Hooge parameter shows a strong dependence on V_{GS}, being $\alpha_H \sim 5 \times 10^{-4}$ for $V_{GS} = 0$V. We suggest that $1/f$ noise sources located in the channel are linked to electron mobility fluctuations, owing to carrier scattering by the depletion regions surrounding dislocations. Besides their high density, screening effects by the channel electrons significantly reduce their effect on the HEMT $1/f$ noise behaviour.

Acknowledgments: This work has been carried out under partial financial support of CICYT PN MAT98-0823-C03-01.

© IEE 1998
17 September 1998
Electronics Letters Online No: 19981597

J.A. Garrido, F. Calle, E. Muñoz, I. Izpura and J.L. Sánchez-Rojas (*Departamento de Ingeniería Electrónica, ETSI Telecomunicación, Ciudad Universitaria, 28040-Madrid, Spain*)

R. Li and K.L. Wang (*Department of Electrical Engineering, U.C.L.A., Los Angeles, CA 90095-1594, USA*)

E-mail: garrido@die.upm.es

References

1 LESTER, S.D., *et al.*: 'High dislocation densities in high efficiency GaN-based light-emitting diodes', *Appl. Phys. Lett.*, 1995, **66**, (10), pp. 1249–1251
2 WEIMANN, N.G., *et al.*: 'Scattering of electrons at threading dislocations in GaN', *J. Appl. Phys.*, 1998, **83**, (7), pp. 3656–3659
3 LEVINSHTEIN, M.E., *et al.*: 'Low-frequency noise in GaN/AlGaN heterojunctions', *Appl. Phys. Lett.*, 1998, **72**, (23), pp. 3053–3055
4 LEVINSHTEIN, M.E., *et al.*: 'AlGaN/GaN high electron mobility field effect transistors with low $1/f$ noise', *Appl. Phys. Lett.*, 1998, **73**, (8), pp. 1089–1091
5 HIRAKAWA, K., and SAKAKI, H.: 'Mobility of the two-dimensional electron gas at selectively doped *n*-type AlGaAs/GaAs heterojunctions with controlled electron concentration', *Phys. Rev. B*, 1986, **33**, (12), pp. 8291–8303
6 HOOGE, F.N.: '$1/f$ noise', *Physica B*, 1976, **83**, pp. 14–23
7 PY, M.A., and BUEHLMANN, H.J.: 'Evidence for screening effects on the $1/f$ current noise in GaAs/AlGaAs modulation doped field effect transistors', *J. Appl. Phys.*, 1996, **80**, (3), pp. 1583–1593
8 MORRISON, S.R.: '$1/f$ noise from levels in a linear or planar array. III. Trapped carrier fluctuations at dislocations', *J. Appl. Phys.*, 1992, **72**, (9), pp. 4104–4112
9 GARRIDO, J.A., *et al.*: 'Low frequency noise sources in AlGaN/GaN HEMTs'. Proc. Int. Conf. on Quantum $1/f$ noise and other Low Frequency Fluctuations in Electronic Devices, St. Louis, 1998, (to be published)

Sliding mode control for mismatched uncertain systems

Kuo-Kai Shyu, Yao-Wen Tsai and Chiu-Keng Lai

The major difficulties in sliding mode control (SMC) design are the chattering phenomenon and a system with mismatched uncertainties. An effective design procedure is proposed to alleviate these two difficulties, while retaining the benefits achieved in conventional SMC design.

Introduction: In a sliding mode, if the controlled system satisfies the invariance condition [1], the system behaviour is independent of the uncertainties and disturbances.

However, in the sliding mode control method, two major problems should be considered. First, the chattering phenomenon is highly undesirable because it may excite high-frequency unmodelled plant dynamics. Secondly, if the invariance conditions are not satisfied, the system behaviour in the sliding mode is not only governed by the sliding surface, but also determined by the mismatched uncertainties. To solve this problem, a method has been presented [2] which uses sliding mode control for a class of systems with mismatched uncertainties. However, this method needs to satisfy other matching conditions for a reduced-order system. Furthermore, the chattering problem is not considered.

In this Letter, we consider a class of uncertain systems in which the invariance condition is not satisfied. Several important design procedures are presented. With a continuous control law, the existence and reachability of a sliding mode (the hitting phase) is established. In the sliding mode, the method guarantees asymptotic stability (the sliding phase) even if the system has mismatched uncertainties. Moreover, the chattering phenomenon is removed.

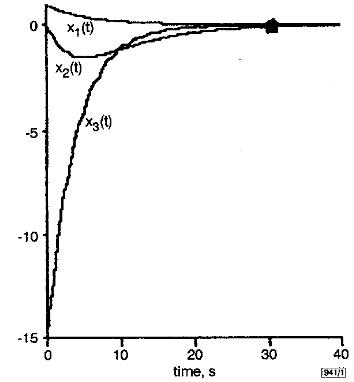

Fig. 1 *State response: $x_1(t)$, $x_2(t)$ and $x_3(t)$*

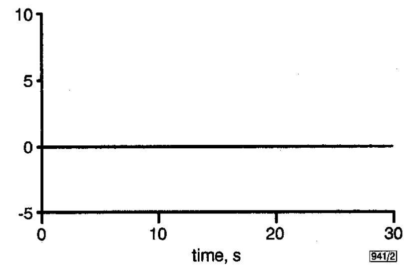

Fig. 2 *Sliding function $\sigma(t)$*

System formulation: Consider the following uncertain systems:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \qquad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, and the continuous function $f(x, t)$ represents the uncertainties with the matched part and mismatched part, i.e. the invariance condition is not satisfied. Note that $f(x, t)$ is uniformly bounded with respect to time t, and locally uniformly bounded with respect to state x.

We denote the sliding surface by $\sigma(t) = 0$, where the sliding function $\sigma(t) = Sx(t)$ is an m-state vector and S has full rank m such that SB is nonsingular. The following assumptions are needed:

(i) A1: There exists a known non-negative continuous function $\rho(\cdot)$ such that $\|f(x, t)\| \leq \rho(x, t)$, where $\|\cdot\|$ denotes the standard Euclidean norm.

(ii) A2: The pair (A, B) is controllable and matrix B has full rank.

Hitting phase: We can now give a continuous control input which drives the state trajectories of the system (eqn. 1) onto the sliding surface $\sigma(t) = 0$ in the state space, and the system remains in it thereafter.

Theorem 1: Suppose that the uncertain system (eqn. 1) satisfies assumptions A1 and A2. Let the control input be

$$u(t) = -(SB)^{-1}[SAx(t) + P\sigma(t)] - \bar{\rho}(x,t)\frac{\mu(x,t)}{\|\sigma^T(t)SB\|(\|\mu(x,t)\| + \varepsilon e^{-\alpha t})} \qquad (2)$$

where $P \in R^{m\times m}$ is a positive symmetric matrix, $\varepsilon, \alpha > 0$, $\bar{\rho}(x,t) = \|\sigma^T(t)S\|\rho(x, t)$ and $\mu(x, t) = (\sigma^T(t)SB)^T\bar{\rho}(x, t)/\|\sigma^T(t)SB\|$. Then the state trajectories will hit the sliding surface $\sigma(t) = 0$ subject to any initial condition.

Proof of theorem 1: In the hitting phase $\sigma^T(t)\sigma(t) > 0$; using the Lyapunov function candidate $V(t) = \sigma^T(t)\sigma(t)/2$, we obtain

Authorized licensed use limited to: IEEE Xplore. Downloaded on November 3, 2008 at 06:05 from IEEE Xplore. Restrictions apply.

$$\dot{V}(t) = \sigma^T(t)\dot{\sigma}(t)$$

$$= \sigma^T(t)\left[-P\sigma(t) - SB\bar{\rho}(x,t)\frac{\mu(x,t)}{\|\sigma^T(t)SB\|(\|\mu(x,t)\| + \varepsilon e^{-\alpha t})} + Sf(x,t)\right]$$

$$\leq -\sigma^T(t)P\sigma(t) - \frac{\|\mu(x,t)\|^2}{\|\mu(x,t)\| + \varepsilon e^{-\alpha t}} + \|\sigma^T(t)S\|\rho(x,t)$$

Since $\|\sigma^T(t)S\|\rho(x,t) = \|\mu(x,t)\|$, we have

$$\dot{V}(t) \leq -\sigma^T(t)P\sigma(t) + \frac{\|\mu(x,t)\|\varepsilon e^{-\alpha t}}{\|\mu(x,t)\| + \varepsilon e^{-\alpha t}}$$
$$\leq -\sigma^T(t)P\sigma(t) + \varepsilon e^{-\alpha t}$$

Now define $w(t) = \sigma^T(t)P\sigma(t)$, we have $0 \leq V(t) = V(0) + \int_0^t \dot{V}(\tau)d\tau \leq V(0) + \int_0^t [-w(\tau) + \varepsilon e^{-\alpha\tau}]d\tau = V(0) - \int_0^t w(\tau)d\tau + (\varepsilon/\alpha)(1 - e^{-\alpha t})$. Taking the limit as t approaches infinity on both sides of this inequality, we have $\lim_{t\to\infty} \int_0^t w(\tau)d\tau \leq V(0) + \varepsilon/\alpha < \infty$. According to the Barbalat Lemma [3], we obtain $\lim_{t\to\infty} w(t) = 0$. That is $\sigma(t) \to 0$ as $t \to \infty$. Hence the theorem is proved.

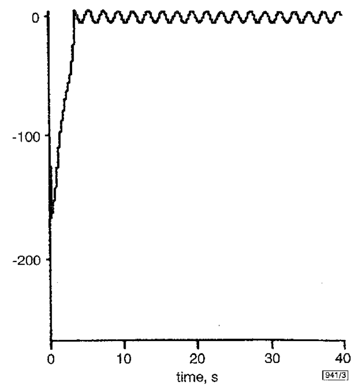

Fig. 3 *Control input u(t)*

Sliding phase: In this Section, we derive some conditions such that the system of eqn. 1 on the sliding surface is asymptotically stable even though the invariance condition does not hold. First, the results [4] for determining the sliding surface are as follows.

Consider the system $\dot{x} = Ax + Bu$, let $J = diag\{\lambda_1, \lambda_2, ..., \lambda_{n-m}\}$, where the eigenvalues λ_j, $j = 1, 2, ..., n - m$ are real, distinct. By assumption A2, matrices $W \in R^{n\times(n-m)}$ and $N \in R^{m\times n}$ exist such that $[A + BN]W = WJ$. If $SW = 0$, we have $Range(W) \cap Range(B) = \{0\}$ because SB is invertible. Hence $[W\ B]$ is nonsingular. The inverse $[W\ B]$ has the form $[(W^g)^T\ (B^g)^T]^T$, where W^g and B^g denote the generalised inverses of W and B, respectively.

Selecting $S = B^g$ and a transformation matrix T such that $y(t) = Tx(t)$, where $T = [(W^g)^T\ (S)^T]^T \in R^{n\times n}$ with $T^{-1} = [W\ B]$, the transformed state $y(t)$ is partitioned as

$$y(t) = \begin{bmatrix} z(t) \\ \sigma(t) \end{bmatrix}$$

where $z(t) = W^g x$ and $\sigma(t) = Sx$. Let

$$TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad (3)$$

then eqn. 1 can be rewritten in the form

$$\begin{cases} \dot{z}(t) = A_{11}z(t) + A_{12}\sigma(t) + \bar{f}(z(t),t) \\ \dot{\sigma}(t) = A_{21}z(t) + A_{22}\sigma(t) + u(t) + Sf(Wz + B\sigma, t) \end{cases} \qquad (4)$$

where the order-reduced uncertainty $\bar{f}(z(t), t)$ has the form $\bar{f}(z(t), t) = W^g f(Wz + B\sigma, t)$ with $f(Wz + B\sigma, t) = f(x, t)$. Theorem 2 will show that a system in the sliding phase is asymptotically stable.

Theorem 2: If $\bar{f}(z(t), t)$ satisfies the uniform Lipschitz condition $\|\bar{f}(z^1(t), t) - \bar{f}(z^2(t), t)\| \leq k\|z^1(t) - z^2(t)\|$ where $0 \leq k < 0.5\,\lambda_{min}(\bar{Q})/\|\bar{P}\|$ with $\bar{P}$, $\bar{Q} \in R^{(n-m)\times(n-m)}$ are symmetric, positive-definite matrices satisfying the Lyapunov equation $A_{11}^T\ \bar{P} + \bar{P}\ A_{11} = -\bar{Q}$, then the uncertain system (eqn. 1) on the sliding surface $\sigma(t) = 0$ is asymptotically stable.

Proof of theorem 2: In the sliding mode, since $\sigma(t) = 0$ and $\dot{\sigma}(t) = 0$, it can be seen by eqn. 4 that $\dot{z}(t) = A_{11}z(t) + \bar{f}(z(t), t)$.

Using a Lyapunov function candidate $\bar{V}(t) - z^T(t)\bar{P}z(t)$, we then have

$$\dot{\bar{V}}(t) = \dot{z}^T(t)\bar{P}z(t) + z^T(t)\bar{P}\dot{z}(t)$$
$$= -z^T(t)\bar{Q}z(t) + 2z^T(t)\bar{P}\bar{f}(z(t),t) \qquad (5)$$

By using eqn. 5 and the fact that $z^T(t)\bar{Q}z(t) \geq \lambda_{min}(\bar{Q})\|z(t)\|^2$ and $\|\bar{P}\bar{f}(z(t), t)\| \leq k\|\bar{P}\|\ \|z(t)\|$, we have $\dot{\bar{V}}(t) \leq -\lambda_{min}(\bar{Q})\|\ z(t)\|^2 + 2k\|\bar{P}\|\ \|z(t)\|^2$. Hence $\dot{\bar{V}}(t)$ is negative. The proof is completed.

Example: To illustrate the design technique, consider the system [1, 2]

$$A = \begin{bmatrix} -0.03 & 0.01 & 0.01 \\ -0.05 & -0.15 & 0.05 \\ -0.09 & 0.03 & -0.17 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and $f(x, t) = \Delta Ax(t) + f_1(t)$ where

$$\Delta A =$$
$$\begin{bmatrix} 0.11{+}0.44\sin(3.14t) & 0.01{+}0.004\cos(3.14t) & 0.008{+}0.002\sin(6.28t) \\ 0.55{+}0.220\sin(3.14t) & 0.05{+}0.020\cos(3.14t) & 0.040{+}0.010\sin(6.28t) \\ 0.50\sin(3.14t) & 0 & 0 \end{bmatrix}$$

$$f_1(t) = \begin{bmatrix} 0 \\ 0 \\ 5\sin(3.14t) \end{bmatrix}$$

Then we have $\|f(x,t)\| \leq 0.934\|x(t)\| + 5 = \rho(x,t)$.

Select the poles in the sliding mode to be $\lambda_1 = -0.14$, $\lambda_2 = -0.26$. Following the sliding phase method, we design the sliding function $\sigma(x) = Sx = [15\ 1.4\ 1]x$. For the control input (eqn. 2), we select $P = 1$, $\varepsilon = 9$, and $\alpha = 0.01$. Fig. 1 shows the state response subjected to the initial condition $x(0) = [1\ 0\ 0]^T$.

Fig. 2 displays the variation in the sliding function with respect to time. The corresponding control input is shown in Fig. 3. It is seen from these Figures that the major problems in conventional SMC design such as the chattering phenomenon and the effect of undesirable mismatched uncertainty are both solved by this proposed design procedure.

© IEE 1998

12 October 1998

Electronics Letters Online No: 19981546

Kuo-Kai Shyu, Yao-Wen Tsai and Chiu-Keng Lai (*Department of Electrical Engineering, National Central University, Chung-Li 320, Taiwan, Republic of China*)

E-mai: kkshyu@ee.ncu.edu.tw

References

1 DRAŽENOVIĆ, B.: 'The invariance conditions in variable structure systems', *Automatica,* 1969, **5**, pp. 287–295

2 KWAN, C.M.: 'Sliding mode control of linear systems with mismatched uncertainties', *Automatica,* 1995, **31**, pp. 303–307

3 SLOTINE, J.-J.E., and LI, W.: 'Applied nonlinear control' (Prentice-Hall, Englewood Cliffs, NJ, 1991)

4 HUI, S., and ŻAK, S.H.: 'Robust control synthesis for uncertain/nonlinear dynamical systems', *Automatica,* 1992, **28**, pp. 289–298

Authorized licensed use limited to: IEEE Xplore. Downloaded on November 3, 2008 at 06:05 from IEEE Xplore. Restrictions apply.