

國立勤益技術學院九十二學年度研究所招生初試試題卷

所別：生產系統工程與管理研究所 組別：精密製造科技 身份別：一般生  
科目：工程數學 准考證號碼： (考生自填)

考生注意事項：

- 一、考試時間 100 分鐘
- 二、不得帶字典、翻譯機、計算機等
- 三、答前，請先核對試題，答案卷（試卷）與准考證上之所組別與考試科目是否相符。
- 四、請依照題目順序在答案卷作答。

1. Solve the following ODE:

(a)  $y' - y + y^2 = 0$  (8 %)

(b)  $x^2 y'' + 7xy' + 13y = 0$ . (8 %)

(c)  $y'' - 2y' + y = xe^x$  (10 %)

2. Solving the equation  $my'' + cy' + ky = 0$  by discussing the relations of the constants  $m$ ,  $c$ , and  $k$ . (12 %)

3. Prove: if  $A$  is a real, symmetric matrix, then the eigenvectors associated with distinct eigenvalues of  $A$  are orthogonal. (12 %)

4. (a) Find  $A^{100} = ?$  where  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ . (10%)

(b) Sketch the graph of the equation of  $2x^2 + 2xy + 2y^2 = 9$ . (10%)

5. Evaluate  $I = \int_{1-i}^{1+i} \frac{dz}{z(z-1)}$  for all possible values. (12 %)

6. The Laurent's series of  $f(z)$  in  $1 < |z| < \infty$  is (8 %)

$$\frac{1}{z} - \frac{1}{z^2} + \frac{1}{z^3} - \frac{1}{z^4} + \dots$$

and find  $f\left(\frac{1}{3}\right)$ .

7. Using the residue theorem to evaluate the following integral

$$\int_0^{\infty} \frac{dx}{(x^2 + 1)^2}. \quad (10 \%)$$