

國立勤益科技大學九十六學年度研究所碩士班招生筆試試題卷  
 所別：工業工程與管理系碩士班  
 組別：乙組

科目：作業研究

准考證號碼：□□□□□□□□ (考生自填)

Operations Research

1. True/False: Indicate by "O" = "true" or "X" = "false." (Each 4 points, total 20 points)

- \_\_\_ (1) If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.
- \_\_\_ (2) The optimal value of a primal minimization LP problem is less than or equal to the objective value of every dual feasible solution.
- \_\_\_ (3) A Poisson process is "memoryless."
- \_\_\_ (4) If a random variable  $T$  has an exponential distribution, then  $P\{T > 2 \mid T \geq 1\} = P\{T > 2\}$ .
- \_\_\_ (5) If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must equal zero.

2. Simplex Method (Each 4 points, total 20 points)

$$\begin{aligned} \text{maximize } z &= 3x_1 + 2x_2 + 5x_3 \\ \text{subject to } &x_1 + 2x_2 + x_3 \leq 430 \\ &3x_1 + 2x_3 \leq 460 \\ &x_1 + 4x_2 \leq 420 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

The associated optimum tableau is as following:

Basic	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	solution
$z$	4	0	0			0	1350
$x_2$	-1/4	1	0	1/2	-1/4	0	
$x_3$	3/2	0	1	0	1/2	0	
$x_6$	2	0	0	-2	1	1	

Fill out the blank cells in the above table.

3. Integer Programming Model Formulation (10 points)

The NCUT is to form a committee to handle the students' complaints. The committee must include at least one female, one male, one student, and one faculty. Eight individuals (identified by the letters  $a$  to  $h$ ) have been nominated. The mix of these individuals in the different categories is given as:

Category	Individuals
Females	$a, b, c, d$
Males	$e, f, g, h$
Students	$b, c, f,$
Faculty	$a, d, e, g, h$

Formulate this problem as an integer linear program.

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4. A product has two similar brands (A, B) on market. The preference transition matrix of customers for each week is as follows:

	A	B
A	0.4	0.6
B	0.8	0.2

- (1) If a customer buys brand A this week, what is the probability that he will buy brand A again two weeks later? (10 points)
- (2) What are the market shares of these two brands when the market reaches a steady state? (10 points)
- (3) Suppose each week (there are 52 weeks per year) the total market sales of this product is 1,000,000 and the net profit for brand A is estimated 5%. Is it worth for brand A spending 300,000 to buy a promotion advertising so that the preference transition matrix of customers for each week will be changed as follows: (10 points)

	A	B
A	0.7	0.3
B	0.9	0.1

5. A gasoline station has one pump. Cars arrive at the station according to a Poisson process at a mean rate of 15 per hour. However, if the pump already is being used, these potential customers may balk (drive to another service station). In particular, if there are  $n$  cars already at the service station, the probability that an arriving potential customer will balk is  $n/3$  for  $n=1, 2, 3$ . The time required to serve a car has an exponential distribution with a mean of 4 minutes.

- (1) Develop the balance equations and solve these equations to find the steady-state probability distribution of the number of cars at the station. (10 points)
- (2) Find the expected waiting time (including service) for those cars that stay. (10 points)