國立勤益技術學院九十三學年度研究所招生初試試題卷

所別：生產系統工程與管理所　組別：產業決策組　身分別：一般生

科目：作業研究　　准考證號碼：□□□□□□□□□（考生自填）

考生注意事項：
一、考試時間 100 分鐘。

一、***True/False:*** Indicate by "O" = "true" or "X" ="false." (Each 3 points, total 30 points)

___ 1. If the "float" of an activity of a project is positive, then the activity cannot be "critical" in the schedule.

___ 2. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

___ 3. In a transportation problem, if the current dual variables $U_2=3$ and $V_4=1$, and $C_{24}=2$, then the current basic solution cannot be optimal.

___ 4. The optimal value of a primal minimization LP problem is less than or equal to the objective value of every dual feasible solution.

___ 5. A Poisson process is "memoryless."

___ 6. If a primal minimization LP problem has a cost that is unbounded below, then the dual maximization problem has an objective that is unbounded above.

___ 7. One advantage of the revised simplex method is that it does not require the use of artificial variables.

___ 8. If a random variable T has an exponential distribution, then $P\{T>2 \mid T\geq1\} = P\{T>1\}$.

___ 9. An M/Ek/1 queueing system can be modeled as a continuous-time Markov chain.

___ 10. If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must equal zero.

二、**Integer Programming Model Formulation.** (Each 3 points, total 15 points)

Comquat owns four production plants at which personal computers are produced. In order to use a plant to produce computers, a fixed cost must be paid to set up the production line in that plant. Define the variables:

$y_i = 1$ if the production line has been set up at plant #i; 0 otherwise

x_i = # of computers produced at plant #i

For each restriction, choose a constraint from the list (a) through (l) below.

___ If the production line at plant 2 is set up, then that plant can produce up to 8000 computers; otherwise, none can be produced at that plant.

___ The production lines at plants 2 and 3 cannot both be set up.

___ The total production must be at least 20,000 computers.

___ If the production line at plant 2 is set up, that plant must produce at least 2000 computers.

___ If the production line at plant 2 is not set up, then the production line at plant 3 cannot be set up.

Constraints:

a. $y_2 \leq 8000x_2$	b. $y_1+y_2+y_3+y_4 \leq 3$	c. $y_1+y_2+y_3+y_4 \geq 3$	d. $y_2+y_3 \leq 1$
e. $y_2 \leq 2000x_2$	f. $x_2 \geq 2000y_2$	g. $x_1+x_2+x_3+x_4 \geq 20000$	h. $y_2+y_3 \geq 1$
i. $x_2 \leq 8000y_2$	j. $y_2 \leq y_3$	k. $y_2 \geq y_3$	l. *None of the above*

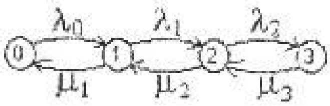
三、**Stochastic Process.** (Total 10 points)

A machine operator has the task of keeping three machines running. Each machine runs for an average of 1 hour before it becomes jammed or otherwise needs the operator's attention. He then spends an average of twelve minutes restoring the machine to running condition. Define a continuous-time Markov chain, the state of the system being the number of machines not running.

1. Specify the value of each of the transition rates: (Each 1 point)

$\lambda_0 =$ ___/hr $\lambda_1 =$ ___/hr $\lambda_2 =$ ___/hr

$\mu_1 =$ ___/hr $\mu_2 =$ ___/hr $\mu_3 =$ ___/hr



___2. Which equation is used to compute the steady-state probability π_0? (2 points)

(a.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu 3}$

(b.) $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}$

(c.) $\pi_0 = 1 \times \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu 3}$

(d.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu 3}$

(e.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}$

(f.) $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu 3}$

(g.) $\frac{1}{\pi_0} = 1 \times \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu 3}$

(h) None of the above

___3. What is the relationship between π_0 and π_1 *for this system*? (2 points)

(a.) $\pi_1 = \pi_0$ (b.) $\pi_1 = 0.1\pi_0$ (c.) $\pi_1 = 0.6\pi_0$

(d.) $\pi_1 = \frac{1}{6}\pi_0$ (e.) $\pi_1 = 3\pi_0$ (f.) None of the above

四、**Simplex and Sensitivity Analysis** (Total 22points)

$$\max imize \ z = 3x_1 + 2x_2 + 5x_3$$

subject to

$$x_1 + 2x_2 + x_3 \le 430$$
$$3x_1 + 2x_3 \le 460$$
$$x_1 + 4x_2 \le 420$$
$$x_1, x_2, x_3 \ge 0$$

the associated optimum tableau is as following:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
z	4	0	0			0	1350
x_2	-1/4	1	0	1/2	-1/4	0	
x_3	3/2	0	1	0	1/2	0	
x_6	2	0	0	-2	1	1	

(a) Filled out the blank cells in the above table (each 2 points, total 10 points)

(b) If the right-hand-side b_3 is decreased from 420 to 320, find the revised optimal solution: x_1 = _____, x_2 = _____, x_3 = _____, z = _____. (each 3 points, total 12 points)

五、**Nonlinear Programming** (Total 23 points)

Consider the following quadratic programming problem

$$\max imize \ f(x) = 8x_1 - x_1^2 + 4x_2 - x_2^2$$

subject to

$$x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$

a. Verify f(x) is a concave function. (6 points)
b. Write down the KKT conditions. (8 points)
c. Derive the optimal solution. (9 points)