

國立勤益技術學院九十五學年度研究所碩士班招生筆試試題卷

所別:精密機械與製造科技研究所

組別:甲、乙、丙

身分別:一般生

科目:工程數學

准考證號碼:

(考生自填)

考生注意事項:

- 一、考試時間100分鐘
- 二、應考人可攜帶不具儲存程式功能之計算機
- 三、無論是否使用計算機，作答時均須詳列解答過程。
- 四、請依照題目順序在答案卷作答。
- 五、共三頁

1. [10%] The austenite state of steel with $750^{\circ}C$ is quenched with $200^{\circ}C$ media by the following Newton's cooling law. During the first 30 minutes, the metal is cooled down to $300^{\circ}C$

$$\frac{dT}{dt} = k(T - 200)$$

where T is temperature of the metal, t is the quench time, and k is the proportional constant.

Determine (1) the constant k (1/min)

(2) the temperature after the first 60 minutes.

2. [15%] Solve the given initial-value problem

$$\begin{aligned}\frac{dx}{dt} &= 2x - 3y \\ \frac{dy}{dt} &= x - 1\end{aligned}$$

$$x(0) = 0, y(0) = 0$$

3. [10%] Matrix

$$A = \begin{bmatrix} 6 & 3 \\ -10 & -5 \end{bmatrix}$$

Determine (1) the eigenvalues and eigenvectors

(2) A^{10}

< 共三頁，第一頁，精機所工程數學 >

4. [10%] Solve the given initial-value problem

$$y'' + 4y = \delta(t - \pi) \quad y(0) = 0, \quad y'(0) = 2$$

where $\delta(t - \pi)$ is an impulse function.

5. [10%] A linear system $Ax = B$

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & -2 \\ 2 & -5 & 7 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 \\ -7 \\ 19 \end{bmatrix}$$

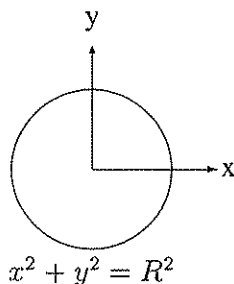
Determine (1) the rank of matrix A

(2) solve the unknowns

(3) sketch the solution in Cartesian coordinates

6. [10%] The homogeneous circular disk of mass m , is located in the $x-y$ plane and described by $x^2 + y^2 = R^2$.

Find the mass moment of inertia (1) I_x (2) I_z



7. [10%] The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2, \quad v = xy + yz + z^2, \quad \text{and} \quad w = -3xz - (z^2/2) + 4.$$

Determine (1) the volumetric dilatation

(2) the rotation velocity.

8. [10%] A plane contains the given points $(1, 3, -1)$, $(0, 1, 0)$, $(0, 1, 1)$,

Determine (1) the normal vector

(2) the equation of the plane

9.[15%] A torque is applied to the free end of a circular shaft and suddenly removed. The shaft is elastic rotating deformation with fixed end at $x = 0$, and free at $x = 1$. Via nondimensionalizing, the governing equation is given by

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2}$$

where θ is the vibrating rotation angle, t is the time variable.

The initial and boundary conditions are

$$\theta(0, t) = 0 \quad \frac{\partial \theta}{\partial x}(1, t) = 0$$

$$\theta(x, 0) = \theta_0 x \quad \frac{\partial \theta}{\partial t}(x, 0) = 0$$

Determine (1) the vibrating mode shape function for the shaft.

(2) the $\theta(x, t)$ at the free end ($x = 1$)

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