

# On the coordination of maintenance scheduling for transportation fleets of many branches of a logistic service provider

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## ARTICLE INFO

### Article history:

Received 10 January 2007

Received in revised form 7 December 2007

Accepted 4 January 2008

### Keywords:

Transportation fleet

Maintenance

Logistic service provider

Scheduling

Search algorithm

## ABSTRACT

In this paper, we study the Transportation Fleet Maintenance Scheduling Problem (TFMSP) for a Logistic Service Provider (LSP) with many sub-companies (or branches). In the literature, Goyal and Gunasekaran's [S.K. Goyal, A. Gunasekaran, determining economic maintenance frequency of a transportation fleet, *International Journal of Systems Science* 23 (4) (1992) 655–659] presented a mathematical model for the TFMSP to determine the economic maintenance frequency of only a single company. However, an LSP usually owns many branches that serve for the operations of transshipments of different areas of a country in most real cases. There exists significant room to reduce average total costs for an LSP if managers coordinate economic maintenance frequencies among branches. Therefore, we were motivated to propose an extended model of the TFMSP with many branches in this study. In order to solve this problem, we first conduct a full analysis on the extended model. By utilizing our theoretical results, we propose an efficient search algorithm that effectively solves an optimal solution for the extended TFMSP. Our numerical results show that the whole transportation fleet system of an LSP can obtain significant cost savings from the coordination policy.

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## 1. Introduction

In this paper, we study the Transportation Fleet Maintenance Scheduling Problem (TFMSP) for a Logistic Service Provider with many sub-companies (or branches). In the literature, Goyal and Gunasekaran's [1] presented a mathematical model for the TFMSP to determine the economic maintenance frequency of only a single company. However, it is a common practice that a logistic service provider usually owns many branches that serve for the operations of transshipments of different areas of a country in most real cases. If managers of the logistic service provider could conduct better coordination among branches, there exists significant room to reduce average total maintenance costs. Therefore, in this study, we were motivated to extend the TFMSP in two aspects: First, we not only derived an extended model of the TFMSP with many branches, but also conducted full theoretical analysis on theoretical properties of the extended model. Second, we proposed a search algorithm that effectively solves an optimal solution for the extended TFMSP.

As mentioned in Goyal and Gunasekaran [1], the problem of determining economic maintenance of machines in a manufacturing system or transportation fleets in a company has been dealt with extensively in management science/operations research/industrial engineering (see [2–8]). But, researchers pay limited attention to the problem of determining operating and maintenance schedules for only a single company (or single manufacturing system).

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Before presenting the extended model of the TFMSP, we first introduce the assumptions made and the notation used later. The logistic service provider has a total of  $B$  branches. There are  $m_i$  groups of vehicles in the  $i$ th branch. We denote the number of vehicles in the  $j$ th group of the  $i$ th branch as  $n_{ij}$ . We assume that all the vehicles need to be maintained periodically. Also, the decision maker plans maintenance schedules of the vehicle groups in some *basic period*, denoted by  $T$ . Maintenance work on a group of vehicles is carried out at a fixed, equal-time interval that is called the *maintenance cycle* for that group of vehicles. The vehicles in the  $j$ th group of the  $i$ th branch are sent for maintenance once in  $k_{ij}$  basic periods where  $k_{ij}$  is a positive integer. (Therefore, the maintenance cycle for the vehicles in the  $j$ th group of the  $i$ th branch is  $k_{ij}T$ .) We note that the extended TFMSP takes into account only planned maintenance, but not unplanned vehicle failures in maintenance scheduling.

We consider two categories of costs in the extended TFMSP, namely, operating cost and maintenance cost. The operating cost of a vehicle depends on the length of the maintenance cycle and it is assumed to increase linearly with respect to time since maintenance on the vehicle. Specifically, the operating cost per unit time at time  $t$  after the last maintenance for a vehicle in the  $j$ th group of the  $i$ th branch is given by  $f_{ij}(t) = a_{ij} + b_{ij}t$  where  $a_{ij}$  is the fixed cost and  $b_{ij}$  indicates the increase in operating cost per unit of time. Also, for each vehicle in the  $j$ th group of the  $i$ th branch, we assume that it takes  $X_{ij}$  units of time for its maintenance work and the utilization factor of a vehicle in the  $j$ th group of the  $i$ th branch on the road is  $Y_{ij}$  where  $X_{ij}$  and  $Y_{ij}$  are known constants. (One may refer to Yanagi [9] for further discussions on the utilization factor of a vehicle.) Therefore, the actual time during which a vehicle can operate is equal to  $Y_{ij}(k_{ij}T - X_{ij})$ , and the total operating cost for a vehicle in the  $j$ th group of the  $i$ th branch is given by

$$\int_0^{Y_{ij}(k_{ij}T - X_{ij})} f_{ij}(t) dt = \int_0^{Y_{ij}(k_{ij}T - X_{ij})} (a_{ij} + b_{ij}t) dt = Y_{ij}(a_{ij} - b_{ij}X_{ij}Y_{ij})k_{ij}T + 0.5b_{ij}Y_{ij}^2k_{ij}^2T^2 - X_{ij}Y_{ij}(a_{ij} - 0.5b_{ij}X_{ij}Y_{ij}). \tag{1}$$

The average fixed cost of maintenance for a vehicle in the  $j$ th group of the  $i$ th branch is given by  $s_{ij}/(k_{ij}T)$ . On the other hand, as maintenance work is carried out at intervals of  $T$ , a fixed cost (denoted by  $S$ ) will be incurred for all vehicle groups scheduled for maintenance in each basic period.

Following the assumptions above, we derived the extended TFMSP as problem  $(P_0)$ .

$$(P_0) \text{ Minimize } Z(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B}, T) = \frac{S}{T} + \sum_{i=1}^B \sum_{j=1}^{m_i} \Phi_{ij}(k_{ij}, T) + C \tag{2}$$

where

$\Phi_{ij}(k_{ij}, T) = \frac{n_{ij}U_{ij}}{k_{ij}T} + n_{ij}V_{ij}k_{ij}T$ ,  $U_{ij} = s_{ij} - X_{ij}Y_{ij}(a_{ij} - 0.5b_{ij}X_{ij}Y_{ij})$  and  $V_{ij} = 0.5b_{ij}Y_{ij}^2$ . Also,  $C = \sum_{i=1}^B \sum_{j=1}^{m_i} n_{ij}Y_{ij}(a_{ij} - b_{ij}X_{ij}Y_{ij})$  is a constant since all the parameters are given in its expression.

Then, solving the problem  $(P_0)$  is equivalent to obtain the optimal solution for the problem  $(P)$  as follows.

$$(P) \Psi(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B}, T) = \inf_{T > 0} \left\{ \frac{S}{T} + \sum_{i=1}^B \sum_{j=1}^{m_i} \Phi_{ij}(k_{ij}, T) \mid k_{ij} \in \mathbb{N}^+, i = 1, \dots, B, j = 1, \dots, m_i \right\}. \tag{3}$$

In the extended TFMSP, the managers of the logistic service provider need to determine  $T$  (i.e., the basic period) and  $(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B})$  (i.e., the frequency of maintenance for vehicles in each group of every branch) so as to minimize total costs incurred per unit time.

We outline the rest of this paper as follows. In Section 2, we survey the literature related to the TFMSP. In Section 3, we present a full theoretical analysis on the optimal cost curve of the problem  $(P)$ . Based on our theoretical results, we derive an effective search algorithm that efficiently solves the extended TFMSP in Section 4. In Section 5, we employ a numerical example to demonstrate implementation of the proposed algorithm. Finally, we address our concluding remarks in Section 6.

## 2. Literature review

Goyal and Gunasekaran [1] proposed their solution approach for the TFMSP based on two equations that were derived by setting the first derivative of the total cost function with respect to decision variables to zero. Then, in term, an initial vector of maintenance frequencies for vehicles in each group can be determined. If the values of maintenance frequencies are not integers, then select the nearest non-zero integer. Once a vector of the maintenance frequencies was determined, we obtained an optimal value of the basic period accordingly. Such an iterative process was kept on until two consecutive vector of the maintenance frequencies were the same.

Later, van Egmond, Dekker and Wildeman [10] presented further discussions on Goyal and Gunasekaran's search procedure. They indicated that the function total cost function is not convex as Goyal and Gunasekaran [1] assumed. And, since the frequencies of maintenance for vehicles in each group of need to be integers, (i.e., the determination of global optimization is not as easy as Goyal and Gunasekaran suggested. They also showed that it did not always obtain the minimum value for the objective function when one rounded multipliers of the basic period to the nearest non-zero integer. Finally,

they indicated that Goyal and Gunasekaran’s search procedure often stopped after its first iteration without obtaining an optimal solution. These three problems explained why Goyal and Gunasekaran’s solution did not guarantee obtaining an optimal solution. In fact, it was often stuck in a local optimal solution. However, van Egmond, Dekker and Wildeman’s [10] only mentioned that one needed to try different starting values to find an optimal solution, but without proposing a new solution approach to solve the TFMSP.

One may notice that the joint replenishment problem (JRP) is actually a special case of the extended TFMSP with  $n_{ij} = 1, a_{ij} = 0.5X_{ij}/Y_{ij}, b_{ij} = 1.0/Y_{ij}$  and  $B = 1$ . Please refer to van Eijs [11], Viswanathan [12,13], Fung and Ma [14] and Lee and Yao [15] for further reference on the JRP. Arkin et al. [16] proved that the JRP is NP-hard, i.e., the JRP is not solvable by polynomial-time algorithms. Therefore, it is obvious that the extended TFMSP is also NP-hard. Probably, because of its difficulty, not many researchers addressed their efforts to propose new solution approaches after van Egmond, Dekker and Wildeman’s [10] study. To the best of the authors’ knowledge, there exists no solution approach that solves the extended TFMSP in the literature. Therefore, we are motivated to propose a new solution approach in this study.

### 3. Theoretical analysis

In this section, we discuss some theoretical results that provide insights into the optimal cost function  $\Psi(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B}, T)$ . Our theoretical results facilitate derivation of the search algorithm presented in Section 4.

We define a new index  $\ell(i, j)$  by combining the subscription of  $i$  and  $j$  into a single index  $\ell$  by  $\ell(i, j) \equiv \sum_i m_{i-1} + j$  where  $i = 1, \dots, B, j = 1, \dots, m_i$  and  $m_0 = 0$ . Also, let  $L$  be the total number of vehicle groups in all branches where  $L = \sum_{i=1}^B m_i$ . For the rest of paper, we use the index  $\ell$  for indicating a particular vehicle group in lieu of  $\ell(i, j)$  to make our presentation more concise. Following the Eq. (3), we have

$$\Psi(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B}, T) = \inf_{T>0} \left\{ \frac{S}{T} + \sum_{\ell=1}^L \Phi_{\ell}(k_{\ell}, T) \mid k_{\ell} \in \mathbb{N}^+, \ell = 1, \dots, L \right\}. \tag{4}$$

From the right-side of (4), we learn that the terms are separable. Therefore, we are motivated to study the properties of  $\Phi_{\ell}(k_{\ell}, T)$  since they shall establish foundation for further investigation of the function  $\Psi(k_{11}, \dots, k_{1m_1}, \dots, k_{B1}, \dots, k_{Bm_B}, T)$ .

**Proposition 1.** For any given  $k_{\ell} \in \mathbb{N}^+$ , the function  $\Phi_{\ell}(k_{\ell}, T)$  satisfies the following properties for  $T > 0$ , where  $\ell \in \{1, \dots, L\}$ .

1.  $\Phi_{\ell}(k_{\ell}, T)$  is strictly convex.
2.  $\Phi_{\ell}(k_{\ell}, T)$  has a minimum for  $T = x_{\ell}^*/k_{\ell}$  with  $x_{\ell}^*$  given by:

$$x_{\ell}^* = \sqrt{U_{\ell}/V_{\ell}} \tag{5}$$

where  $U_{ij} = s_{ij} - X_{ij}Y_{ij}(a_{ij} - 0.5b_{ij}X_{ij}Y_{ij})$  and  $V_{ij} = 0.5b_{ij}Y_{ij}^2$ .

3. The function  $\Phi_{\ell}(k_{\ell}, T)$  obtains its minimal objective function value by

$$2n_{\ell}\sqrt{U_{\ell}V_{\ell}}. \tag{6}$$

**Proof.** We may prove these assertions by simple algebra. ■

Let us define a new function  $g_{\ell}(T)$  by taking the optimal value of  $k_{\ell}$  at any value  $T' > 0$  for the function  $\Phi_{\ell}(k_{\ell}, T)$  as follows.

$$g_{\ell}(T) \equiv \inf_{k_{\ell} \in \mathbb{N}^+} \{ \Phi_{\ell}(k_{\ell}, T') \mid T = T' \in \mathbb{R}^+ \}. \tag{7}$$

Consequently, the problem (P) can be re-written by

$$(P_1) \Gamma(T) = \inf_{T>0} \left\{ S/T + \sum_{\ell=1}^L g_{\ell}(T) \right\} \tag{8}$$

where the function  $\Gamma(T)$  is the optimal objective function value of problem (P<sub>1</sub>) with respect to  $T$ .

Before having further analysis on problem (P<sub>1</sub>), we first graphically display the function curve of  $g_{\ell}(T)$  in Fig. 1. Note that the curve of the  $g_{\ell}(T)$  function is actually their lower envelope.

In the following discussion, we will have further analysis on these two observations and will formally prove them as the base for deriving the theoretical properties for problem (P<sub>1</sub>) later.

#### 3.1. The junction point

We define a “junction point” for  $g_{\ell}(T)$  as a particular value of  $T$  where two consecutive convex curves  $\Phi_{\ell}(k_{\ell}, T)$  and  $\Phi_{\ell}(k_{\ell} + 1, T)$  concatenate. These junction points determine at “what value of  $T$ ” where one should change the value of  $k_{\ell}$  so as to obtain the optimal value for the  $g_{\ell}(T)$  function. We first derive a closed-form for the location of the junction points.

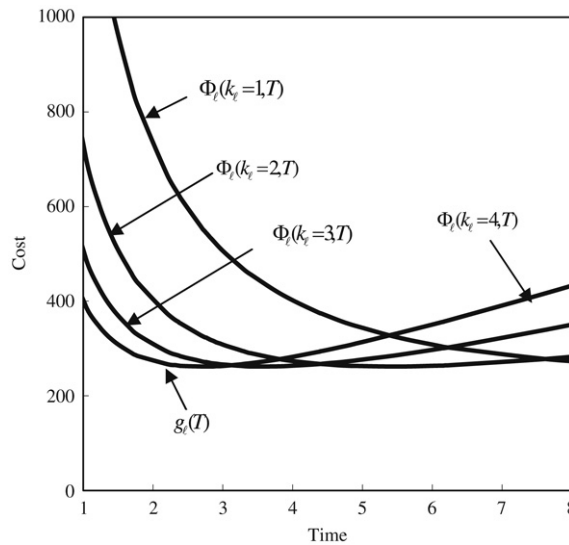


Fig. 1. The function curve of the  $g_l(T)$  function.

We define the difference function  $\Delta_\ell(k, T)$  by

$$\begin{aligned} \Delta_\ell(k, T) &= \Phi_\ell(k + 1, T) - \Phi_\ell(k, T) \\ &= -\frac{n_\ell U_\ell}{T(k + 1)k} + n_\ell V_\ell T. \end{aligned} \tag{9}$$

We note that  $\Delta_\ell(k, T)$  is the cost difference between using  $k$  and  $k + 1$  as its multiplier. Since the first derivative of the function  $\Delta_\ell(k, T)$  is always positive for all  $T > 0$ ,  $\Delta_\ell(k, T)$  is an increasing function with respect to  $T$ . Suppose that the search algorithm proceeds from an upper bound toward smaller values of  $T$ , we evaluate  $\Delta_\ell(k, T)$  from positive values, to zero and finally, to negative values. Let  $w$  be the point where  $\Delta_\ell(k, T)$  reaches zero. Assume that  $k$  is the optimal multiplier for  $T > w$ . This scheme implies that one should keep using  $k$  until it meets  $w$ . From the point  $w$  onwards, the value of  $g_\ell(T)$  can be improved by using  $k + 1$  as its optimal multiplier. We note that  $w$  is the point where two neighboring convex curves  $\Phi_\ell(k_\ell + 1, T)$  and  $\Phi_\ell(k_\ell, T)$  meet. Importantly, such a junction point  $w$  not only provides us with the information on at “what value of  $T$ ” where one should change the value of  $k$  so as to secure the optimal value for the  $g_\ell(T)$  function.

By the rationale discussed above, we derive a closed form to locate the junction points by letting  $\Delta_\ell(k, T) = 0$  as follows.

$$\delta_\ell(k) = \sqrt{\frac{U_\ell}{V_\ell(k + 1)k}} = \sqrt{\frac{2(s_\ell - X_\ell Y_\ell(a_\ell - 0.5b_\ell X_\ell Y_\ell))}{b_\ell Y_\ell^2(k + 1)k}}. \tag{10}$$

Note that  $\delta_\ell(k)$  indicates the location of the  $k$ th junction point of the function  $g_\ell(T)$  (from its right-side). By (10), the following inequality (11) holds

$$\delta_\ell(v) < \dots < \delta_\ell(k + 1) < \delta_\ell(k) \dots < \delta_\ell(2) < \delta_\ell(1) \tag{11}$$

where  $v$  is an (unknown) upper bound on the value of  $k$ .

Theorem 1 is an immediate result from (10) and (11).

**Theorem 1.** Suppose that  $k^*(w^-)$  and  $k^*(w^+)$  are the optimal multipliers of the left-side and right-side convex curves with regard to a junction point  $w$  of the  $g_\ell(T)$  function, then  $k^*(w^-) = k^*(w^+) + 1$ .

The following corollary is a by-product of (11), and it provides an easier way to obtain the optimal maintenance frequency  $k_\ell^*(T) \in \mathbb{N}^+$  for the  $g_\ell(T)$  function for any given  $T > 0$ .

**Corollary 1.** For any given  $T > 0$ , an optimal value of  $k_\ell^*(T) \in \mathbb{N}^+$  for the  $g_\ell(T)$  function is given by

$$k_\ell^*(T) = \left\lceil -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4U_\ell}{V_\ell T^2}} \right\rceil \tag{12}$$

with  $\lceil \cdot \rceil$  denoting the upper-entier function.

**Proof.** For any given  $T > 0$ , an optimal value of  $k \in \mathbb{N}^+$  for the  $g_\ell(T)$  function is such that  $T_\ell^{(k)} \leq T < T_\ell^{(k-1)}$ . Equivalently, the value of  $k$  must satisfy

$$\sqrt{\frac{U_\ell}{V_\ell(k+1)k}} \leq T \quad \text{and} \quad T < \sqrt{\frac{U_\ell}{V_\ell(k-1)k}}.$$

Therefore, we have  $k^2 + k - \frac{U_\ell}{V_\ell T^2} \geq 0$ . Since  $k$  must be positive, we have

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4U_\ell}{V_\ell T^2}} \leq k < \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4U_\ell}{V_\ell T^2}}.$$

Thus, we complete the proof. ■

### 3.2. Some insights into the optimal cost function

Here, we shall utilize our theoretical results on the  $\Phi_\ell(k_\ell, T)$  and  $g_\ell(T)$  functions to gain more insights into the  $\Gamma(T)$  function.

First, Propositions 2 and 3 characterize the  $\Gamma(T)$  function as follows.

**Proposition 2.** *The  $\Gamma(T)$  function is piece-wise convex with respect to  $T$ .*

**Proof.** The first term in the  $\Gamma(T)$  function, i.e.,  $S/T$ , is a convex function with respect to  $T$ . The second term, i.e.,  $\sum_{\ell=1}^L g_\ell(T)$ , is a piece-wise convex function since it is the sum of  $L$  piece-wise convex functions. Since the  $\Gamma(T)$  function is the sum of a convex function and  $L$  piece-wise convex functions, it is obvious piece-wise convex with respect to  $T$ . ■

**Proposition 3.** *All the junction points for vehicle group  $\ell$ , will be inherited by the  $\Gamma(T)$  function. In other words, if  $w$  is a junction point for vehicle group  $\ell$ ,  $w$  must also show as a junction point on the piece-wise convex curve of the  $\Gamma(T)$  function.*

**Proof.** Recall that the function  $\Gamma(T)$  is a separable function where  $\Gamma(T) = \inf_{T>0} \{S/T + \sum_{\ell=1}^L g_\ell(T)\}$ . Without loss of generality, assume that  $w$  is a junction point for vehicle group  $\ell$ , but not a junction point for the other  $(L - 1)$  vehicle groups. Then, there must exist  $\varepsilon > 0$  such that the followings hold.

1. The curve for  $\sum_{j \neq \ell} g_j(T)$  is convex in the interval of  $[w - \varepsilon, w + \varepsilon]$  since each one of  $g_\ell(T)$  is convex in  $[w - \varepsilon, w + \varepsilon]$  where  $j \neq \ell$ ,
2.  $g_\ell(T)$  is convex in the intervals of  $[w - \varepsilon, w]$  and  $[w, w + \varepsilon]$ .
3.  $S/T$  is convex in the intervals of  $[w - \varepsilon, w]$  and  $[w, w + \varepsilon]$ .

Since  $\Gamma(T) = S/T + g_\ell(T) + \sum_{j \neq \ell} g_j(T)$ ,  $\Gamma(T)$  is still convex in the intervals  $[w - \varepsilon, w]$  and  $[w, w + \varepsilon]$ . Therefore,  $w$  is a junction point on the curve of  $\Gamma(T)$ . ■

To make our notation more concise, we define  $\mathbf{k} \equiv (k_1, \dots, k_L)$  to represent a vector of maintenance frequencies.

Theorem 2 is an immediate result of Theorem 1 and Proposition 3.

**Theorem 2.** *Suppose that  $\mathbf{k}(w^-)$  and  $\mathbf{k}(w^+)$ , respectively, are vectors of optimal multipliers for left-side and right-side convex curves with regard to a junction point  $w$  in the plot of the  $\Gamma(T)$  function. Then,  $\mathbf{k}(w^-)$  is secured from  $\mathbf{k}(w^+)$  by changing at least one of  $k_\ell$  by  $k_\ell^*(w^-) = k_\ell^*(w^+) + 1$ .*

## 4. The search algorithm

In this section, we propose a search algorithm that solves the optimal solution for the problem  $(P_1)$  in (8).

Our theoretical results in Section 3 provide us an important foundation to design the proposed search algorithm for solving the problem  $(P_1)$ . Since the proposed algorithm searches along the  $T$ -axis, we shall define the search range by setting a lower bound and an upper bound on the  $T$ -axis, which are denoted by  $T_{\min}$  and  $T_{\max}$ , respectively. We note that the bounds  $T_{\min}$  and  $T_{\max}$  are derived by asserting that the optimal solution in  $[T_{\min}, T_{\max}]$  must be no worse than any solution outside of  $[T_{\min}, T_{\max}]$ . Also, we must utilize our theoretical results on the  $\Gamma(T)$  function, especially, the properties of the junction points.

In the following discussions, we first discuss how to find lower and upper bounds of the search range. Then, we demonstrate how to use the junction points to proceed with the search. Finally, we summarize our proposed search algorithm.

### 4.1. The lower and upper bounds

We derive a lower and an upper bound on the search range by a relaxed problem for the problem  $(P_1)$ . By relaxing the constraints  $k_\ell \in \mathbb{N}^+$  by  $k_\ell \geq 1$  for  $\ell = 1, \dots, L$ , we obtain a relaxation, namely (R), for the problem  $(P_1)$  as follows.

$$(R) \quad \Gamma^{(R)}(k_1, \dots, k_L, T) = \inf_{T>0} \left\{ S/T + \sum_{\ell=1}^L \Phi_\ell(k_\ell, T) \mid k_\ell \geq 1, \ell = 1, \dots, L \right\}. \tag{13}$$

Clearly, for any  $T$ , the problem (R) secures an optimal value no larger than that of the problem  $(P_1)$ . Namely, the optimal cost curve of the problem (R) serves the lower envelope for that of problem  $(P_1)$  as shown in Fig. 2.

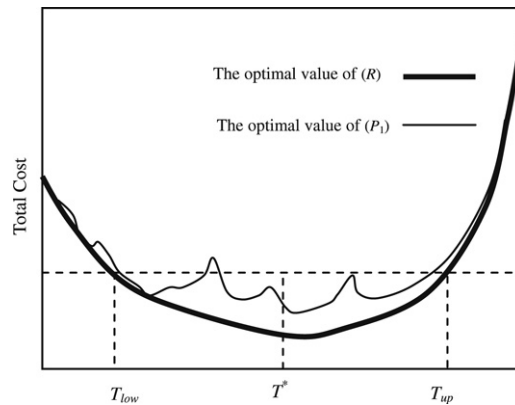


Fig. 2. The optimal cost curves of the problem (P<sub>1</sub>) and the problem (R).

Denote as  $T_p^*$  and  $T_R^*$  the optimal value of  $T$  for the problems (P<sub>1</sub>) and (R), respectively. We observe that solving (R) is equivalent to obtain the optimal solution for the problem (R<sub>1</sub>) as follows.

$$(R_1) h(T) = \inf_{T>0} \left\{ S/T + \sum_{\ell=1}^L g_{\ell}^{(R)}(T) \right\} \tag{14}$$

where

$$g_{\ell}^{(R)}(T) \equiv \inf_{T>0} \{ \Phi_{\ell}(k_{\ell}, T) | k_{\ell} \geq 1, \ell = 1, \dots, L \}. \tag{15}$$

By Proposition 1, it follows that

$$g_{\ell}^{(R)}(T) = \begin{cases} \Phi_{\ell}(1, x_{\ell}^*) & \text{if } T \leq x_{\ell}^* \\ \Phi_{\ell}(1, T) & \text{if } T > x_{\ell}^* \end{cases} \tag{16}$$

where  $x_{\ell}^*$  is expressed in (5). So, the function  $g_{\ell}^{(R)}(T)$  is convex, increasing, and continuously differentiable on  $(0, \infty)$ . Without loss generality, we assume that  $x_1^* \leq x_2^* \leq \dots \leq x_L^*$ , the strictly increasing derivative  $h'(\cdot)$  is given by

$$h'(T) = \begin{cases} \frac{-S}{T^2} & \text{if } T \leq x_1^* \\ \frac{\sum_{\ell=1}^i n_{\ell} V_{\ell} - \left( S + \sum_{\ell=1}^i n_{\ell} U_{\ell} \right)}{T^2} & \text{if } x_i^* \leq T \leq x_{i+1}^*, 1 \leq i \leq L-1 \\ \frac{\sum_{\ell=1}^L n_{\ell} V_{\ell} - \left( S + \sum_{\ell=1}^L n_{\ell} U_{\ell} \right)}{T^2} & \text{if } T \geq x_L^*. \end{cases} \tag{17}$$

By setting the derivative of  $h(\cdot)$  in (17) to zero, we have the following lemma to locate the optimal solution  $T_R^*$  for (R<sub>1</sub>).

**Lemma 1.** Assume without loss generality that  $x_1^* \leq x_2^* \leq \dots \leq x_L^*$ . If it holds that  $\ell^* \equiv \max\{1 \leq \ell \leq L : h'(x_{\ell}^*) < 0\}$ , then the optimal solution  $T_R^*$  of (R<sub>1</sub>) is given by

$$T_R^* = \sqrt{\left( S + \sum_{\ell=1}^{\ell^*} n_{\ell} U_{\ell} \right) / \sum_{\ell=1}^{\ell^*} n_{\ell} V_{\ell}}. \tag{18}$$

Let  $TC(T_R^*)$  be the objective function value of the problem (P) at  $T_R^*$ , i.e.,  $TC(T_R^*) = \Psi(\mathbf{k}(T_R^*), T_R^*)$ . Obviously,  $TC(T_R^*)$  serves as an upper bound on the optimal objective function value of the problem (P<sub>1</sub>). Denote  $T_{low}$  and  $T_{up}$  as the lower and the upper bounds of the search range. In the following lemma, we will show that a lower and an upper bound on  $T_R^*$  are given by the two values of  $T$  where the objective function of (R) is equal to  $TC(T_R^*)$ . The derivation of the bounds is done by the following proposition.

**Proposition 4.** Let  $T_{low}$  and  $T_{up}$  be the smallest and the largest  $T$ , respectively, for which the objective function of (R) is equal to  $TC(T_R^*)$ . Then, the optimal value of  $T$  for the problem (P) must lie between  $T_{low}$  and  $T_{up}$ , i.e.,  $T_p^* \in [T_{low}, T_{up}]$ .

**Proof.** Since the objective function of (R) is strictly convex, we clearly have the results that  $T_{low} \leq T_R^* \leq T_{up}$ . Consequently, the objective function value is larger than  $TC(T_R^*)$  for  $T < T_{low}$ . Since (R) is a relaxation of (P), so that is a lower bound on  $T_p^*$ .

Similarly, we may proof that  $T_p^* \leq T_{up}$ . ■

We note that the one may easily locate the bounds  $T_{low}$  and  $T_{up}$  by some line search methods (see Cormen, et al. [17]) and set the bounds by letting  $T_{min} = T_{low}$  and  $T_{max} = T_{up}$ .

Intuitively, if we may shorten the search range on the  $T$ -axis, we may reduce computational efforts in the proposed search algorithm. Therefore, we are motivated to find another upper bound and another lower bound to possibly shorten the search range.

First, we present another upper bound on the search range by the Common Cycle (CC) approach in which it requires that  $k_\ell = 1$  for all  $\ell$ , i.e., all of the vehicle groups share the same maintenance cycle. We set

$$T_{CC} = \sqrt{\left( S + \sum_{\ell} n_{\ell} U_{\ell} \right) / \sum_{\ell} n_{\ell} V_{\ell}} \tag{19}$$

where  $T_{CC}$  is the optimal maintenance cycle for the CC approach. Next, we will show that it is appropriate to set  $T_{max} = T_{CC}$  in the following lemma.

**Lemma 2.** For the  $\Gamma(T)$  function, there exist no local minima for  $T > T_{CC}$ .

**Proof.** For any given vector  $\mathbf{k}$ , one may obtain its local minimum,  $\tilde{T}(\mathbf{k})$ , by taking the first derivative of the objective function  $\Gamma(T)$  and equating it to zero.

$$\tilde{T}(\mathbf{k}) = \sqrt{\left( S + \sum_{\ell=1}^L \frac{n_{\ell} U_{\ell}}{k_{\ell}} \right) / \sum_{\ell=1}^L n_{\ell} V_{\ell} k_{\ell}}. \tag{20}$$

It is obvious that  $\tilde{T}(\mathbf{k}) \leq T_{CC}$  since  $k_{\ell} \geq 1$  for all  $\ell$ . Therefore, there exists no local minimum for  $T > T_{CC}$ . ■

Denote the optimal objective function value of (P<sub>1</sub>) and the optimal value of the basic period by  $\Gamma^*$  and  $T_p^*$ . Next, we derive a lower bound on the search range in the following lemma.

**Lemma 3.** The value  $\beta_1$  serves as a lower bound for  $T_p^*$

$$\text{where } \beta_1 = \frac{2S}{\Gamma^U} \tag{21}$$

where  $\Gamma^U$  is an upper bound on the optimal objective function value of the problem (P<sub>1</sub>).

**Proof.** For any given vector  $\mathbf{k}$ , by substituting its local minimum  $\tilde{T}(\mathbf{k})$  into the objective function of the problem (P<sub>1</sub>) in (8), one shall obtain its optimal objective function value by

$$\Gamma(\mathbf{k}, T) = 2 \sqrt{\left( S + \sum_{\ell=1}^L n_{\ell} U_{\ell} k_{\ell} \right) \left( \sum_{\ell=1}^L n_{\ell} V_{\ell} k_{\ell} \right)}. \tag{22}$$

By the expressions of  $\tilde{T}(\mathbf{k})$  in (20), it follows that  $\Gamma^* T_p^* > 2S$ , so  $T_p^* > 2S/\Gamma^*$ . Given  $\Gamma^U$  is an upper bound on the optimal objective function value of the problem (P<sub>1</sub>), it obviously holds that  $T_p^* > 2S/\Gamma^U$  since  $\Gamma^U \geq \Gamma^*$ . ■

Note that we need an upper bound  $\Gamma^U$  to obtain  $\beta_1$  as indicated in Eq. (21). The lower the value of  $\Gamma^U$ , the tighter the lower bound  $\beta_1$ . Here, we have an easy way to obtain a good value of  $\Gamma^U$ . First, we shall locate  $T_0 = \min_{\ell} \sqrt{0.5 U_{\ell} / V_{\ell}}$ . Denote  $\mathbf{k}^*(T) \equiv (k_1^*(T), k_2^*(T), \dots, k_L^*(T))$  as the vector of optimal maintenance frequency with respect to a given value of  $T$ . Then, we obtain the optimal  $\mathbf{k}^*(T_0)$  corresponding to  $T_0$  by (12). Since the objective function value of any feasible solution serves as an upper bound on  $\Gamma^*$ , we have an upper bound by  $\Gamma^U = \Gamma(\mathbf{k}^*(T_0), T_0)$  from Eq. (22). So, a lower bound is obtained by  $2S/\Gamma^U$ .

Following the discussions above, we obtain the search range  $[T_{min}, T_{max}]$  by setting  $T_{min} = \max(T_{low}, \beta_1)$  and  $T_{max} = \min(T_{up}, T_{CC})$ . Next, we are going to elaborate the proposed mechanism for searching all the local optimum existing in the interval  $[T_{min}, T_{max}]$ .

#### 4.2. Proceed with the search by the junction points

Note that the proposed algorithm proceeds with the search from the upper bound  $T_{max}$  to lower values of  $T$  until it meets the lower bound  $T_{min}$ . Denote  $\mathbf{k}^*(T)$  as the vector of optimal multipliers at  $T$ . Before starting the search, we first obtain  $\mathbf{k}^*(T_{max})$  by (12) in Corollary 1. Then, by Propositions 2 and 3, each junction point  $\delta_{\ell}(k_{\ell})$  provides the information that one should change the optimal maintenance multiplier of the vehicle group  $\ell$  from  $k_{\ell}$  to  $(k_{\ell} + 1)$  at  $\delta_{\ell}(k_{\ell})$  to obtain the optimal value for the  $\Gamma(T)$  function. Therefore, during the search, we need to keep an  $L$ -dimensional vector  $\mathbf{\Delta} \equiv (\delta_1(k_1), \dots, \delta_L(k_L))$  where  $\ell = 1, \dots, L$ . The vector  $\mathbf{\Delta}$  records the location of the next junction point where each vehicle group should change its optimal maintenance multiplier. Since the algorithm searches toward lower values of  $T$ , one shall change the multiplier

**Table 1**  
The data set of the demonstrative example

	$n_{ij}$	$s_{ij}$	$X_{ij}$	$Y_{ij}$	$a_{ij}$	$b_{ij}$
Branch 1	10	60	0.75	0.92	25	9
	20	40	0.95	0.95	40	8
Branch 2	24	110	0.85	0.87	55	7.5
	10	75	0.65	0.90	30	6.5
	8	65	0.92	0.93	45	8
S	200					

for the particular vehicle group with the *largest* value of  $\delta_\ell(k_\ell)$  to correctly update the vector of optimal multipliers. Let  $T_c$  be the current value of  $T$  where the search algorithm reaches. Denote  $\pi$  as the index for the vehicle group with the largest value of  $\delta_\ell(k_\ell)$ , i.e.,

$$\pi = \arg \min_\ell \{ \delta_\ell(k_\ell) < T_c \}. \tag{23}$$

When proceeding with the search from  $T_c$ , we need to update the vector of optimal multipliers at  $\delta_\pi(k_\pi)$  by

$$\mathbf{k}^*(\delta_\pi(k_\pi)) \equiv (\mathbf{k}^*(T_c) \setminus \{k_\pi\}) \cup \{k_\pi + 1\} \tag{24}$$

where “\” denotes set subtraction.

Note that **Theorem 2** implies that the vector of optimal multipliers  $\mathbf{k}^*$  is invariant in each convex sub-interval (i.e., between a pair of consecutive junction points) on the  $\Gamma(T)$  function. Hence, this step actually obtains the vector of optimal multipliers for all the values of  $T \in (\delta_\pi(k_\pi), T_c)$ . Then, we should check if the local minimum for  $\mathbf{k}^*(T_c)$  exists in the convex sub-interval  $(\delta_\pi(k_\pi), T_c)$  since such a local minimum could be a candidate for the optimal solution. (For any given set of  $\mathbf{k}$ , one may obtain its local minimum,  $\tilde{T}(\mathbf{k})$ , by Eq. (20).)

### 4.3. The proposed algorithm

We are now ready to enunciate the proposed search algorithm as follows.

1. Obtain the lower bound and the upper bound of the search range by:
  - (a) Compute the value of  $T_R^*$  by (18) and secure the values of  $T_{low}$  and  $T_{up}$  by line search methods.
  - (b) Obtain  $T_{CC}$  by (19), compute  $T_0 = \min_\ell \sqrt{0.5U_\ell/V_\ell}$  and obtain  $\mathbf{k}^*(T_0)$  (i.e., the optimal vector of multipliers corresponding to  $T_0$ ) by (12).
  - (c) We calculate  $\Gamma^U = \Gamma(\mathbf{k}^*(T_0), T_0)$  by Eq. (22). Then, we have another lower bound by  $\beta_1 = 2S/\Gamma^U$ .
  - (d) Set  $T_{min} = \max(T_{low}, \beta_1)$  and  $T_{max} = \min(T_{up}, T_{CC})$ .
2. Set  $T_c = T_{max}$  and obtain  $\mathbf{k}^*(T_c)$  by (12). Also, set  $TC^* = TC(\mathbf{k}^*(T_{CC}), T_{CC})$  and  $\mathbf{K}^* = \mathbf{k}^*(T_{CC})$  to start the search.
3. If  $T_c \leq T_{min}$ , then go to step 5.
4. Proceed with the search to the next convex sub-interval:
  - (a) Set  $\pi = \arg \min_\ell \{ \delta_\ell(k_\ell) < T_c \}$  and  $\mathbf{k}^*(\delta_\pi(k_\pi)) \equiv (\mathbf{k}^*(T_c) \setminus \{k_\pi\}) \cup \{k_\pi + 1\}$ . Then, let  $T_c = \delta_\pi(k_\pi)$ .
  - (b) Calculate  $\tilde{T}(\mathbf{k}^*(T_c))$  by (20) and compute  $TC(\mathbf{k}^*(T_c), \tilde{T}(\mathbf{k}^*(T_c)))$ .
  - (c) If  $TC^* > TC(\mathbf{k}^*(T_c), \tilde{T}(\mathbf{k}^*(T_c)))$ , set  $TC^* = TC(\mathbf{k}^*(T_c), \tilde{T}(\mathbf{k}^*(T_c)))$ ,  $\mathbf{K}^* = \mathbf{k}^*(T_c)$ , and  $T^* = T_c$ .
  - (d) Go to Step 3.
5. Output the optimal solution  $(\mathbf{K}^*, T^*)$  with the corresponding minimal cost  $TC^*$ .

## 5. Numerical experiments

In the first part of this section, we employ a numerical example to demonstrate implementation of the proposed search algorithm. Then, we conduct sensitivity analysis on parameters of the extend model for the TFMSP to gain more managerial insights into the benefit from coordination of maintenance scheduling among the branches of a logistic service provider.

### 5.1. A demonstrative example

In this section, we use a two-branch example to demonstrate the implementation of the proposed search algorithm. In this example, there is a total of two groups and three groups of vehicles in the first and the second branch of the logistic service provider, respectively. The data set for this example is given in **Table 1**.

In the first step, we compute the value of  $T_R^*$  and find the bounds  $T_{low}$  and  $T_{up}$ . We locate the optimal  $T_R^*$  of the problem (R<sub>1</sub>) by  $T_R^* = 2.180$ . We use (12) in **Corollary 1** to get the vector of optimal maintenance frequencies  $\mathbf{k}(T_R^*) = (k_{11}, k_{12}, k_{21}, k_{22}, k_{23}) = (2, 1, 2, 2, 1)$  to obtain a feasible solution for the problem (P) at  $T_R^*$ . Therefore, we have the objective



**Table 2**  
The 7 local optimum obtained by the proposed search algorithm

$m_1$		$m_2$			$T(\mathbf{k}^*(T_c))$	$TC(\mathbf{k}^*(T_c), \bar{T}(\mathbf{k}^*(T_c)))$
$k_{11}$	$k_{12}$	$k_{21}$	$k_{22}$	$k_{23}$		
1	1	1	1	1	3.800	\$4179.00
1	1	2	1	1	2.860	4142.71
1	1	2	2	1	2.580	4106.47
2	1	2	2	1	2.316	4107.73
2	1	2	2	2	2.167	4117.19
2	1	3	2	2	1.831	4109.24
2	1	3	3	2	1.725	4101.12

function value by  $TC(T_R^*) = TC(\mathbf{k}(T_R^*), T_R^*) = \$ 1716.76$ . Next, we locate the bounds  $T_{low}$  and  $T_{up}$  by some line search method by finding the two values of  $T$  where the objective function of  $(R_1)$  equals to  $TC(T_R^*)$ . Then, we obtain the search range by  $T_{low} = 1.540$  and  $T_{up} = 3.072$ . Next, we have  $\beta_1 = 0.220$  secured by  $\beta_1 = 2S/\Gamma^U$  where  $\Gamma^U = TC(\mathbf{k}^*(T_0), T_0) = \$ 1788.05$  and  $T_0 = \min_{\ell} \sqrt{0.5U_{\ell}/V_{\ell}} = 0.996$ . Let  $\mathbf{K}^* = \mathbf{k}^*(T_0)$ ,  $T^* = T_0$  and  $\Gamma^* = \Gamma^U$ . Also, we get  $T_{CC} = 3.800$  by (19). Therefore, we have  $T_{min} = \max(T_{low}, \beta_1) = 1.540$  and  $T_{max} = \min(T_{up}, T_{CC}) = 3.800$ .

Here, we start the search with  $T_c = T_{max} = 3.800$ . Since  $T_c > T_{min}$ , we proceed with the search to the next convex sub-interval by setting  $\pi = \arg \min_i \{\delta_i(k_i) < T_c\} = 3$ . We locate the next junction point at  $\delta_3(k_3) = 3.546$ . With the vector of optimal multipliers being  $\mathbf{k}^*(T_c) = (1, 1, 2, 1, 1)$ , we obtain the local minimum  $\bar{T}(\mathbf{k}^*(T_c)) = 2.864$  and the corresponding optimal objective function value  $TC(\mathbf{k}^*(T_c), \bar{T}(\mathbf{k}^*(T_c))) = \$ 4142.71$ . We proceed to the next convex sub-interval by setting  $\pi = \arg \min_i \{\delta_i(k_i) < T_c\} = 4$ . Then let  $T_c = \delta_4(k_4) = 3.335$ . Therefore, we move to the new junction point by letting with the set of optimal maintenance frequency as  $\mathbf{k}^*(\delta_{\pi}(k_{\pi})) \equiv (\mathbf{k}^*(T_c) \setminus \{k_4\}) \cup \{k_4 + 1\} = (1, 1, 2, 2, 1)$ . We obtain the local minimum by  $\bar{T}(\mathbf{k}^*(T_c)) = 2.5795$ . The optimal objective function value corresponding to this local minimum is  $TC(\mathbf{k}^*(T_c), \bar{T}(\mathbf{k}^*(T_c))) = \$4106.47$ .

In this example, we visit in total only 7 convex sub-intervals before the search algorithm terminates. The optimal solution is obtained by  $T^* = 1.7254$  and  $\mathbf{K}^* = (2, 1, 3, 3, 2)$  with the optimal annual total cost given by \$4101.12. Table 2 summarizes the vectors of optimal multipliers for these 7 convex sub-intervals and their corresponding local minima obtained before the search terminates.

We may employ Goyal and Gunasekaran’s [1] approach (which is abbreviated as G&G for the rest of paper) to solve the TFMSp for two branches independently. We obtain the optimal solution for the first branch at  $T^* = 2.680$  with  $\mathbf{K}^* = (1, 1)$  and the optimal objective function value being \$1386.80. Also, the optimal solution for the second branch is given by  $T^* = 4.738$  and  $\mathbf{K}^* = (1, 1, 1)$  with the optimal objective function value being \$2779.46. The total annual cost of the solution obtained from the G&G approach (without coordination) is around 1.59% larger than the optimal solution from the extended TFMSp with coordination.

In order to observe the benefit from coordination among branches, we should compare our solution with solving the TFMSp for two branches independently using the proposed search algorithm. For the first branch, we solve the optimal solution by  $T^* = 1.956$  and  $\mathbf{K}^* = (2, 1)$  with the optimal objective function being \$1,376.11. We have the optimal solution at  $T^* = 4.738$  with  $\mathbf{K}^* = (1, 1, 1)$  and the optimal objective function value being \$2,779.46 for the second branch. Therefore, this example shows that coordination of maintenance scheduling among branches leads to a cost saving of 1.33%.

### 5.2. Sensitivity analysis

This section presents our sensitivity analysis on parameters of the extended TFMSp model. We would like to observe how parameters in the model affect cost saving from the coordination of maintenance scheduling among vehicle groups in different branches (which is abbreviated as the coordination policy later).

Here, we conduct our sensitivity analysis based on the demonstrative example presented in Section 5.1. To analyze the sensitivity of a parameter, we will observe the change in magnitude of cost saving from the coordination policy after perturbing the value of that particular parameter. In our numerical experiments, we have four levels of perturbation in parameters of the extended TFMSp model, namely, replacing the parameter with 50%, 75%, 125%, and 150% of its original value, respectively. Also, we change the parameter only one at a time (but keeping other parameters the same) to avoid that the confounding effects from different parameters could make our analysis difficult for interpretation.

To facilitate our discussion later, we first define some notations. We denote  $TC^{G\&G}$  (no-coordination) as the objective function value of the solution obtained from the G&G approach without coordination. Since the last names of the authors are Huang and Yao, we use H&Y as the abbreviation for the proposed search algorithm. We denote  $TC^{H\&Y}$  (coordination) and  $TC^{H\&Y}$  (no-coordination) as the objective function values of solutions obtained by the proposed search algorithm for scenarios with and without using the coordination policy, respectively.

Next, we employ the optimal solution from the proposed search algorithm as a benchmark for comparison. We define  $CS^{G\&G}$  as a measure for the cost saving comparing to the solution obtained from the G&G approach without coordination as

**Table 3**

A summary of our sensitivity analysis

Parameters	Perturbed values (%)	The cost saving from coordination	
		$CS^{G\&G}$ (%)	$CS^{H\&Y}$ (%)
$S$	50	1.58	0.96
	75	1.62	1.18
	125	1.69	1.62
	150	1.90	1.90
$s_{ij}$	50	2.34	1.20
	75	1.90	1.15
	125	1.13	1.06
	150	0.92	0.92
$X_{ij}$	50	0.91	0.91
	75	1.15	1.15
	125	1.92	1.72
	150	2.47	2.27
$n_{ij}$	50	2.38	2.38
	75	1.95	1.74
	125	1.61	1.22
	150	1.59	1.09
$a_{ij}$	50	1.25	1.25
	75	1.37	1.37
	125	1.94	1.51
	150	2.04	1.77
$b_{ij}$	50	1.48	1.08
	75	1.69	1.21
	125	1.81	1.42
	150	1.86	1.48

shown in Eq. (25).

$$CS^{G\&G} = \frac{TC^{G\&G}(\text{no-coordination}) - TC^{H\&Y}(\text{coordination})}{TC^{H\&Y}(\text{coordination})} \cdot 100\%. \quad (25)$$

Similarly, we may define  $CS^{H\&Y}$  as a measure for the cost saving compared with the solution obtained from the coordination policy (while both solving the extended TFMSP model using the proposed search algorithm).

Table 3 summarizes the results of our sensitivity analysis. For example, the parameter  $S$  (i.e., the fixed cost incurred for all vehicle groups scheduled for maintenance in each basic period) is the first parameter shown in Table 3. The measure of the cost saving from coordination  $CS^{G\&G}$  increases from 1.58% to 1.90% when the value of  $S$  increases from 50% to 150% of its original value. One may have a similar observation on the measure  $CS^{H\&Y}$ .

From Table 3, we may have several interesting findings that may provide more managerial insights into the coordination policy as follows.

1. The larger the value of  $S$ , the more significant the cost saving from the coordination policy. This observation matches with our intuition since when the fixed cost incurred in each basic period is large, the LSP may gain more cost savings from coordination among branches.
2. The larger the values of  $s_{ij}$ , the less the cost saving from the coordination policy. We note that when the fixed cost for an individual vehicle group is significant, the LSP would pay more attention to the maintenance scheduling of individual vehicle groups within a branch but have less emphasis on coordination among branches.
3. The larger the values of  $X_{ij}$ , the more significant the cost saving from the coordination policy. We note that a larger value of  $X_{ij}$  implies a higher proportion of time for maintenance work in a maintenance cycle. Therefore, since maintenance scheduling becomes an important issue in such a case, the coordination policy could lead to more cost savings.
4. The larger the values of  $n_{ij}$ , the less the cost saving from the coordination policy. When the values of  $n_{ij}$  are large, the cost saving from coordination is not considerable since the costs incurred within a branch are more significant than fixed costs among branches.
5. The larger the values of  $a_{ij}$  and  $b_{ij}$ , the more significant the cost saving from the coordination policy. When the values of  $a_{ij}$  and  $b_{ij}$  are large, our numerical results indicate that the value of basic period usually becomes longer in such cases. If the managers did not coordinate among the branches in an LSP, the operating cost of vehicle groups becomes more significant.
6. The value of cost saving is more sensitive to parameters  $X_{ij}$  and  $s_{ij}$ , but less sensitive to parameters  $S$  and  $n_{ij}$ .

## 6. Concluding remarks

In this paper, we study the Transportation Fleet Maintenance Scheduling Problem (TFMSP) for a Logistic Service Provider (LSP) with many branches. In order to solve this problem, we formulate an extended model of the TFMSP with many branches and conduct a full analysis on the extended model in this study. Also, by utilizing our theoretical results, we propose an efficient search algorithm that effectively solves an optimal solution for the extended TFMSP. Our numerical results show that the whole transportation fleet system of an LSP can obtain significant cost savings from the coordination policy.

In the area of supply chain management, researchers have devoted their efforts to analyze and emphasize the benefit from the coordination of production and inventory policies among the firms in supply chains. To the best of the authors' knowledge, this paper presents the first work on the coordination of maintenance scheduling among the branches in an LSP. Hopefully, our study may draw attention from the interested readers and invite more researchers' to investigate their efforts on this topic.

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