國立勤益科技大學九十七學年度研究所碩士班招生筆試試題卷

所別：工業工程與管理系碩士班　　組別：乙組(產業決策資訊組)

科目：作業研究

准考證號碼：□□□□□□□□（考生自填）

考生注意事項：

一、考試時間 100 分鐘。

二、可以使用本校所提供之計算機

1. ***True/False:*** Indicate by "O" = "true" or "X" ="false." (Each 2 points, total 20 points)

__ 1. The system **AX** = **b** has no solution if **A** is singular and **b** is independent of **A**.

__ 2. The "minimum ratio test" is used to determine the pivot row in the simplex method.

__ 3. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible.

__ 4. The slack variable and the dual variable for a constraint cannot both be positive.

__ 5. The optimal values of the primal and dual LP problems, if they exist, must be equal.

__ 6. If you increase the right-hand-side of a "greater-than-or-equal" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

__ 7. If a primal minimization LP problem has a cost which is unbounded below, then the dual maximization problem has an objective which is unbounded above.

__ 8. The assignment problem is a special case of a transportation problem.

__ 9. Using the "revised" simplex method usually requires fewer pivots than the "ordinary" simplex method in order to find the optimal solution of an LP.

__ 10. In a "balanced" transportation problem, the number of sources equals the number of destinations.

2. LP Duality. (Each 4 points, total 20 points)

Consider the following **primal** LP problem:

Maximize $Z = 5x_1 + 7x_2 + 10x_3$

Subject to
$$2x_1 - x_2 + 5x_3 \le 10 \quad \text{----- (1)}$$
$$x_1 + 3x_2 \le 15 \quad \text{----- (2)}$$
$$x_1, x_2, x_3 \ge 0$$

Suppose that the optimal basis $\mathbf{B} = (\mathbf{P}_3, \mathbf{P}_2) = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$, and its inverse is $\mathbf{B}^{-1} = \begin{pmatrix} \frac{3}{15} & \frac{1}{15} \\ 0 & \frac{5}{15} \end{pmatrix}$. Let y_1 and y_2 be dual variables associated with constraints (1) and (2) respectively. Determine the optimal $\mathbf{X}^*$ and $\mathbf{Y}^*$. That is, $x_1 =$ _____, $x_2 =$ _____, $x_3 =$ _____, $y_1 =$ _____, $y_2 =$_____.

3. For each of the following functions, show whether it is a convex, concave, or neither. (Each 4 points, total 20 points)

a. $f(x) = 10x - x^2$
b. $f(x) = x^4 + 6x^2 + 12x$
c. $f(x) = 2x^3 - 3x^2$
d. $f(x) = x^4 + x^2$
e. $f(x) = x^3 + x^4$

4. Repair King maintain a certain type of copy machine in a big city. The breakdowns of the copy machine follow a Poisson probability distribution with a mean rate of 0.5 per hour. Suppose the repair time for the breakdowns follow an exponential probability distribution with a mean service rate of 1 machine per hour. (Each 4 points, total 20 points)

a. What is the probability that no requests for the repair are in the system?
b. What is the average number of machines that will be waiting for repair?
c. What is the average waiting time in minutes before repair begins?
d. What is the average waiting time in minutes in the system?
e. What is the probability that a new arrival has to wait for repair?

5. The PC store's policy is to place an order for delivery at the start of the following week. If the inventory level is less than 3 PCs, then the new replenishment orders up to 5 units. According to historical data, the estimated demand per week shows as follows.

Demand	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

a. Express the situation as a Markov chain. (10 points)
b. Determine the long-run probability that no order will be placed in any week. (10 points)