國立勤益技術學院九十四學年度研究所碩士班招生筆試試題卷

所別：生產系統工程與管理研究所　　組別：乙組

身分別：一般生

科目：作業研究　　准考證號碼：□□□□□□□□

（考生自填）

考生注意事項：

一、考試時間 100 分鐘。

試題一：**Two-Phase Method**. (Each 3 points, total 15 points)

Solve the following LP problem with two-phase approach.

maximize $Z = 3x_1 - x_2$

st $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

(1). The initial solution from phase one is x_1:____, x_2:____

(2). The optimal solution from phase two is x_1:____, x_2:____. Objective value Z:=_____

試題二：**Linear Programming Duality and Sensitivity Analysis**. (Total 20 points)

A company uses three resources to produce three products. The data is given in the following.

Resources	Products			Resources availability
	x_1	x_2	x_3	
Material I	1	1	1	100
Labor	10	4	5	600
Material II	2	2	6	300
Profit per unit	10	6	4	

(1). Formulate the above problem as a LP model to maximize the total profit. (Each 4 points, total 8 points)

Objective function:_____

Constraints:

_____

The final tableau of the simplex method for the LP model above is as following.

c_j		10	6	4	0	0	0	b
c_B	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
6	x_2	0	1	5/6	10/6	-1/6	0	200/3
10	x_1	1	0	1/6	-4/6	1/6	0	100/3
0	x_6	0	0	4	-2	0	1	100
$c_j - z_j$		0	0	-16/6	-20/6	-4/6	0	2200/3

(2). At what value of c_3 will encourage the company to produce product x_3. (2 points)

$c_3 \geq$ _____

(3). The unit profit c_1 of product x_1 can be changed in what range of value without changing the optimal solution. (4 points) _____ $\leq c_1 \leq$ _____

(4). Identify the shadow price for each of the three resources. (Each 2 points, total 6 points)

Shadow price for Material I, Labor, Material II = (_____, _____, _____)

試題三：**Nonlinear Programming Problem**. (Total 15 points)

Consider the following problem

$$\min \quad f(x, y) = x^2 + y^2 - 2$$
$$st \quad g(x, y) = x^2 - y^2 - 2 = 0$$

(1) Show that $f(x, y)$ is a convex function. (8 points)

(2) Find the optimal solution by Lagrangian multiplier method. (7 points)

試題四：**Integer Programming Model Formulation.** (Each 2 points, total 10 points)

The board of directors of a large manufacturing firm is considering the set of investments shown below: Let R_i be the annual revenue (in $millions) from investment i and C_i the cost (in $millions) to make investment i. The board wishes to maximize total annual revenues and invest no more than a total of 50 million dollars.

Investment	Revenue	Cost	
# i	R_i	C_i	Condition
1	1	5	*None*
2	2	8	Only if #1
3	3	12	*None*
4	4	18	Must if #1 and #2
5	5	24	Not if both #3 and #4
6	6	27	*None*
7	7	30	Only if both #3 and #6

Define the variables:

$X_i = 1$ if investment i is selected, else 0

a. Formulate this problem as a 0-1 integer LP problem.

b. Add a constraint or constraints to enforce the condition "Investment #2 can be selected only if #1 is selected".

c. Add a constraint or constraints to enforce the condition "Investment #4 *must* be selected if both #1 & #2 are selected".

d. Add a constraint or constraints to enforce the condition "Investment #5 *cannot* be selected if both #3 & #4 are selected".

e. Add a constraint or constraints to enforce the condition "Investment #7 only if both #3 and #6 are selected".

試題五：**Stochastic Process**. (Each 3 points, total 15 points)

A parking lot consists of four spaces. Cars making use of these spaces arrive according to a Poisson process at the rate of *eight cars per hour*. Parking time is exponentially distributed with mean of *30 minutes*. Cars who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves, but may get impatient and leave before a parking space opens up. Assume that the time that a driver is willing to wait has exponential distribution with an average of *15 minutes*. The temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, … 6.

The steady state probability distribution of the number of cars in the system is:

n	0	1	2	3	4	5	6
π_n	0.02	0. 08	0. 18	0.24	0.24	0.16	0.08

__ 1. What is the fraction of the time that there is at least one empty space? *(Choose nearest value!)*

a. 10% c. 30% e. 50% g. 70%

b. 20% d. 40% f. 60% h. 80%

__ 2. What is the average number of cars in the lot? *(Choose nearest value!)*

a. 1 c. 2 e. 3 g. 4

b. 1.5 d. 2.5 f. 3.5 h. 4.5

__ 3. What is the average number of cars waiting? *(Choose nearest value!)*

a. 0.1 c. 0.3 e. 0.5 g. 0.7

b. 0.2 d. 0.4 f. 0.6 h. 0.8

__ 4. What is the average arrival rate (keeping in mind that the arrival rate is zero when n=6)? *(Choose nearest value!)*

a. 5/hr c. 7/hr e. 9/hr g. 11/hr

b. 6/hr d. 8/hr f. 10/hr h. 12/hr

__ 5. According to Little's Law, what is the average time that a car waits for a parking space? *(Choose nearest value!)*

a. 0.025 hr c. 0.075 hr e. 0.25 hr g. 0.75 hr

b. 0.05hr d. 0.1 hr f. 0.5 hr h. 1 hr.

試題六：**Dynamic Programming.** (Each 2 points, total 10 points)

Suppose that there are 15 matches originally on the table, and you are challenged by your friend to play this game. Each player must pick up either 1, 2, 3, or 4 matches, with the player who picks up the last match paying $1.

Define **F(i)** to be the **minimal cost** to you (either $1 or $0) if

• it is your turn to pick up matches, and

• **i** matches remain on the table.

Thus, F(1) = 1, since you are forced to pick up the last match; F(2) = 0 (since you can pick up one match, forcing your opponent to pick up the last match), etc.

1. What is the value of F(3)? ____
2. What is the value of F(4)? ____
3. What is the value of F(6)? ____
4. What is the value of F(15)? ____

____5. If you are allowed to decide whether you or your friend should take the first turn, what is your optimal decision?

a. You take first turn c. You are indifferent about this choice

b. Friend takes first turn d. You refuse to play the game

試題七：**Markov Chains.** (Each 3 points, total 15 points)

The Green Valley Christmas Tree Farm owns a plot of land with 5000 evergreen trees. Each year they allow individuals to select and cut Christmas trees. However, they protect small trees (usually less than 4 feet tall) so that they will grow and be available for sale in future years. Currently 2000 trees are classified as protected trees, while the remaining 3000 are available for cutting. However, even though a tree is available for cutting in a given year, it possibly might not be selected for cutting until future years. While most trees not cut in a given year live until the next year (protected or unprotected), approximately 15% are lost to disease. Each year, approximately 50% of the unprotected trees are cut, and 40% of the protected trees surviving from the previous year have matured sufficiently to be made available for cutting.

Define a Markov chain model of the system consisting of a **single** tree, with states (1) protected, (2) unprotected, (3) dead, (4) cut & sold. The transition probability matrix is

$$P = \begin{bmatrix} 0.51 & 0.34 & 0.15 & 0 \\ 0 & 0.425 & 0.075 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The following computations were performed:

$$\begin{bmatrix} 0.49 & -0.34 \\ 0 & 0.575 \end{bmatrix}^{-1} = \begin{bmatrix} 2.0408 & 1.2067 \\ 0 & 1.7391 \end{bmatrix}$$

$$\begin{bmatrix} 2.0408 & 1.2067 \\ 0 & 1.7391 \end{bmatrix} \begin{bmatrix} 0.15 & 0 \\ 0.075 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.3966 & 0.6034 \\ 0.1304 & 0.8696 \end{bmatrix}$$

1. What are the absorbing states of this model? _____
2. What is the probability that a tree which is protected is eventually sold? _____
3. What is the probability that a protected tree eventually dies of disease? _____
4. How many of the farm's 5000 trees are expected to be sold eventually? _____
5. If a tree is initially protected, what is the expected number of years until it either is sold or dies? _____