乏晰競爭式學習網路於影像壓縮之應用

The Application of Fuzzy Competitive Learning Network to Image Compression

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向量量化為影像壓縮中有效技術，本篇論文中，應用競爭式學習網路(Competitive Learning Network)**及乏晰**(Fuzzy)**推論策略於影像壓縮技術上。所應用的壓縮技術即為向量量化**(Vector Quantization)。**其目的在於建立一編碼簿**(Codebook)**使得介於訓練向量與編碼簿中之編碼向量**(Codevector)**的平均誤差最小。根據實驗模擬結果顯示，基於最小平方誤差準則的乏晰競爭式學習網路所產生的影像壓縮編碼簿，其重建品質比一般** Lloyd **演算法所得的結果提昇了約** 1dB **至** 1.5dB。

關鍵詞：*乏晰群集演算法，神經網路，影像壓縮。*

Abstract

Vector quantization has been shown to be an effective technique for image compression. In this paper, an unsupervised parallel approach called the Fuzzy Competitive Learning Network (FCLN) for vector quatization in image compression is proposed. The goal is to apply an unsupervised scheme based on a neural network using the fuzzy clustering technique so that on-line learning and parallel implementation for codebook design are feasible. In FCLN, the codebook design is conceptually considered as a clustering problem. Here, it is a kind of neural network model imposed by the fuzzy clustering strategy working toward minimizing an objective function defined as the average distortion measure between any two training vectors within the same class. For an image of n training vectors and c interesting objects, the proposed FCLN would consist of n input and c output neurons. The experimental results show that a promising codebook can be obtained using the fuzzy competitive learning neural network based on least squares criteria in comparison with the generalized Lloyd algorithm.

Keywords: *Fuzzy clustering algorithm, Neural network, Image compression.*

I. Introduction

Vector quantization is a popular method for image compression. A number of vector quantization algorithms for image compression have been proposed [1-7]. Clustering or codebook design is an essential process in vector quantization. The purpose of vector quantization is to create a codebook such that the average distortion between training vectors and codevectors in the codebook is minimized. The minimization of the average distortion measure is widely performed by a gradient descent based iteration algorithm known as the Generalized Lloyd Algorithm (GLA) [1]. In accordance with the cluster centers obtained in the previous iteration and the nearest neighbor rule, the GLA performs a positive improvement to update the codebook at every iteration.

Neural networks have been demonstrated capable of performing vector quatization [8-9]. In this paper, a vector quantizer design approach based on the Fuzzy Competitive Learning Network (FCLN) for image compression is presented. In FCLN, an original competitive learning neural network is modified and the fuzzy c-mean clustering method is added. With this structure, the image compression based on vector quantization can then be regarded as an optimization problem defined as the scatter energy function depicted in this network. The training vectors are then partitioned into different feasible classes when the scatter energy function is converged.

The structure of the FCLN is constructed as a two layer network with the output neurons representing the number of codevectors (classes) and the input neurons representing the training vectors from images. In a simulated study, the FCLN is demonstrated to have the capability for vector quantization in image compression and shows the promising results in comparison with the GLA method.

The rest of this paper is organized as follows. Section II reviews the fuzzy c-means clustering algorithm. Section III presents the fuzzy competitive learning network. Experimental results are given in Section IV. Finally, conclusions are drawn in Section V.

II. Fuzzy C-Means Clustering Algorithm

Clustering is a process for classifying training samples in such a way that samples within a cluster are more similar to one another than samples belonging to different clusters. In many fields, such as segmentation, pattern recognition and vector quatization, clustering is an indispensable step. Many methods have been applied to clustering problem such as hard clustering approaches and fuzzy reasoning strategies.

The FCM clustering algorithm was first introduced by Dunn [10], the related formulations and algorithms were extended by Bezdek [11]. The FCM approach, like the conventional clustering techniques, minimizes an objective function in the least squared error sense. For class number c ($c \geq 2$), sample number n and fuzzification parameter $m\ (1 \leq m < \infty)$, the algorithm chooses $u_j : X \rightarrow [0,1]$ so that $\sum_j u_j = 1$ and $w_j \in R^d$ for $j = 1,2,\ldots,c$ to minimize the objective function

$$J_{FCM} = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n}\left(u_{i,j}\right)^{m}\left\|x_i - w_j\right\|^2 , \tag{1}$$

where $u_{i,j}$ is the value of jth membership grade on ith sample x_i. The cluster centroids $w_1,...,w_j,...,w_c$ can be regarded as prototypes for the clusters represented by the membership grades. For the purpose of minimizing the objective function, the cluster centroids and membership grades are chosen so that a high degree of membership occurs for samples close to the corresponding cluster centroids. The steps of the FCM algorithm are listed below.

FCM Algorithm

Step 1. Initialize the cluster centroids $w_j\left(2 \le j \le c\right)$, fuzzification parameter m ($1 \le m < \infty$), and the value $e > 0$. Give a fuzzy c-partition $U^{(0)}$ and t=1.

Step 2. Calculate the membership matrix $U^{(t)} = [u_{i,j}]$ by

$$u_{i,j} = \frac{\left(\frac{1}{\left(d_{i,j}\right)^2}\right)^{1/(m-1)}}{\sum_{l=1}^{c}\left(\frac{1}{\left(d_{l,i}\right)^2}\right)^{1/(m-1)}} , \tag{2}$$

where $d_{i,j}$ is the Euclidean distance between the training sample x_i and the class centroid w_j.

Step 3. Update the class centroids by

$$w_j = \frac{1}{\sum_{i=1}^{n}\left(u_{i,j}\right)^m}\sum_{i=1}^{n}\left(u_{i,j}\right)^m x_i \tag{3}$$

Step 4. Compute $\Delta = \max\left(\left|U^{(t+1)} - U^{(t)}\right|\right)$. If $\Delta > e$, then t=t+1 and go to step 2 ; otherwise go to step 5.

Step 5. Find the results for the cluster centroids.

III. Fuzzy Competitive Learning Network for VQ

Suppose an image is divided into n blocks (vectors of pixels) and each block occupies $\ell \times \ell$ pixels. A vector quantizer is a technique that maps the Euclidean $\ell \times \ell$-dimensional space $\mathbf{R}^{\ell\times\ell}$ into a set $\{\mathbf{w}_j, j = 1,2,...,c\}$ of points in $\mathbf{R}^{\ell\times\ell}$, called a codebook. A vector quantizer approximates a training vector as close as possible by one of the codevectors in a codebook. The average distortion

$E[d(\mathbf{x}_i, \mathbf{w}_j)]$ between an input sequence of training vectors $\{\mathbf{x}_i, i = 1,2,...,n\}$ and its corresponding output sequence of codevectors $\{\mathbf{w}_j, j = 1,2,...,c\}$ is defined as

$$D = E[d(\mathbf{x}_i, \mathbf{w}_j)] = \frac{1}{n}\sum_{i=1}^{n} d(\mathbf{x}_i, \mathbf{w}_j) \tag{4}$$

The distortion measure $d(\mathbf{x}_i, \mathbf{w}_j)$ is defined as the squared Euclidean distance between vectors $\mathbf{x}_i$ and $\mathbf{w}_j$. Thus a vector quantizer is optimal if the average distortion is at the minimum value.

The fuzzy comp etitive learning network has the same architecture proposed by Jou and Lin et al. [12-13]. It occupies two-dimensional fuzzy-relation synaptic weights matrix $U = [u_{i,j}]$ between output neurons $\mathbf{W} = \{\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_c\}$ and input samples $\mathbf{X}$. It has a single layer of output neurons, each of which is fully connected to the input nodes. A two-dimentional image is divided into n blocks (a block represents a training vector that captures $\ell \times \ell$ pixels) and mapped to the two-dimensional neural network. In which, an image divided n training vectors are directly fed into n inputs and classified into c interesting classes (The class centroid is represented a proper codevectors in codebook). If the number of codevectors c in a codebook is defined in advance, then the network dimension in this paper would consist of n inputs and c output neurons. Each vector was trained with the nearest neighbor condition and iteratively updating the neurons' weights.

Using the within-class scatter matrix criteria, the clustering problem can be mapped into a two-layer neural network with the fuzzy reasoning for vector quantization in image compression. The object function for this network, J_{FCLN}, is similar to that for the J_{FCM} as

$$J_{FCLN} = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}^{m}\left\|\mathbf{x}_i - \mathbf{w}_j\right\|^2 \tag{5}$$

Gradient descent on the objective function (5), yields

$$\left\langle \Delta\mathbf{w}_j \right\rangle = -h\frac{\partial\left(J_{FCLN}\right)}{\partial\mathbf{w}_j}$$

$$= h\sum_{i=1}^{n}\left[u_{i,j}^{m}\left(\mathbf{x_i} - \mathbf{w_j}\right) - \frac{1}{2}m\left(u_{i,j}\right)^{m-1}\left\|\mathbf{x_i} - \mathbf{w_j}\right\|^2\frac{\partial u_{i,j}}{\partial\mathbf{w_j}}\right] \tag{6}$$

From Eq. (2), the derivative of $u_{i,j}$ with respect to $\mathbf{w}_j$ could be obtained as

$$\frac{\partial u_{i,j}}{\partial\mathbf{w_j}} = \frac{2u_{i,j}\left(1 - u_{i,j}\right)\left(\mathbf{x_i} - \mathbf{w_j}\right)}{(m-1)\left(\left\|\mathbf{x_i} - \mathbf{w_j}\right\|^2\right)} \tag{7}$$

Replacing the $\dfrac{\partial u_{i,j}}{\partial \mathbf{w_j}}$ in Eq. (6) by Eq. (7), the gradient descent on the objective function with fuzzy units can be updated as

$$\left\langle \Delta \mathbf{w}_j \right\rangle = h \sum_{i=1}^{n} u_{i,j}^{m} \left(\mathbf{x}_i - \mathbf{w}_j \right) \left[1 - \frac{m}{m-1} \left(1 - u_{i,j} \right) \right] \tag{8}$$

Like the definition of the standard competitive learning updated rule:

$$\Delta \mathbf{w}_j = h \left(\mathbf{x}_i - \mathbf{w}_j \right) u_{i,j} \tag{9}$$

and

$$\mathbf{w}_j (t+1) = \mathbf{w}_j (t) + \Delta \mathbf{w}_j (t) \tag{10}$$

where h is the learning-rate parameter, and the update rule is also written as follows:

$$\Delta \mathbf{w}_j = h u_{i,j}^{m} \left(\mathbf{x}_i - \mathbf{w}_j \right) \left[1 - \frac{m}{m-1} \left(1 - u_{i,j} \right) \right] \tag{11}$$

Therefore, the FCLN-based vector quatizer with within-class scatter matrix is performed as follows:

<u>*FCLN algorithm*</u>

Step 1: Initialize the codevectors, fuzzification parameter $m(1 \le m < \infty)$, learning rate h, constant v, and the value $e > 0$. Give a fuzzy c-partition $U^{(0)}$ and t=1.

Step 2: Find the $\mathbf{w}_j^{(t)}$ with $U^{(t-1)}$ using Eq. (12). Calculate the membership matrix $U^{(t)} = [u_{i,j}]$ with $\mathbf{w}_j^{(t)}$ using Eq. (13).

$$\mathbf{w}_j = \frac{\sum_{i=1}^{n} {u_{i,j}}^{m} \mathbf{x}_i}{\sum_{i=1}^{n} {u_{i,j}}^{m}} \tag{12}$$

$$u_{i,j} = \left(\sum_{\ell=1}^{c} \frac{\left(\left\| \mathbf{x}_i - \mathbf{w}_j \right\|^2 \right)^{1/(m-1)}}{\left(\left\| \mathbf{x}_i - \mathbf{w}_\ell \right\|^2 \right)^{1/(m-1)}} \right)^{-1}, \qquad i = 1,2,\ldots,c \tag{13}$$

Step 3: Sequentially select a neuron to update all the weights (cluster centroids) with competitive learning according to Eqs. (10) and (11).

Step 4: Compute $\Delta = \max\left(\left| U^{(t+1)} - U^{(t)} \right| \right)$. If $\Delta > e$, then t=t+1 and go to step 2; otherwise go to step 5.

Step 5: Generate a codebook using the centroids of c classes.

VI. Experimental Results

The quality of the image reconstructed from the designed codebooks was compared with those using the FCLN and GLA algorithms using real image data. The training vectors were extracted from $256 \times 256 \times 8$ real images, each of which is divided into 4×4 blocks to generate 4096 non-overlapping 16-D training vectors. Three codebooks of size 64, 128, and 256 were built by these training vectors. This results compression rates of 6/16=0.375, 7/16=0.438, and 8/16=0.5 bits per pixel respectively. The resulting images were evaluated subjectively by the peak signal to noise ratio (PSNR) that is defined for images of size $N \times N$ as

$$PSNR = 10 \log_{10} \frac{255 \times 255}{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(x_{ij} - \hat{x}_{ij} \right)^2} \tag{14}$$

where x_{ij} and $\hat{x}_{ij}$ are the pixel gray levels from the original and reconstructed images and 255 is the peak gray level, respectively. Fig. 1 shows the test images with their reconstructed images from a codebook of size c=256 using FCLN. Table 1 shows the PSNR of the various images reconstructed from three codebooks of size c=64, 128, and 256 designed by the GLA and the proposed FCLN algorithms. From experimental results, the reconstructed images obtained from the presented FCLN are superior to those obtained from the GLA algorithm.

V. Conclusions

In this paper, a two-layer competitive neural network based on the within-class scatter matrix using the fuzzy c-means strategy for vector quantization in image compression is proposed. The goal is apply an unsupervised scheme based on a neural network using the fuzzy clustering technique so that on-line learning and parallel implementation for codebook design are feasible. Instead of iteratively updating the codebook in the conventional algorithm such as GLA, the codebook is updated just one time after the last iteration in the FCLN. It has been also demonstrated that the reconstructed images obtained from the FCLN based on within-class scatter energy are smoother than those obtained from the GLA algorithm in experimental results. In addition, this proposed algorithm lends itself admirably to parallel implementation and has great potential in the real-time applications.

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Table 1 PSNR of the various images reconstructed from the codebook of size=64, 128, and 256 respectively designed by the GLA and FCLN algorithms.

Image/Algorithm \ Codebook Size		64	128	256
Boy-girl	GLA	28.26	29.40	31.20
	FCLN	29.69	30.95	32.02
Girl	GLA	27.68	28.51	29.69
	FCLN	28.93	29.80	30.66
Pepper	GLA	24.82	25.60	26.78
	FCLN	25.83	27.03	27.34
F16	GLA	24.10	25.29	26.34
	FCLN	25.21	26.35	27.37

Fig 1. Test images and their reconstructed images using the FCLN with codebook of size c=256; (left column) Original images, (right column) reconstructed images