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Statistical testing for assessing the performance of lifetime index of electronic components with exponential distribution

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Abstract *The electronics industry has heavily prioritized enhancing the quality, lifetime and conforming rate (conforming to specifications) of electronic components. Various methods have been developed for assessing quality performance. In practice, process capability indices (PCIs) are used as a means of measuring process potential and performance. Moreover, most PCIs have been developed or investigated under the assumption that electronic components have a lifetime with a normal distribution. However, PCIs for non-normal distributions have seldom been discussed. Nevertheless, the lifetime of electronic components generally may possess an exponential, gamma or Weibull distribution and so forth. Under an exponential distribution, some properties of the PCIs and their estimators differ from those in a normal distribution. To utilize the PCIs more reasonably and accurately in assessing the lifetime performance of electronic components, this study constructs a uniformly minimum variance unbiased (UMVU) estimator of their lifetime performance index under an exponential distribution. The UMVU estimator of the lifetime performance index is then utilized to develop the hypothesis testing procedure. The purchasers can then employ the testing procedure to determine whether the lifetime of the electronic components adheres to the required level. Manufacturers can also utilize this procedure to enhance process capability.*

Introduction

Process capability analysis is an effective means of measuring process performance and potential capability. In the manufacturing industry, process capability indices (PCIs) are utilized to assess whether product quality meets the required level. For instance, Kane (1986) developed two indices, C_p and C_{pk}, which are widely utilized by the industry. Boyles (1991) noted that the these indices are conforming rate-based indices and, hence, are independent of the

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target value T. Consequently, these indices may fail to account for the extent to which the process center deviates from the target value. Moreover, Chan *et al.* (1988) and Pearn *et al.* (1992) developed two more advanced indices C_{pm} and C_{pmk} to overcome the shortcomings of C_{p} and C_{pk}. C_{p}, C_{pk}, C_{pm} and C_{pmk} measure the target-the-better type quality characteristics which generally have bilateral tolerances. Besides these four PCIs, additional PCIs have been developed for assessing the target-the-better type quality characteristics, but these are essentially merely modifications of the four basic indices (Boyles, 1991; Pearn and Chen, 1995). Besides the PCIs of bilateral tolerances, Montgomery (1985) and Kane (1986) provided the PCIs, such as C_L, C_{PL} and C_{PU}, for unilateral tolerances, where C_{PU} denotes the index measuring the smaller-the-better type quality characteristics, C_L and C_{PL} represent the indices that measure the larger-the-better type quality characteristics. All of the above PCIs are derived under the assumption that the quality characteristics are normally distributed. However, some quality characteristics are not normally distributed, and PCIs for non-normal distributions are derived by Clements (1989), Chang and Lu (1994), Pearn and Chen (1997). These studies utilized the percentiles as estimates for the process mean and standard deviation to calculate the PCIs and concluded that the non-normal PCIs are effective for distributions of any shape. However, if the quality characteristic possesses a specific non-normal distribution, the estimators of the distribution mean and standard deviation can be derived. PCIs can then be estimated using the estimates of the mean and standard deviation. Consequently, these estimated PCIs may be more accurate than the PCIs estimated using percentile approaches.

Epstein and Sobel (1953), Meyer (1965), Anderson *et al.* (1990) as well as Keller *et al.* (1994) noted that the lifetime of electronic components frequently possesses an exponential, gamma or Weibull distribution and so on. Since the lifetime of electronic components exhibits the larger-the-better quality characteristic of time orientation, Montgomery (1985) recommended using the capability index (lifetime performance index) for evaluating the lifetime performance of electronic components. Assuming exponential distribution in this paper, some properties of C_L and estimators of the process mean and standard deviation differ from those with a normal distribution. To more reasonably and accurately utilize C_L in assessing the lifetime performance of electronic components under the assumption of exponential distribution, this study constructs the UMVU estimator of C_L. The UMVU estimator of C_L is then utilized to construct the hypothesis testing procedure. The testing procedure can be employed by purchasers to assess whether the lifetime of electronic components meets the required level. Manufacturers can also utilize this procedure to enhance the process capability. This study also investigates the relationship between the process capability index and the conforming rate of electronic components, and tabulates the conforming rate under exponential distribution.

The rest of this paper is organized as follows. The next section introduces some properties of the lifetime performance index under the assumption of exponential distribution. The third section discusses the relationship between the lifetime performance index and conforming rate. The section then presents the estimator of the lifetime performance index and its statistical properties. The section develops a hypothesis testing procedure for the lifetime performance index. Conclusions are finally made in the last section.

Lifetime performance index

Generally, the lifetime of different electronic components varies. Assume that the lifetime (X) of electronic components is a random variable, and possesses an exponential distribution with mean λ unit times. The unit of time may be hours, days, weeks, months or some other unit. Clearly, a longer lifetime implies a better product quality. Hence, the lifetime of electronic components is a larger-the-better type quality characteristic. The lifetime is generally required to exceed L unit times (i.e. L is the lower specification limit) to both be economically profitable and satisfy customers. Montgomery (1985) developed a capability index C_L for properly measuring the larger-the-better type quality characteristic. C_L is defined as follows:

$$C_L = \frac{\mu - L}{\sigma}, \tag{1}$$

where μ denotes the process mean, σ represents the process standard deviation, and L is the lower specification limit.

To assess the lifetime performance of electronic components, C_L can be defined as the lifetime performance index. Under the assumption of exponential process distribution, the probability density function of the lifetime X can be expressed as:

$$f_X(x) = \frac{1}{\lambda} e^{(\frac{-x}{\lambda})}, \tag{2}$$

where $X > 0$ and $\lambda > 0$.

The exponential distribution has several important properties, as follows:

- $$\mu = E(X) = \lambda \quad \text{and} \quad \sigma = \sqrt{\operatorname{Var}(X)} = \lambda. \tag{3}$$
- The cumulative distribution function of X is given by:
$$F_x(t) = 1 - e^{(-t/\lambda)}, \quad t \geq 0. \tag{4}$$
- The failure rate function is defined by:

$$r(t) = \frac{fx(t)}{1 - F_X(t)} = \frac{1}{\lambda}. \tag{5}$$

From Equation (3), lifetime performance index can be rewritten as:

$$C_L = \frac{\mu - L}{\sigma} = \frac{\lambda - L}{\lambda} = 1 - \frac{L}{\lambda}. \tag{6}$$

When the mean lifetime of electronic components $\lambda > L$, then $C_L > 0$. From Equation (5), we can see that the larger the λ, the smaller the failure rate and the larger the lifetime performance index. Conversely, when $\lambda < L$, then $C_L < 0$. The smaller the λ, the larger the failure rate and the smaller C_L is. Therefore, the index reasonably and accurately represents the lifetime performance of electronic components.

Conforming rate

If the lifetime of an electronic component, X, exceeds the lower specification limit (i.e. $X \geq L$), then the component is defined as a conforming product. Otherwise, the component is defined as a nonconforming product. The ratio of conforming products is known as the conforming rate, and can be defined as:

$$P_r = \Pr(X \geq L) = e^{C_L - 1}, \quad -\infty < C_L < 1. \tag{7}$$

Obviously, a one-to-one mathematical relationship exists between the lifetime index C_L and conforming rate P_r, and it is only true under single parameter sufficiently specifying the true distribution of lifetimes. The larger the index value C_L, the larger the conforming rate P_r. Table I lists various C_L values and the corresponding conforming rates P_r.

For the C_L values which are not listed in Table I, the conforming rate P_r can be obtained through interpolation. The conforming rate can be calculated by

Lifetime index (C_L)	Conforming rate (P_r)	Lifetime index (C_L)	Conforming rate (P_r)
–∞	0.00000	0.40	0.54881
–2.00	0.04979	0.45	0.57695
–1.50	0.08208	0.50	0.60653
–1.00	0.13534	0.55	0.63763
–0.50	0.22313	0.60	0.67032
0.00	0.36788	0.65	0.70469
0.05	0.38674	0.70	0.74082
0.10	0.40657	0.75	0.77880
0.15	0.42741	0.80	0.81873
0.20	0.44933	0.85	0.86071
0.25	0.47237	0.90	0.90484
0.30	0.49659	0.95	0.95123
0.35	0.52205	1.00	1.00000

Table I. Lifetime index $C_L = 0$ (0.05) 1.0 vs conforming rate dividing the number of conforming components by the total number of electronic components sampled. To accurately estimate the conforming rate, Montgomery (1985) suggested that the sample size must be large. However, a large sample size is usually not practical from the perspective of cost, since collecting the lifetime data of electronic components involves damaging the components. In fact, a large sample is not necessary in measuring the index value. Since a one-to-one mathematical relationship exists between lifetime performance index C_L and conforming rate P_r, the conforming rate of electronic components can still be estimated precisely. Therefore, utilizing the one-to-one relationship between C_L and P_r, lifetime performance index can be a flexible and effective tool not only for evaluating product quality, but also for estimating the conforming rate P_r.

Estimation of lifetime performance index

Because the population mean and standard deviation of the lifetime of electronic components are generally unknown, they must be estimated. Let X_j represent the lifetime of the jth sample (electronic component), then $(X_1, X_2, \ldots, X_n)$ is a random sample taken from the exponential distribution with mean of λ units of time. If the sample mean is used to estimate the population mean λ, then the intuitive estimator of index C_L can be written as:

$$\hat{C}_L = 1 - \frac{L}{\bar{X}} = 1 - \frac{nL}{X}, \tag{8}$$

where $X = \sum_{j=1}^{n} X_j$.

Because X is the sum of n random variables (X_j) taken from an exponential distribution with mean λ unit times, hence X possesses a gamma distribution with parameters n and λ, and the probability density function of X can be expressed as follows:

$$f_x(x) = \frac{1}{\Gamma(n)\lambda^n} x^{n-1} e^{-x/\lambda}, \quad x > 0. \tag{9}$$

The expectation of $\hat{C}_L$ can then be derived as:

$$E(\hat{C}_L) = E\left(1 - \frac{nL}{X}\right) = 1 - nLE(X^{-1}). \tag{10}$$

Because

$$E(X^{-1}) = \int_0^\infty X^{-1} f(X) dX$$

$$= \frac{\Gamma(n-1)\lambda^{n-1}}{\Gamma(n)\lambda^n} \int_{-\infty}^{\infty} \frac{1}{\Gamma(n-1)\lambda^{n-1}} X^{(n-1)-1} e^{-X/\lambda} dX = \frac{1}{\lambda(n-1)},$$

the expectation of $\hat{C}_L$ can be expressed as:

$$E(\hat{C}_L) = 1 - \left(\frac{n}{n-1}\right)\left(\frac{L}{\lambda}\right). \tag{11}$$

$\hat{C}_L$ is not an unbiased estimator of C_L, since

$$E(\hat{C}_L) = 1 - \left\{\frac{n}{n-1}\right\}\left(\frac{L}{\lambda}\right) \neq C_L.$$

But when n approaches ∞, $E(\hat{C}_L)$ approaches C_L. Hence, $\hat{C}_L$ is a consistent and unbiased estimator of C_L when n becomes very large. $\hat{C}_L$ can be modified as below:

$$\hat{C}_L^{\prime} = 1 - \frac{(n-1)L}{X}. \tag{12}$$

Therefore, $\hat{C}_L^{\prime}$ is not only the unbiased estimator of C_L (i.e. $E(\hat{C}_L^{\prime}) = C_L$), but is also a function of the complete and sufficient statistic $\bar{X}$. Therefore, $\hat{C}_L^{\prime}$ is the best estimator (i.e. minimum variance unbiased estimator (UMVU estimator)) of C_L.

Let $Y=\hat{C}_L^{\prime}$, then the probability density function of $\hat{C}_L^{\prime}$ can be derived as follows (see Appendix 1 for details):

$$f_Y(y) = \frac{n(n+1)\lambda^2}{(n-1)L}\left\{\frac{1}{\Gamma(n+2)\lambda^{n+2}}\left[\frac{(n-1)L}{1-y}\right]^{(n+2)-1} e^{\frac{-(n-1)L}{(1-y)\lambda}}\right\},$$

$$-\infty < y < 1. \tag{13}$$

Meanwhile, the rth moment of $\hat{C}_L^{\prime}$ can be derived as below (see Appendix 2 for details):

$$E(\hat{C}_L^{\prime})^r = \sum_{i=0}^{r} C_i^r \left\{(-1)^i\left[(n-1)\frac{L}{\lambda}\right]^i \frac{\Gamma(n-1)}{\Gamma(n)}\right\}. \tag{14}$$

By the rth moment of $\hat{C}_L^{\prime}$, the expectation value and variance of $\hat{C}_L^{\prime}$ can be obtained as:

$$E(\hat{C}_L^{\prime}) = 1 - \frac{L}{\lambda} = C_L, \tag{15}$$

$$\mathrm{Var}(\hat{C}_L^{\prime}) = \frac{1}{n-2}\left(\frac{L}{\lambda}\right)^2, \quad n > 2. \tag{16}$$

Testing procedure for the lifetime performance index

Owing to the sampling error, the point estimate of the electronic component lifetime index C_L cannot be employed directly to determine whether the lifetime of electronic components meets the requirements. Thus, a statistical testing procedure is needed to objectively assess whether the lifetime index adheres to the required level. Assuming that the required index value of lifetime performance is larger than or equal to c, where c denotes the target value, then the hypothesis testing procedure for testing $H_0 : C_L \leq c$ (the process is not capable) vs $H_1 : C_L > c$ (the process is capable) can be developed. By using $\hat{C}_L'$, the best estimator of C_L, as the test statistic, the rejection region can be expressed as $\{\hat{C}_L' \mid \hat{C}_L' > C_0\}$. Under the specified significance level α, the critical value can be calculated as follows:

$$\Pr\{\hat{C}_L' > C_0 | C_L = c\} = \alpha,$$

i.e.

$$\Pr\left\{1 - \frac{(n-1)L}{X} > C_0 | C_L = c\right\} = \alpha,$$

$$\Pr\left\{X > \frac{(n-1)L}{1 - C_0} | \lambda = \frac{L}{1-c}\right\} = \alpha. \tag{17}$$

Let $W=X/\lambda$, then W is distributed as $\Gamma(n, 1)$ (see Appendix 3 for details). Substituting $W=X/\lambda$ in Equation (17), the following result can be obtained:

$$\Pr\left\{W > \frac{1-c}{1-C_0} \times (n-1)\right\} = \alpha, \tag{18}$$

$$\Pr\left\{W \leq \frac{1-c}{1-C_0} \times (n-1)\right\} = 1 - \alpha. \tag{19}$$

From Equations (19), utilizing *GAMINV*$(1-\alpha, n)$ function which represents the lower $1-\alpha$ quantile of $\Gamma(n, 1)$, then

$$\frac{1-c}{1-C_0} \times (n-1) = \mathrm{GAMINV}(1-\alpha, n)$$

is obtained. Thus, the following critical value can be derived:

$$C_0 = 1 - \frac{(1-c)(n-1)}{\mathrm{GAMINV}(1-\alpha, n)}. \tag{20}$$

Tables II and III list the critical values C_0 for $n = 2(1)50$ and $c = 0.1(0.1)0.9$ at $\alpha = 0.01$ and $\alpha = 0.05$, respectively. A statistical software package, namely the

n	$c = 0.1$	$c = 0.2$	$c = 0.3$	$c = 0.4$	$c = 0.5$	$c = 0.6$	$c = 0.7$	$c = 0.8$	$c = 0.9$
2	0.864	0.879	0.895	0.910	0.925	0.940	0.955	0.970	0.985
3	0.786	0.810	0.833	0.857	0.881	0.905	0.929	0.952	0.976
4	0.731	0.761	0.791	0.821	0.851	0.881	0.910	0.940	0.970
5	0.690	0.724	0.759	0.793	0.828	0.862	0.897	0.931	0.966
6	0.657	0.695	0.733	0.771	0.809	0.847	0.886	0.924	0.962
7	0.629	0.671	0.712	0.753	0.794	0.835	0.876	0.918	0.959
8	0.606	0.650	0.694	0.737	0.781	0.825	0.869	0.912	0.956
9	0.586	0.632	0.678	0.724	0.770	0.816	0.862	0.908	0.954
10	0.569	0.617	0.665	0.713	0.760	0.808	0.856	0.904	0.952
11	0.553	0.603	0.653	0.702	0.752	0.801	0.851	0.901	0.950
12	0.539	0.591	0.642	0.693	0.744	0.795	0.846	0.898	0.949
13	0.527	0.579	0.632	0.684	0.737	0.790	0.842	0.895	0.947
14	0.515	0.569	0.623	0.677	0.731	0.785	0.838	0.892	0.946
15	0.505	0.560	0.615	0.670	0.725	0.780	0.835	0.890	0.945
16	0.495	0.551	0.607	0.663	0.720	0.776	0.832	0.888	0.944
17	0.486	0.543	0.600	0.658	0.715	0.772	0.829	0.886	0.943
18	0.478	0.536	0.594	0.652	0.710	0.768	0.826	0.884	0.942
19	0.470	0.529	0.588	0.647	0.706	0.765	0.823	0.882	0.941
20	0.463	0.523	0.582	0.642	0.702	0.761	0.821	0.881	0.940
21	0.456	0.517	0.577	0.637	0.698	0.758	0.819	0.879	0.940
22	0.450	0.511	0.572	0.633	0.694	0.755	0.817	0.878	0.939
23	0.444	0.506	0.567	0.629	0.691	0.753	0.815	0.876	0.938
24	0.438	0.501	0.563	0.625	0.688	0.750	0.813	0.875	0.938
25	0.433	0.496	0.559	0.622	0.685	0.748	0.811	0.874	0.937
26	0.428	0.491	0.555	0.618	0.682	0.746	0.809	0.873	0.936
27	0.423	0.487	0.551	0.615	0.679	0.743	0.808	0.872	0.936
28	0.418	0.483	0.547	0.612	0.677	0.741	0.806	0.871	0.935
29	0.414	0.479	0.544	0.609	0.674	0.739	0.805	0.870	0.935
30	0.409	0.475	0.541	0.606	0.672	0.737	0.803	0.869	0.934
31	0.405	0.471	0.537	0.604	0.670	0.736	0.802	0.868	0.934
32	0.401	0.468	0.534	0.601	0.667	0.734	0.800	0.867	0.933
33	0.398	0.465	0.532	0.598	0.665	0.732	0.799	0.866	0.933
34	0.394	0.461	0.529	0.596	0.663	0.731	0.798	0.865	0.933
35	0.391	0.458	0.526	0.594	0.661	0.729	0.797	0.865	0.932
36	0.387	0.455	0.523	0.592	0.660	0.728	0.796	0.864	0.932
37	0.384	0.452	0.521	0.589	0.658	0.726	0.795	0.863	0.932
38	0.381	0.450	0.519	0.587	0.656	0.725	0.794	0.862	0.931
39	0.378	0.447	0.516	0.585	0.654	0.724	0.793	0.862	0.931
40	0.375	0.444	0.514	0.583	0.653	0.722	0.792	0.861	0.931
41	0.372	0.442	0.512	0.581	0.651	0.721	0.791	0.860	0.930
42	0.370	0.440	0.510	0.580	0.650	0.720	0.790	0.860	0.930
43	0.367	0.437	0.508	0.578	0.648	0.719	0.789	0.859	0.930
44	0.364	0.435	0.506	0.576	0.647	0.717	0.788	0.859	0.929
45	0.362	0.433	0.504	0.575	0.645	0.716	0.787	0.858	0.929
46	0.359	0.431	0.502	0.573	0.644	0.715	0.786	0.858	0.929
47	0.357	0.429	0.500	0.571	0.643	0.714	0.786	0.857	0.929
48	0.355	0.427	0.498	0.570	0.642	0.713	0.785	0.857	0.928
49	0.353	0.425	0.497	0.568	0.640	0.712	0.784	0.856	0.928
50	0.351	0.423	0.495	0.567	0.639	0.711	0.784	0.856	0.928

Table II.
Critical value C_0 for $n = 2(1)50$ and $c = 0.1(0.1)0.9$ at $\alpha = 0.01$

Table III.
Critical value C_0 for $n = 2(1)50$ and $c = 0.1(0.1)0.9$ at $\alpha = 0.05$

n	$c = 0.1$	$c = 0.2$	$c = 0.3$	$c = 0.4$	$c = 0.5$	$c = 0.6$	$c = 0.7$	$c = 0.8$	$c = 0.9$
2	0.810	0.831	0.852	0.874	0.895	0.916	0.937	0.958	0.979
3	0.714	0.746	0.778	0.809	0.841	0.873	0.905	0.936	0.968
4	0.652	0.690	0.729	0.768	0.807	0.845	0.884	0.923	0.961
5	0.607	0.650	0.694	0.738	0.782	0.825	0.869	0.913	0.956
6	0.572	0.620	0.667	0.715	0.762	0.810	0.857	0.905	0.952
7	0.544	0.595	0.645	0.696	0.747	0.797	0.848	0.899	0.949
8	0.521	0.574	0.627	0.681	0.734	0.787	0.840	0.894	0.947
9	0.501	0.557	0.612	0.667	0.723	0.778	0.834	0.889	0.945
10	0.484	0.542	0.599	0.656	0.713	0.771	0.828	0.885	0.943
11	0.469	0.528	0.587	0.646	0.705	0.764	0.823	0.882	0.941
12	0.456	0.517	0.577	0.638	0.698	0.758	0.819	0.879	0.940
13	0.445	0.506	0.568	0.630	0.691	0.753	0.815	0.877	0.938
14	0.434	0.497	0.560	0.623	0.686	0.748	0.811	0.874	0.937
15	0.424	0.488	0.552	0.616	0.680	0.744	0.808	0.872	0.936
16	0.416	0.480	0.545	0.610	0.675	0.740	0.805	0.870	0.935
17	0.407	0.473	0.539	0.605	0.671	0.737	0.802	0.868	0.934
18	0.400	0.467	0.533	0.600	0.667	0.733	0.800	0.867	0.933
19	0.393	0.461	0.528	0.595	0.663	0.730	0.798	0.865	0.933
20	0.387	0.455	0.523	0.591	0.659	0.727	0.796	0.864	0.932
21	0.381	0.449	0.518	0.587	0.656	0.725	0.794	0.862	0.931
22	0.375	0.444	0.514	0.583	0.653	0.722	0.792	0.861	0.931
23	0.370	0.440	0.510	0.580	0.650	0.720	0.790	0.860	0.930
24	0.365	0.435	0.506	0.576	0.647	0.718	0.788	0.859	0.929
25	0.360	0.431	0.502	0.573	0.644	0.716	0.787	0.858	0.929
26	0.356	0.427	0.499	0.570	0.642	0.714	0.785	0.857	0.928
27	0.351	0.423	0.496	0.568	0.640	0.712	0.784	0.856	0.928
28	0.347	0.420	0.492	0.565	0.637	0.710	0.782	0.855	0.927
29	0.344	0.416	0.489	0.562	0.635	0.708	0.781	0.854	0.927
30	0.340	0.413	0.487	0.560	0.633	0.707	0.780	0.853	0.927
31	0.336	0.410	0.484	0.558	0.631	0.705	0.779	0.853	0.926
32	0.333	0.407	0.481	0.555	0.630	0.704	0.778	0.852	0.926
33	0.330	0.404	0.479	0.553	0.628	0.702	0.777	0.851	0.926
34	0.327	0.402	0.476	0.551	0.626	0.701	0.776	0.850	0.925
35	0.324	0.399	0.474	0.549	0.624	0.700	0.775	0.850	0.925
36	0.321	0.397	0.472	0.547	0.623	0.698	0.774	0.849	0.925
37	0.318	0.394	0.470	0.546	0.621	0.697	0.773	0.849	0.924
38	0.316	0.392	0.468	0.544	0.620	0.696	0.772	0.848	0.924
39	0.313	0.390	0.466	0.542	0.619	0.695	0.771	0.847	0.924
40	0.311	0.388	0.464	0.541	0.617	0.694	0.770	0.847	0.923
41	0.309	0.385	0.462	0.539	0.616	0.693	0.770	0.846	0.923
42	0.306	0.383	0.461	0.538	0.615	0.692	0.769	0.846	0.923
43	0.304	0.381	0.459	0.536	0.613	0.691	0.768	0.845	0.923
44	0.302	0.380	0.457	0.535	0.612	0.690	0.767	0.845	0.922
45	0.300	0.378	0.456	0.533	0.611	0.689	0.767	0.844	0.922
46	0.298	0.376	0.454	0.532	0.610	0.688	0.766	0.844	0.922
47	0.296	0.374	0.453	0.531	0.609	0.687	0.765	0.844	0.922
48	0.294	0.373	0.451	0.529	0.608	0.686	0.765	0.843	0.922
49	0.292	0.371	0.450	0.528	0.607	0.686	0.764	0.843	0.921
50	0.291	0.369	0.448	0.527	0.606	0.685	0.764	0.842	0.921

statistical analysis system (SAS), is utilized to calculate the critical value C_0 (see Appendix 4 for the SAS control lines used in calculating C_0).

The proposed testing procedure about C_L can be organized as follows (Kane, 1986):

1. *Step 1*. Determine the lower lifetime limit L for the electronic components, performance index value c and sample size n.
2. *Step 2*. Specify a significance level α.
3. *Step 3*. Take a sample of size n and calculate the sample mean $\bar{X} = X/n$, where $X = \sum_{j=1}^{n} X_j$, and calculate the value of test statistic $\hat{C}_L'$.
4. *Step 4*. Obtain the critical value C_0 from Tables II or III, according to the c value, sample size n and α value selected in Step 1.
5. *Step 5*. Compare $\hat{C}_L'$ with C_0, and draw a conclusion. The decision rules are provided as follows:
   - If $\hat{C}_L' \leq C_0$, it is concluded that the lifetime performance index or conforming rate of the electronic components do not meet the required level.
   - If $\hat{C}_L' > C_0$, it is concluded that the lifetime performance index or conforming rate of electronic components meets the required level.

Based on the proposed testing procedure, the capability of the process for manufacturing electronic components is easy to assess. The following example illustrates the use of the testing procedure. To deal with customers' serious concerns regarding quality and reliability, the conforming rate of electronic components is required to exceed 80 per cent. Referring to Table I, a C_L value of 0.80 is obtained. Thus, in Step 1, the performance index value is set at $c = 0.80$. Furthermore, assume that a sample of size $n = 20$ is obtained and L is known. By specifying the significance level $\alpha = 0.01$, the test statistic $\hat{C}_L'$ can be calculated from the sample data. In Step 4, the critical value $C_0 = 0.881$ is obtained from Table II. Finally, Step 5 compares the value of $\hat{C}_L'$ with 0.881 and draws a conclusion about the hypotheses. If $\hat{C}_L' \leq 0.881$, it is concluded that the true lifetime performance index of the products does not meet the required level, or that the process is not capable. Otherwise, the process is concluded to be capable.

Conclusion

Process capability indices are widely employed by manufacturers to assess the performance and potential of their processes. However, when the quality characteristic possesses a specific non-normal distribution (such as exponential distribution), the PCI estimators obtained by percentile approaches are biased and the probability density functions of the PCI estimators are very difficult to obtain. This study constructs a best estimator $\hat{C}_L'$ of the lifetime performance index for electronic components, and develops a testing procedure for the lifetime performance index under the exponential distribution.

The proposed testing procedure is easily applied and can effectively evaluate whether the true performance of electronic components meets requirements. Additionally, this study provides a table of the lifetime performance index with its corresponding conforming rate. Hence, for any specified conforming rate, a corresponding C_L value can be obtained, and the hypotheses of the proposed testing procedure can also be expressed in terms of the conforming rate.

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Appendix 1

Let

$$Y = \hat{C}_L' = \left[1 - \frac{(n-1)L}{X}\right]$$

then

$$X = \frac{(n-1)L}{1-Y}$$

and

$$\frac{dX}{dY} = \frac{(n-1)L}{(1-Y)^2}.$$

Utilizing transformations of random variables, the probability density function of Y can be obtained as follows:

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right| = f_X\left(\frac{(n-1)L}{1-y}\right)\left|\frac{(n-1)L}{(1-y)^2}\right|$$

$$= \frac{1}{\Gamma(n)\lambda''}\left(\frac{(n-1)L}{1-y}\right)^{n-1} e^{\left(\frac{-(n-1)L}{(1-y)}\lambda\right)} \times \frac{(n-1)L}{(1-y)^2}$$

$$= \frac{n(n+1)\lambda^2}{(N-1)L}\left\{\frac{1}{\Gamma(n+2)\lambda^{n+2}}\left[\frac{(n-1)L}{1-y}\right]^{(n+2)-1} e^{\left(\frac{-(n-1)L}{(1-y)}\lambda\right)}\right\}.$$

Appendix 2

The rth moment of $\hat{C}_L'$ is derived as follows:

$$E(\hat{C}_L')^r = E\left(1 - \frac{(n-1)L}{X}\right)^r = \sum_{i=0}^{r} C_i^r\left\{(-1)^i[(n-1)L]^i\ E(X^{-i})\right\}$$

Because

$$E(X)^{-i} = \int_0^\infty \frac{1}{\Gamma(n)\lambda^n} X^{(n-i)-1} e^{-X/\lambda} dX = \frac{\Gamma(n-i)}{\Gamma(n)\lambda^i},$$

then rth moment of $\hat{C}_L'$ can be obtained as follows:

$$E(\hat{C}_L')^r = \sum_{i=0}^{r} C_i^r\left\{(-1)^i\left[(n-1)\frac{L}{\lambda}\right]^i \frac{\Gamma(n-1)}{\Gamma(n)}\right\}.$$

Appendix 3

Because X conforms to $\Gamma(n, \lambda)$, let $W = X/\lambda$, and the distribution of W can be derived through transformations of random variables as below:

$$f_W(w) = f_X(x)\left|\frac{dx}{dw}\right| = f_X(\lambda w)\lambda = \frac{1}{\Gamma(n)\lambda^n}(\lambda w)^{(n-1)} e^{\frac{-\lambda w}{\lambda}} \times \lambda = \frac{1}{\Gamma(n)} w^{n-1} e^{-w}.$$

Clearly, W is conforming to $\Gamma(n, 1)$.

Appendix 4

options replace ps = 58 ls = 78 nodate;

```
data lifetime;
do alpha = 0.01, 0.05;        /* significant level */
     do n = 2 to 50 by 1;
         do c = 0.1 to 0.9 by 0.1;
             C0 = 1 − (1 − c) ∗ (n − 1)/GAMINV(1 − α, n);        /* GAMINV is a function of SAS */
             label alpha = “significance level”,     n = “sample size”
                 c = “required index value c”,     C0 = “critical value”;
             output;
         end;
     end;
end;
format C0 5.3;
proc print data = lifetime label;
var alpha n c C0;
run;
```