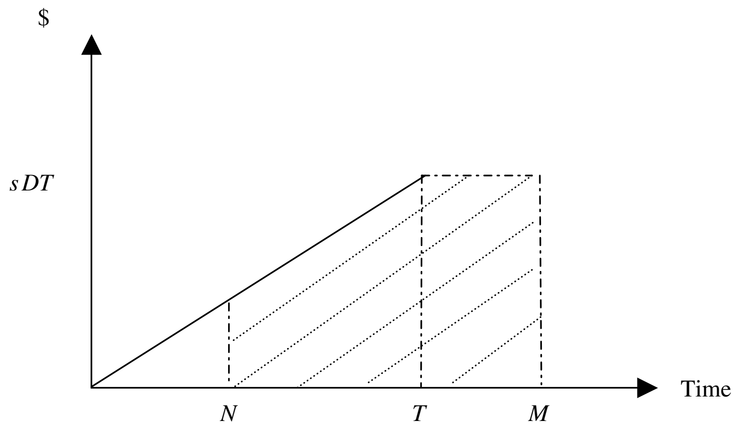

Fig. 4. The total accumulation of interest when $N \leqslant T \leqslant M$.

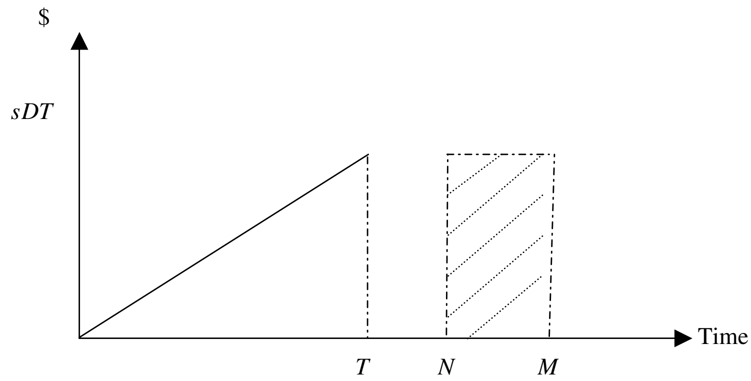

Fig. 5. The total accumulation of interest when $T \leqslant N$.

We show that the annual total relevant cost, TRC(T), is given by

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T \geqslant M, \quad \text{(a)} \\ \mathrm{TRC}_2(T) & \text{if } N \leqslant T \leqslant M, \quad \text{(b)} \\ \mathrm{TRC}_3(T) & \text{if } W/D < T \leqslant N, \quad \text{(c)} \\ \mathrm{TRC}_4(T) & \text{if } 0 < T \leqslant W/D, \quad \text{(d)} \end{cases} \tag{1}$$

where

$$\mathrm{TRC}_1(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT} + \frac{cI_\mathrm{p}D(T-M)^2}{2T} - \frac{sI_\mathrm{e}D(M^2-N^2)}{2T}, \tag{2}$$

$$\mathrm{TRC}_2(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT} - \frac{sI_\mathrm{e}D(2MT-N^2-T^2)}{2T}, \tag{3}$$

$$\mathrm{TRC}_3(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT} - sI_\mathrm{e}D(M-N) \tag{4}$$

and

$$\mathrm{TRC}_4(T) = \frac{A}{T} + \frac{DTh}{2} - sI_eD(M-N). \tag{5}$$

Since $\mathrm{TRC}_1(M) = \mathrm{TRC}_2(M)$, $\mathrm{TRC}_2(N) = \mathrm{TRC}_3(N)$ and $\mathrm{TRC}_3(W/D) = \mathrm{TRC}_4(W/D)$, $\mathrm{TRC}(T)$ is continuous and well defined on $T > 0$. All $\mathrm{TRC}_1(T)$, $\mathrm{TRC}_2(T)$, $\mathrm{TRC}_3(T)$, $\mathrm{TRC}_4(T)$ and $\mathrm{TRC}(T)$ are defined on $T > 0$.

Case II: Suppose that $M \geqslant W/D \geqslant N$.
If $M \geqslant W/D \geqslant N$, Eqs. 1(a)–(d) will be modified as

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T \geqslant M, & \text{(a)} \\ \mathrm{TRC}_2(T) & \text{if } W/D < T \leqslant M, & \text{(b)} \\ \mathrm{TRC}_5(T) & \text{if } N \leqslant T \leqslant W/D, & \text{(c)} \\ \mathrm{TRC}_4(T) & \text{if } 0 < T \leqslant N. & \text{(d)} \end{cases} \tag{6}$$

When $N \leqslant T \leqslant W/D$, the annual total relevant cost, $\mathrm{TRC}_5(T)$, consists of the following elements:

(1) Annual ordering cost $= \frac{A}{T}$.
(2) In this case, the order quantity is not larger than retailer's storage capacity. So the retailer will not necessary to rent warehouse to storage items. Hence
Annual stock-holding cost $= \frac{DTh}{2}$.
(3) In this case, no interest charges are paid for the items kept in stock.
(4) Annual interest earned $= sI_eD(2MT - N^2 - T^2)/2T$.
Combining above elements, we get

$$\mathrm{TRC}_5(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{sI_eD(2MT - N^2 - T^2)}{2T}. \tag{7}$$

Since $\mathrm{TRC}_1(M) = \mathrm{TRC}_2(M)$, $\mathrm{TRC}_2(W/D) = \mathrm{TRC}_5(W/D)$ and $\mathrm{TRC}_5(N) = \mathrm{TRC}_4(N)$, $\mathrm{TRC}(T)$ is continuous and well defined on $T > 0$. All $\mathrm{TRC}_1(T)$, $\mathrm{TRC}_2(T)$, $\mathrm{TRC}_5(T)$, $\mathrm{TRC}_4(T)$ and $\mathrm{TRC}(T)$ are defined on $T > 0$.

Case III: Suppose that $W/D \geqslant M \geqslant N$.
If $W/D \geqslant M \geqslant N$, Eqs. 1(a)–(d) will be modified as

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T > W/D, & \text{(a)} \\ \mathrm{TRC}_6(T) & \text{if } M \leqslant T \leqslant W/D, & \text{(b)} \\ \mathrm{TRC}_5(T) & \text{if } N \leqslant T \leqslant M, & \text{(c)} \\ \mathrm{TRC}_4(T) & \text{if } 0 < T \leqslant N. & \text{(d)} \end{cases} \tag{8}$$

When $M \leqslant T \leqslant W/D$, the annual total relevant cost, $\mathrm{TRC}_6(T)$, consists of the following elements:

(1) Annual ordering cost $= \frac{A}{T}$.
(2) In this case, the order quantity is not larger than retailer's storage capacity. So the retailer will not necessary to rent warehouse to storage items. Hence
Annual stock-holding cost $= \frac{DTh}{2}$.

(3) Cost of interest charges for the items kept in stock per year $= \frac{cI_pD(T-M)^2}{2T}$.
(4) Annual interest earned $= sI_eD(M^2 - N^2)/2T$.
Combining above elements, we get

$$\mathrm{TRC}_6(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_pD(T-M)^2}{2T} - \frac{sI_eD(M^2-N^2)}{2T}. \quad (9)$$

Since $\mathrm{TRC}_1(W/D) = \mathrm{TRC}_6(W/D)$, $\mathrm{TRC}_6(M) = \mathrm{TRC}_5(M)$ and $\mathrm{TRC}_5(N) = \mathrm{TRC}_4(N)$, $\mathrm{TRC}(T)$ is continuous and well defined on $T > 0$. All $\mathrm{TRC}_1(T)$, $\mathrm{TRC}_6(T)$, $\mathrm{TRC}_5(T)$, $\mathrm{TRC}_4(T)$ and $\mathrm{TRC}(T)$ are defined on $T > 0$.

3. Determination of the optimal cycle time *T**

In this section, we shall determine optimal cycle time for the above three situations under minimizing annual total relevant cost using algebraic method.

Case I: Suppose that $M \geqslant N \geqslant W/D$.
Then, we can rewrite

$$\begin{aligned}\mathrm{TRC}_1(T) &= \frac{2A + \frac{W^2}{D}(k-h) + D[M^2(cI_p - sI_e) + N^2sI_e]}{2T} + \frac{DT(k+cI_p)}{2} - [W(k-h) + cDMI_p] \\ &= \frac{D(k+cI_p)}{2T}\left[T - \sqrt{\frac{2A + \frac{W^2}{D}(k-h) + D[M^2(cI_p - sI_e) + N^2sI_e]}{D(k+cI_p)}}\right]^2 \\ &\quad + \left\{\sqrt{D(k+cI_p)\left[2A + \frac{W^2}{D}(k-h) + D(M^2(cI_p - sI_e) + N^2sI_e)\right]} \right. \\ &\quad \left. - [W(k-h) + cDMI_p]\right\}. \end{aligned} \quad (10)$$

From Eq. (10) the minimum of $\mathrm{TRC}_1(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_1^* is

$$T_1^* = \sqrt{\frac{2A + \frac{W^2}{D}(k-h) + D[M^2(cI_p - sI_e) + N^2sI_e]}{D(k+cI_p)}} \quad \text{if } 2A + \frac{W^2}{D}(k-h) + D[M^2(cI_p - sI_e) + N^2sI_e] > 0. \quad (11)$$

Therefore,

$$\mathrm{TRC}_1(T_1^*) = \left\{\sqrt{D(k+cI_p)\left[2A + \frac{W^2}{D}(k-h) + D(M^2(cI_p - sI_e) + N^2sI_e)\right]} - [W(k-h) + cDMI_p]\right\}. \quad (12)$$

Similarly, we can derive $\mathrm{TRC}_2(T)$ without derivatives as follows:

$$\begin{aligned}\mathrm{TRC}_2(T) &= \frac{2A+\frac{W^2}{D}(k-h)+sDN^2I_e}{2T}+\frac{DT(k+sI_e)}{2}-[W(k-h)+sDMI_e]\\ &= \frac{D(k+sI_e)}{2T}\left[T-\sqrt{\frac{2A+\frac{W^2}{D}(k-h)+sDN^2I_e}{D(k+sI_e)}}\right]^2\\ &\quad+\left\{\sqrt{D(k+sI_e)\left[2A+\frac{W^2}{D}(k-h)+sDN^2I_e\right]}-[W(k-h)+sDMI_e]\right\}. \qquad (13)\end{aligned}$$

From Eq. (13) the minimum of $\mathrm{TRC}_2(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_2^* is

$$T_2^* = \sqrt{\frac{2A+\frac{W^2}{D}(k-h)+sDN^2I_e}{D(k+sI_e)}}. \qquad (14)$$

Therefore,

$$\mathrm{TRC}_2(T_2^*) = \left\{\sqrt{D(k+sI_e)\left[2A+\frac{W^2}{D}(k-h)+sDN^2I_e\right]}-[W(k-h)+sDMI_e]\right\}. \qquad (15)$$

Likewise, we can derive $\mathrm{TRC}_3(T)$ algebraically as follows:

$$\begin{aligned}\mathrm{TRC}_3(T) &= \frac{2A+\frac{W^2}{D}(k-h)}{2T}+\frac{kDT}{2}-[W(k-h)+sDI_e(M-N)]\\ &= \frac{kD}{2T}\left[T-\sqrt{\frac{2A+\frac{W^2}{D}(k-h)}{kD}}\right]^2\\ &\quad+\left\{\sqrt{kD\left[2A+\frac{W^2}{D}(k-h)\right]}-[W(k-h)+sDI_e(M-N)]\right\}. \qquad (16)\end{aligned}$$

From Eq. (16) the minimum of $\mathrm{TRC}_3(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_3^* is

$$T_3^* = \sqrt{\frac{2A+\frac{W^2}{D}(k-h)}{kD}}. \qquad (17)$$

Therefore,

$$\mathrm{TRC}_3(T_3^*) = \left\{\sqrt{kD\left[2A+\frac{W^2}{D}(k-h)\right]}-[W(k-h)+sDI_e(M-N)]\right\}. \qquad (18)$$

At last, we can derive $\mathrm{TRC}_4(T)$ algebraically as follows:

$$\mathrm{TRC}_4(T) = \frac{A}{T} + \frac{DTh}{2} - sI_eD(M-N) = \frac{Dh}{2T}\left[T - \sqrt{\frac{2A}{Dh}}\right]^2 + \left[\sqrt{2ADh} - sDI_e(M-N)\right]. \quad (19)$$

From Eq. (19) the minimum of $\mathrm{TRC}_4(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_4^* is

$$T_4^* = \sqrt{\frac{2A}{Dh}}. \quad (20)$$

Therefore,

$$\mathrm{TRC}_4(T_4^*) = \left[\sqrt{2ADh} - sDI_e(M-N)\right]. \quad (21)$$

Eq. (11) gives the optimal value of T^* for the case when $T \geqslant M$ so that $T_1^* \geqslant M$. We substitute Eq. (11) into $T_1^* \geqslant M$, then we can obtain that

$$T_1^* \geqslant M \quad \text{if and only if} \quad -2A - \frac{W^2}{D}(k-h) + DM^2(k+sI_e) - sDN^2I_e \leqslant 0.$$

Similarly, Eq. (14) gives the optimal value of T^* for the case when $N \leqslant T \leqslant M$ so that $N \leqslant T_2^* \leqslant M$. We substitute Eq. (14) into $N \leqslant T_2^* \leqslant M$, then we can obtain that

$$T_2^* \leqslant M \quad \text{if and only if} \quad -2A - \frac{W^2}{D}(k-h) + DM^2(k+sI_e) - sDN^2I_e \geqslant 0$$

and

$$N \leqslant T_2^* \quad \text{if and only if} \quad -2A - \frac{W^2}{D}(k-h) + DN^2k \leqslant 0.$$

Likewise, Eq. (17) gives the optimal value of T^* for the case when $W/D < T \leqslant N$ so that $W/D < T_3^* \leqslant N$. We substitute Eq. (17) into $W/D < T_3^* \leqslant N$, then we can obtain that

$$T_3^* \leqslant N \quad \text{if and only if} \quad -2A - \frac{W^2}{D}(k-h) + DN^2k \geqslant 0$$

and

$$W/D < T_3^* \quad \text{if and only if} \quad -2A + \frac{W^2}{D}h < 0.$$

Finally, Eq. (20) gives the optimal value of T^* for the case when $T \leqslant W/D$ so that $T_4^* \leqslant W/D$. We substitute Eq. (20) into $T_4^* \leqslant W/D$, then we can obtain that

$$T_4^* \leqslant W/D \quad \text{if and only if} \quad -2A + \frac{W^2}{D}h \geqslant 0.$$

Furthermore, we let

$$\Delta_1 = -2A - \frac{W^2}{D}(k-h) + DM^2(k+sI_e) - sDN^2I_e, \quad (22)$$

$$\Delta_2 = -2A - \frac{W^2}{D}(k-h) + DN^2k \quad (23)$$

and

$$\Delta_3 = -2A + \frac{W^2}{D}h. \tag{24}$$

Eqs. (22)–(24) imply that $\Delta_1 \geqslant \Delta_2 \geqslant \Delta_3$. From above arguments, we can obtain following results.

Theorem 1. *Suppose that* $M \geqslant N \geqslant W/D$, *then*

(A) *If* $\Delta_3 \geqslant 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_4^*)$ *and* $T^* = T_4^*$.
(B) *If* $\Delta_2 \geqslant 0$ *and* $\Delta_3 < 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_3^*)$ *and* $T^* = T_3^*$.
(C) *If* $\Delta_1 > 0$ *and* $\Delta_2 < 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_2^*)$ *and* $T^* = T_2^*$.
(D) *If* $\Delta_1 \leqslant 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_1^*)$ *and* $T^* = T_1^*$.

Case II: Suppose that $M \geqslant W/D \geqslant N$.

If $M \geqslant W/D \geqslant N$, we know TRC($T$) as follows from Eqs. 6(a)–(d):

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T \geqslant M, \\ \mathrm{TRC}_2(T) & \text{if } W/D < T \leqslant M, \\ \mathrm{TRC}_5(T) & \text{if } N \leqslant T \leqslant W/D, \\ \mathrm{TRC}_4(T) & \text{if } 0 < T \leqslant N. \end{cases}$$

From Eq. (7), we can derive $\mathrm{TRC}_5(T)$ without derivatives as follows:

$$\begin{aligned}\mathrm{TRC}_5(T) &= \frac{2A + sDN^2I_e}{2T} + \frac{DT(h+sI_e)}{2} - sDMI_e \\ &= \frac{D(h+sI_e)}{2T}\left[T - \sqrt{\frac{2A + sDN^2I_e}{D(h+sI_e)}}\right]^2 \\ &\quad + \left[\sqrt{D(h+sI_e)(2A + sDN^2I_e)} - sDMI_e\right].\end{aligned} \tag{25}$$

From Eq. (25) the minimum of $\mathrm{TRC}_5(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_5^* is

$$T_5^* = \sqrt{\frac{2A + sDN^2I_e}{D(h+sI_e)}}. \tag{26}$$

Therefore,

$$\mathrm{TRC}_5(T_5^*) = \left[\sqrt{D(h+sI_e)(2A + sDN^2I_e)} - sDMI_e\right]. \tag{27}$$

Similar to the above procedure in Case I. We substitute Eq. (11) into $T_1^* \geqslant M$, then we can obtain that

$$T_1^* \geqslant M \quad \text{if and only if} \quad -2A - \frac{W^2}{D}(k-h) + DM^2(k+sI_e) - sDN^2I_e \leqslant 0.$$

Substituting Eq. (14) into $W/D < T_2^* \leqslant M$, then we can obtain that

$$T_2^* \leqslant M \quad \text{if and only if} \; -2A - \frac{W^2}{D}(k-h) + DM^2(k+sI_e) - sDN^2I_e \geqslant 0$$

and

$$W/D < T_2^* \quad \text{if and only if} \; -2A + \frac{W^2}{D}(h+sI_e) - sDN^2I_e < 0.$$

Substituting Eq. (26) into $N \leqslant T_5^* \leqslant W/D$, then we can obtain that

$$T_5* \leqslant W/D \quad \text{if and only if} -2A + \frac{W^2}{D}(h+sI_e) - sDN^2I_e \geqslant 0$$

and

$$N \leqslant T_5^* \quad \text{if and only if} \; -2A + DN^2h \leqslant 0.$$

Substituting Eq. (20) into $T_4^* \leqslant N$, then we can obtain that

$$T_4^* \leqslant N \quad \text{if and only if} \; -2A + DN^2h \geqslant 0.$$

Furthermore, we let

$$\Delta_4 = -2A + \frac{W^2}{D}(h+sI_e) - sDN^2I_e \tag{28}$$

and

$$\Delta_5 = -2A + DN^2h. \tag{29}$$

Eqs. (22), (28) and (29) imply that $\Delta_1 \geqslant \Delta_4 \geqslant \Delta_5$. From above arguments, we can obtain following results.

Theorem 2. *Suppose that* $M \geqslant W/D \geqslant N$, *then*

(A) *If* $\Delta_5 \geqslant 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_4^*)$ *and* $T^* = T_4^*$.
(B) *If* $\Delta_4 \geqslant 0$ *and* $\Delta_5 < 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_5^*)$ *and* $T^* = T_5^*$.
(C) *If* $\Delta_1 > 0$ *and* $\Delta_4 < 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_2^*)$ *and* $T^* = T_2^*$.
(D) *If* $\Delta_1 \leqslant 0$, *then* $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_1^*)$ *and* $T^* = T_1^*$.

Case III: Suppose that $W/D \geqslant M \geqslant N$.

If $W/D \geqslant M \geqslant N$, we know TRC(T) as follows from Eqs. 8(a)–(d).

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T > W/D, \\ \mathrm{TRC}_6(T) & \text{if } M \leqslant T \leqslant W/D, \\ \mathrm{TRC}_5(T) & \text{if } N \leqslant T \leqslant M, \\ \mathrm{TRC}_4(T) & \text{if } 0 < T \leqslant N. \end{cases}$$

From Eq. (9), we can derive $\mathrm{TRC}_6(T)$ without derivatives as follows:

$$\begin{aligned}\mathrm{TRC}_6(T) &= \frac{2A + D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}]}{2T} + \frac{DT(h + cI_\mathrm{p})}{2} - cDMI_\mathrm{p} \\ &= \frac{D(h + cI_\mathrm{p})}{2T}\left[T - \sqrt{\frac{2A + D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}]}{D(h + cI_\mathrm{p})}}\right]^2 \\ &\quad + \left\{\sqrt{D(h + cI_\mathrm{p})[2A + D(M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e})]} - cDMI_\mathrm{p}\right\}. \end{aligned} \tag{30}$$

From Eq. (30) the minimum of $\mathrm{TRC}_6(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_6^* is

$$T_6^* = \sqrt{\frac{2A + D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}]}{D(h + cI_\mathrm{p})}} \quad \text{if } 2A + D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}] > 0. \tag{31}$$

Therefore,

$$\mathrm{TRC}_6(T_6^*) = \left\{\sqrt{D(h + cI_\mathrm{p})[2A + D(M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e})]} - cDMI_\mathrm{p}\right\}. \tag{32}$$

Similar to the above procedures in Case I and Case II. We substitute Eq. (11) into $T_1^* > W/D$, then we can obtain that

$$T_1^* > W/D \quad \text{if and only if } \; -2A + \frac{W^2}{D}(h + cI_\mathrm{p}) - D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}] < 0.$$

Substituting Eq. (31) into $M \leqslant T_6^* \leqslant W/D$, then we can obtain that

$$T_6^* \leqslant W/D \quad \text{if and only if } \; -2A + \frac{W^2}{D}(h + cI_\mathrm{p}) - D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}] \geqslant 0$$

and

$$M \leqslant T_6^* \quad \text{if and only if } \; -2A + DM^2(h + sI_\mathrm{e}) - DN^2 sI_\mathrm{e} \leqslant 0.$$

Substituting Eq. (26) into $N \leqslant T_5^* \leqslant M$, then we can obtain that

$$T_5^* \leqslant M \quad \text{if and only if } \; -2A + DM^2(h + sI_\mathrm{e}) - DN^2 sI_\mathrm{e} \geqslant 0$$

and

$$N \leqslant T_5^* \quad \text{if and only if } \; -2A + DN^2 h \leqslant 0.$$

Substituting Eq. (20) into $T_4^* \leqslant N$, then we can obtain that

$$T_4^* \leqslant N \quad \text{if and only if } \; -2A + DN^2 h \geqslant 0.$$

Furthermore, we let

$$\Delta_6 = -2A + \frac{W^2}{D}(h + cI_\mathrm{p}) - D[M^2(cI_\mathrm{p} - sI_\mathrm{e}) + N^2 sI_\mathrm{e}] \tag{33}$$

and

$$\Delta_7 = -2A + DM^2(h + sI_e) - DN^2 sI_e. \tag{34}$$

Eqs. (29), (33) and (34) imply that $\Delta_6 \geqslant \Delta_7 \geqslant \Delta_5$. From above arguments, we can obtain following results.

Theorem 3. *Suppose that $W/D \geqslant M \geqslant N$, then*

(A) *If $\Delta_5 \geqslant 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_4^*)$ and $T^* = T_4^*$.*
(B) *If $\Delta_7 \geqslant 0$ and $\Delta_5 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_3^*)$ and $T^* = T_5^*$.*
(C) *If $\Delta_6 \geqslant 0$ and $\Delta_7 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_6^*)$ and $T^* = T_6^*$.*
(D) *If $\Delta_6 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_1^*)$ and $T^* = T_1^*$.*

4. Special cases

4.1. Huang's model

When $k = h$, it means that the RW unit stock-holding cost and the OW unit stock-holding cost are equal. It implies that the retailer's OW storage capacity is unlimited. And $s = c$, it means that the selling price per item and the purchasing cost per item are equal. Therefore, when $k = h$ and $s = c$, we let

$$\mathrm{TRC}_7(T) = \frac{A}{T} + \frac{DTh}{2} + cI_pD(T - M)^2/2T - cI_eD(M^2 - N^2)/2T, \tag{35}$$

$$\mathrm{TRC}_8(T) = \frac{A}{T} + \frac{DTh}{2} - cI_eD(2MT - N^2 - T^2)/2T \tag{36}$$

and

$$\mathrm{TRC}_9(T) = \frac{A}{T} + \frac{DTh}{2} - cI_eD(M - N). \tag{37}$$

Eqs. 1(a)–(d), 6(a)–(d), and 8(a)–(d) will be reduced as follows:

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_7(T) & \text{if } T \geqslant M, & \text{(a)} \\ \mathrm{TRC}_8(T) & \text{if } N \leqslant T \leqslant M, & \text{(b)} \\ \mathrm{TRC}_9(T) & \text{if } 0 < T \leqslant N. & \text{(c)} \end{cases} \tag{38}$$

Eqs. 38(a)–(c) will be consistent with Eqs. 1(a)–(c) in Huang [21], respectively. Hence, Huang [21] will be a special case of this paper.

4.2. Teng's model

When $N = 0$, it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his/her customer. That is one level of trade credit. Therefore, when $k = h$ and $N = 0$, we let

$$\mathrm{TRC}_{10}(T) = \frac{A}{T} + \frac{DTh}{2} + cI_{\mathrm{p}}\left[\frac{D(T-M)^2}{2}\right]\Big/T - sI_{\mathrm{e}}\left(\frac{DM^2}{2}\right)\Big/T \tag{39}$$

and

$$\mathrm{TRC}_{11}(T) = \frac{A}{T} + \frac{DTh}{2} - sI_{\mathrm{e}}\left[\frac{DT^2}{2} + DT(M-T)\right]\Big/T. \tag{40}$$

Eqs. 1(a)–(d), 6(a)–(d) and 8(a)–(d) will be reduced as follows:

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_{10}(T) & \text{if } M \leqslant T, & \text{(a)} \\ \mathrm{TRC}_{11}(T) & \text{if } 0 < T \leqslant M. & \text{(b)} \end{cases} \tag{41}$$

Eqs. 41(a) and (b) will be consistent with Eqs. (1) and (2) in Teng [1], respectively. Hence, Teng [1] will be a special case of this paper.

4.3. Goyal's model

When $k = h$, $s = c$ and $N = 0$, we let

$$\mathrm{TRC}_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + cI_{\mathrm{p}}\left[\frac{D(T-M)^2}{2}\right]\Big/T - cI_{\mathrm{e}}\left(\frac{DM^2}{2}\right)\Big/T \tag{42}$$

and

$$\mathrm{TRC}_{13}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_{\mathrm{e}}\left[\frac{DT^2}{2} + DT(M-T)\right]\Big/T. \tag{43}$$

Eqs. 1(a)–(d), 6(a)–(d) and 8(a)–(d) will be reduced as follows:

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_{12}(T) & \text{if } M \leqslant T, & \text{(a)} \\ \mathrm{TRC}_{13}(T) & \text{if } 0 < T \leqslant M. & \text{(b)} \end{cases} \tag{44}$$

Eqs. 44(a) and (b) will be consistent with Eqs. (1) and (4) in Goyal [2], respectively. Hence, Goyal [2] will be a special case of this paper.

5. Numerical example

To illustrate the results obtained in this paper, let us apply the proposed method to efficiently solve the following numerical example. For convenience, the values of the parameters are selected randomly.

From Table 1, we can observe the optimal cycle time with various parameters of W, k and s, respectively. The following inferences can be made based on Table 1.

(1) The optimal cycle time will not decrease when retailer's storage capacity W is increasing. It means that the retailer will order more quantity since the retailer owns larger storage space to storage more items.

Table 1
The optimal cycle time with various values of W, k and s

k (\$/unit/ year)	$s = \$50$/unit				$s = \$100$/unit				$s = \$150$/unit			
	Δ_1	Δ_2	Δ_3	T^*	Δ_1	Δ_2	Δ_3	T^*	Δ_1	Δ_2	Δ_3	T^*
$W = 100$ units, ($W/D = 0.03333$ year)												
5	<0	<0	<0	$T_1^* = 0.10285$	>0	<0	<0	$T_2^* = 0.08819$	>0	<0	<0	$T_2^* = 0.08$
10	>0	<0	<0	$T_2^* = 0.0876$	>0	<0	<0	$T_2^* = 0.07914$	>0	<0	<0	$T_2^* = 0.07387$
15	>0	<0	<0	$T_2^* = 0.07817$	>0	<0	<0	$T_2^* = 0.07286$	>0	<0	<0	$T_2^* = 0.06927$
$W = 250$ units, ($W/D = 0.08333$ year)												
	Δ_1	Δ_4	Δ_5	T^*	Δ_1	Δ_4	Δ_5	T^*	Δ_1	Δ_4	Δ_5	T^*
5	<0	<0	<0	$T_1^* = 0.10729$	>0	<0	<0	$T_2^* = 0.092$	>0	>0	<0	$T_5^* = 0.08309$
10	<0	<0	<0	$T_1^* = 0.10103$	>0	<0	<0	$T_2^* = 0.0901$	>0	>0	<0	$T_5^* = 0.08309$
15	>0	<0	<0	$T_2^* = 0.09718$	>0	<0	<0	$T_2^* = 0.08889$	>0	>0	<0	$T_5^* = 0.08309$
$W = 400$ units ($W/D = 0.13333$ year)												
	Δ_6	Δ_7	Δ_5	T^*	Δ_6	Δ_7	Δ_5	T^*	Δ_6	Δ_7	Δ_5	T^*
5	>0	<0	<0	$T_6^* = 0.11127$	>0	>0	<0	$T_5^* = 0.09309$	>0	>0	<0	$T_5^* = 0.08309$
10	>0	<0	<0	$T_6^* = 0.11127$	>0	>0	<0	$T_5^* = 0.09309$	>0	>0	<0	$T_5^* = 0.08309$
15	>0	<0	<0	$T_6^* = 0.11127$	>0	>0	<0	$T_5^* = 0.09309$	>0	>0	<0	$T_5^* = 0.08309$

Example: Let $A = \$150$/order, $c = \$50$/unit, $D = 3000$ units/year, $h = \$3$/unit/year, $I_p = \$0.15/\$$/year, $I_e = \$0.12/\$$/year, $M = 0.1$ year, $N = 0.05$ year.

(2) When the RW unit stock-holding cost k is increasing, the optimal cycle time will not increase. The retailer will order less quantity to avoid renting expensive warehouse to storage these exceeding items.
(3) And last, we can find the optimal cycle time will decrease when the unit selling price s is increasing. This result implies that the retailer will order less quantity to take the benefits of the trade credit more frequently.

6. Conclusions

This paper is incorporated both Huang's model [21] and Teng's model [1] by considering the retailer's storage space limited. We provide three easy-to-use theorems to help the retailer in accurately and quickly determining the optimal cycle time. Then, Huang's model [21], Teng's model [1] and Goyal's model [2] are deduced as special cases. Finally, a numerical example is given to illustrate all theorems developed in this paper and we can obtain a lot of managerial insights: (1) the retailer will order more quantity when retailer's storage capacity is larger since the retailer owns larger storage space to storage more items; (2) the retailer will order less quantity to avoid renting expensive warehouse to storage these exceeding items when the RW unit stock-holding cost is expensive; (3) the retailer will order less quantity to take the benefits of the trade credit more frequently when the larger the differences between the unit selling price and the purchasing price per item.

The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, quantity discounts, time value of money, finite time horizon, finite replenishment rate and others.

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