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Fuzzy logic combining controller design for chaos control of a rod-type plasma torch system

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ABSTRACT

The nonlinear behavior analysis and chaos suppression control for a rod-type plasma torch system was discussed in the paper. The scenarios for the possible non-linear behavior in the plasma torch dynamics were also obtained with respect to the variation of system parameter μ via the numerical simulations, which might provide a guide for finding non-linear phenomena in the practical application of the plasma torch. From the bifurcation diagram, it shows that the plasma torch dynamics exit undesired chaotic behavior. In order to suppress the irregular chaotic motion, a fuzzy logic controller (FLC) that combines a sliding mode controller (SMC) and a state feedback controller (SFC) with guaranteed closed loop stability is designed. Each rule in this FLC has an SMC or an SFC in the consequent part. The role of the FLC is to schedule the final control under different antecedents. It is guaranteed that under the proposed control law, the rod-type plasma system with undesired chaotic motion can asymptotically stabilize to the unstable equilibrium point i.e. zero state. More importantly, the controller thus design can keep the advantages and remove the disadvantage of the two conventional controllers. Numerical simulations show the high performance of this method for chaos elimination in rod-type plasma torch system.

1. Introduction

In the past 20 years, the study of thermal plasma technology have attracted increasing attention due to its potential applications to plasma spraying, plasma chemical vapor deposition (CVD), arc plasma welding, plasma waste destruction, plasma synthesis of advanced ceramics, etc. (Boulos, Fauchais, & Pfender, 1994; Nicholson, 1983; Sturrock, 1994). One of the major issues concerned the effectiveness of these processes is the properties of the plasma-generating devices, in particular, the plasma fluctuations always associated with such devices. The fluctuating behavior of the plasma jet is identified as one of the most important characteristics, which is correlated with the underlying physical phenomena of the plasma source and determines the performance of the plasma processing. In the plasma spray process, the arc instabilities and jet fluctuations may lead to a non-uniform heating and transport of the injected powder particles and, consequently, affect the coating qualities. Moreover, the electrode erosion, low thermal efficiency, and unreliable performance of the plasma devices, directly or indirectly attributed to the lack of control on these fluctuations, have been the main challenge for the further development of thermal plasma technology and have inhibited its more potential industrial applications. Whatever from the point of view of academic research or of engineering application, a better understanding of the fundamental mechanisms and processes involved in the plasma source and its application is still necessary and indispensable (Ghorui & Das, 2004; Ghorui, Sahasrabudhe, Muryt, & Das, 2004).

Recently, a schematic of plasma torch has been proposed to study the existence of fluctuation in the practical experiments. The earlier concept indicated that the possibly observed fluctuation is a random behavior so that it is unpredictable. However, from dynamical analysis and experimental results, Ghorui et al. claimed that the inherent fluctuation appearing in plasma devices is a chaotic dynamical behavior (Ghorui, Sahasrabudhe, Muryt, Das, & Venkatramani, 2000). The study of chaotic systems has recently attracted lots of attention (Bagheri & Moghaddam, 2009; Chen, Chang, Yan, & Liao, 2008). Chaos is a more exotic form of steady-state response. Thus, chaotic behavior is unpredictable since chaotic system is more sensitive and its behavior strongly depends on system's initial values.

Recently, controlling this kind of complex dynamical systems has attracted considerable attention within the engineering society. Nowadays, many methods and techniques have been developed in chaos control (Yau, 2004). Among these methods, sliding mode control is often to be used because of its better robust character. Sliding-mode control (SMC) (Utkin, 1997) provides an effective alternative to deal with uncertain chaotic systems, and has

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been successfully applied in controlling chaos (Yan, 2004; Yang, Chen, & Yau, 2002). In the traditional SMC, it is assumed that the control can be switched from one value to another infinite fast, and this is impossible due to finite time delays and limitations in practical system. This non-ideal switching result in an undesirable phenomenon called chattering. The boundary layer approach is introduced to eliminate chattering around the switching surface and the control discontinuity within this thin boundary layer is smoothed out. If systems uncertainties are large, the sliding-mode controller would require a high switching gain with a thicker boundary layer to eliminate the higher chattering effect resulting. However, if we continuously increase the boundary layer thickness, we are actually reducing the feedback system to a system without sliding mode. To tackle these difficulties, a simple but more general methodology of fuzzy logic control is applied to deal with the chattering phenomenon in SMC.

In this paper, we will combine a sliding mode controller (SMC) and a state feedback controller (SFC) into a signal FLC to control the rod-type plasma torch system under undesired chaotic motion. The resulting closed-loop system has fast response, due to the SMC. Still, when the states are near the sliding plane, the FLC will gradually be dominated by the SFC to avoid chattering. As a result, the advantage of two conventional controllers can be kept by this combined controller. Finally, we present the numerical simulation results to illustrate the effectiveness of the proposed control scheme to stabilize the chaotic rod-type plasma torch system to the unstable equilibrium point.

The paper is organized as follows. The nonlinear dynamics of a rod-type plasma torch system is studied in Section 2. In Section 3, the FLC that combines a sliding mode controller (SMC) and a state feedback controller (SFC) with guaranteed closed loop stability is designed. Numerical analysis is carried on in Section 4 to verify the analytical results. Concluding remarks are given in Section 5 to summarize the major results.

2. System description and problem formulation

In the paper, we consider a class of the third-order nonlinear systems which was motivated by the amplitude equation being applied to the detection of possible occurrences of bifurcation phenomena in the thermal plasma (Ghorui et al., 2000). According to the theory of triple convection, the thermal arc plasma equation was proposed as follows:

$$\dddot{F} + \Omega_2 \ddot{F} + \Omega_1 \dot{F} + \Omega_0 F = \pm F^3. \tag{1}$$

The coefficients of Eq. (1) are known to depend on thermo-physical properties such as arc current, flow velocity of plasma gas, and the plasma torch device. To focus on the study of dynamical behavior in plasma torch, we adopt the parameter values of system (8) from the work by Ghorui et al. (Ghorui et al., 2000) and rewrite system (8) as given in the following equation:

$$\dddot{F} + \Omega_2 \ddot{F} + \Omega_1 \dot{F} + \Omega_0 F = -F^3, \tag{2}$$

with $\Omega_0 = \mu, \Omega_1 = 50$, and $\Omega_2 = 1$, where μ denotes the bifurcation parameter. Here, we only discuss the case of $-F^3$ appearing on the right hand side of (9). It is not difficult to extend the study to the other case of $+F^3$ appearing on the right hand side of (9). Details are not given.

The system (2) can then be rewritten state space equations as given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -\mu x_1 - 50x_2 - x_3 - x_1^3. \end{aligned} \tag{3}$$

The equilibrium points computed from (3) are obtained as $(x_1,x_2,x_3) = (0,0,0)$ and $(x_1,x_2,x_3) = (\pm\sqrt{-\mu},0,0)$ for $\mu \leqslant 0$. Now, we consider the stability of system (3) at the system equilibrium. Denote $(x_1^e,0,0)$ the system equilibrium. Here, we have $x_1^e = 0$ or $x_1^e = \pm\sqrt{-\mu}$. It is not difficult to have the Jacobian matrix at the equilibrium point $(x_1^e,0,0)$ of system (3) as

$$J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3(x_1^e)^2 - \mu & -50 & -1 \end{pmatrix}. \tag{4}$$

This gives the corresponding characteristic equation as

$$\lambda^3 + \lambda^2 + 50\lambda + \mu + 3(x_1^e)^2 = 0. \tag{5}$$

By using Routh–Hurwitz stability criteria, we then have the stability results for system (3) are as: (i) The origin of the system (3) is asymptotically stable for $0 < \mu < 50$, and (ii) The equilibrium point $(\pm\sqrt{\mu},0,0)$ for $\mu < 0$ of system (3) is asymptotically stable for $-25 < \mu < 0$.

In order to understand the rich behaviors of rod-type plasma torch system relative to parameter μ, the bifurcation diagrams of system states versus μ are shown in Fig. 1. As indicated by the map in Fig. 1, with the decrease of μ, the rod-type plasma torch system undergoes single-periodic, multi-periodic and aperiodic motions. When $\mu = -130$, the system time responses and phase plane trajectory of (x_1,x_2,x_3) are shown in Fig. 2. It shows that the dynamics of rod-type plasma torch system displaced a very complex behavior at this condition. The maximum Lyapunov exponent is computed on purpose to identify if the system is in a state

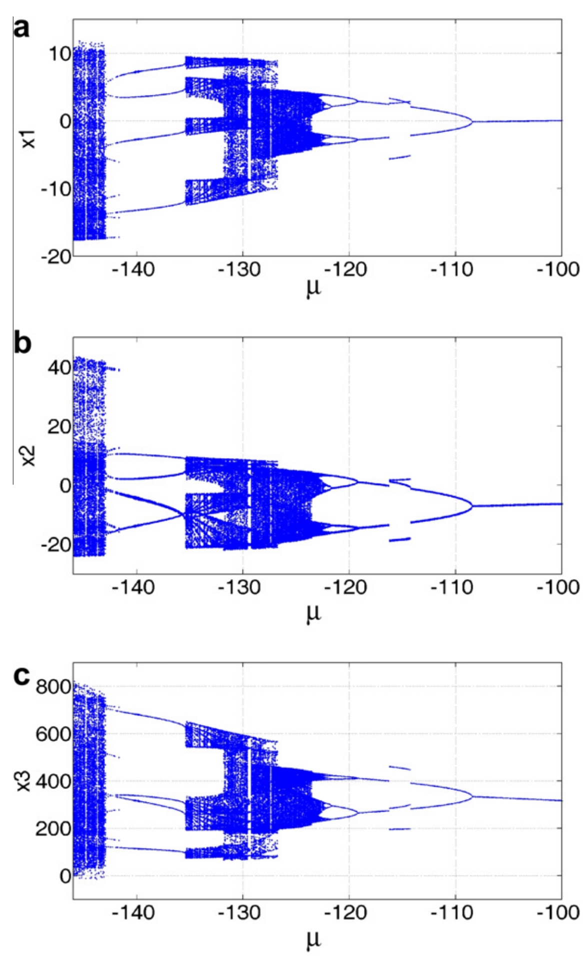

Fig. 1. The bifurcation diagrams of system states x_1, x_2 and x_3 versus μ.

of chaotic motion. Fig. 3 reports the diagram of the maximum Lyapunov exponent. It can be seen that the steady-state value of the maximum Lyapunov exponent is positive, which confirms the chaotic nature of the motion at this operation condition.

In the above discussions, it is known that (0,0,0) is an unstable equilibrium point under $\mu < 0$. In the following, the control technique is considered to stabilize the rod-type plasma torch system (3) to the unstable equilibrium point (0,0,0) when the system (3) is in a state of undesired chaotic motion with $\mu < 0$. It means that the undesired chaotic motion will be suppressed. In order to reach this object, a control input u is added in Eq. (3). Therefore, the controlled rod-type plasma torch system is shown in the follows:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -\mu x_1 - 50x_2 - x_3 - x_1^3 + u(t). \end{aligned} \tag{6}$$

The goal of this paper is that for any given chaotic rod-type plasma torch system, such as (6), a controller is designed such that the asymptotic stability of the resulting system (6) can be achieved in the sense that

$$\|[x_1\ x_2\ x_3]\| \to 0 \text{ as } t \to \infty,$$

where $\|\cdot\|$ is the Euclidean norm of a vector.

3. Fuzzy Logic controller design

In this section, a sliding mode controller (SMC) and a state feedback controller (SFC) will be combined into a signal FLC by applying the proposed designed approach. This FLC will be used to stabilize system (3) to the unstable equilibrium point (0,0,0) when the system (3) is in a state of undesired chaotic motion. When the states are far from the origin of the state plane, the SMC will take a major part of control to give a fast transient response. However, when the states are approaching the equilibrium value (the origin of the state plane), the SMC will gradually be replaced by SFC, in order to avoid chattering.

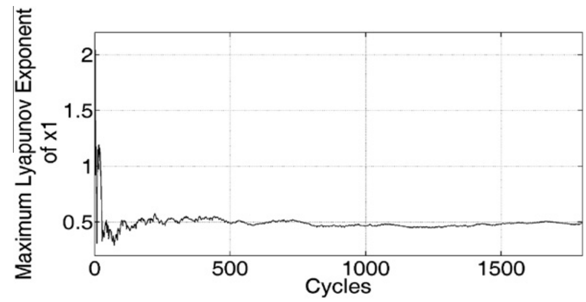

Fig. 3. The maximum Lyapunov exponent of uncontrolled system state x_1.

Let the control input

$$u(t) = u_1(t) + u_2(t) \quad \text{and} \quad u_1(t) = x_1^3, \tag{7}$$

then the controlled rod-type plasma torch system can be written in the following form:

$$\dot{x} = f(x) + b(x)u_2, \tag{8}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad f(x) = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\mu & -50 & -1 \end{bmatrix} x,$$

$$b(x) = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

3.1. SMC

Generally speaking, using the SMC technique to control a chaotic system involves two major steps. The first step is to select an appropriate switching surface which can guarantee the stability of the equivalent dynamics in the sliding mode such that the

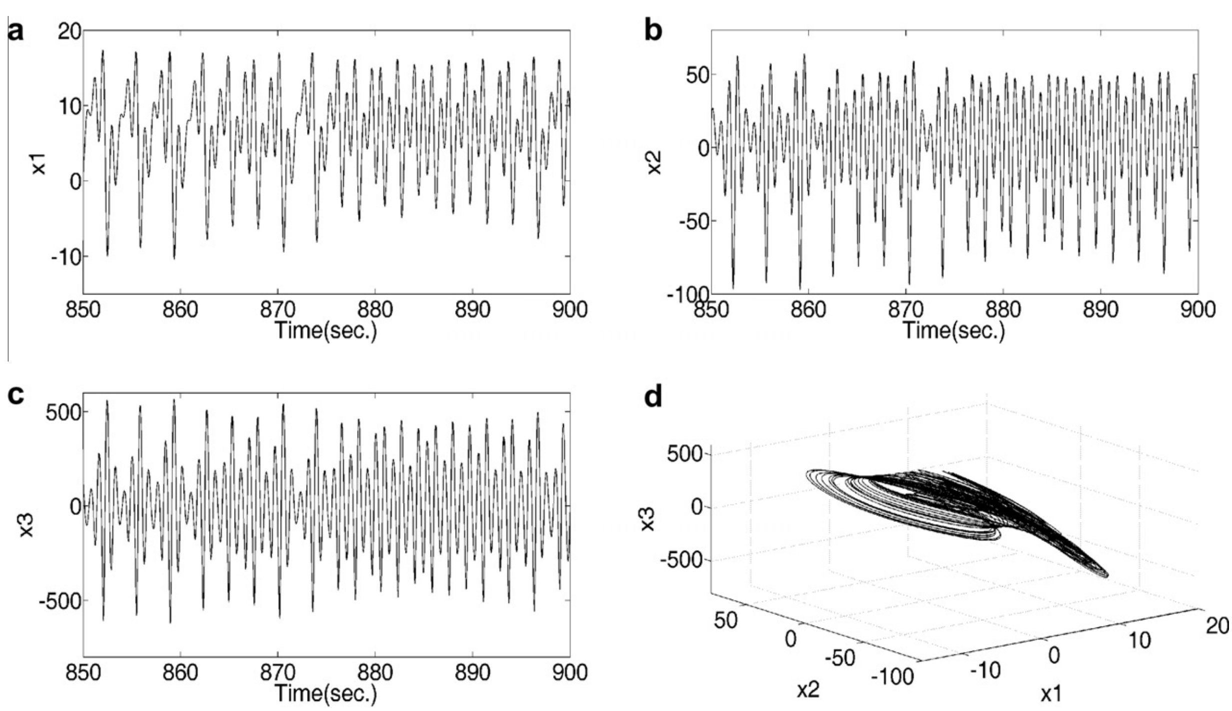

Fig. 2. Time responses of the rod-type plasma torch system states (a) x_1, (b) x_2, (c) x_3 and phase plane trajectory (d) x_1 versus x_2 under the parameter $\mu = -130$.

dynamics (8) can converge to zero. The second step is to determine a SMC to guarantee the hitting of the switching surfaces. As mentioned above, we first need to design a proper switching surface to ensure the stability of the system in the sliding mode. To reach this goal, a switching sliding surface is defined as

$$s = cx, \tag{9}$$

where $c = [1\ 1\ 0.1]$. It is obvious that the sliding plane is stable. An equivalent control u_{eq} can be obtained by considering

$$\dot{s} = c\dot{x} = 0, \tag{10}$$
$$\Rightarrow c[f(x) + b(x) \cdot u_{eq}] = 0,$$
$$\Rightarrow u_{eq} = -(cb(x))^{-1} cf(x). \tag{11}$$

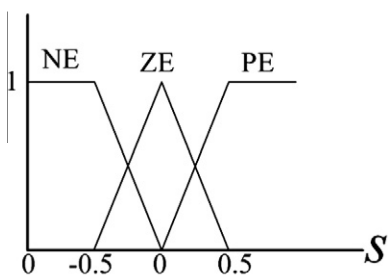

Fig. 4. Fuzzy levels of *s* and their membership functions.

The final control is realized as $u_2 = u_{eq} + u_{sw}$, where $u_{sw} = -(cb(x))^{-1} k \cdot sign(s)$, k is a positive constant, and

$$sign(s) = \begin{cases} 1, & \text{if } s > 0, \\ 0, & \text{if } s = 0, \\ -1, & \text{if } s < 0. \end{cases}$$

Defining a Lyapunov function

$$V = \frac{1}{2}s^2. \tag{12}$$

We have

$$\begin{aligned} \dot{V} &= s\dot{s} = s[cf(x) + cb(x)(-(cb(x))^{-1}(cf(x) + k \cdot sign(s)))] \\ &= -s \cdot k \cdot sign(s) = -k \cdot |s| \leqslant 0. \end{aligned} \tag{13}$$

It confirming the presence of reaching condition $(s \cdot \dot{s} < 0)$, that is, the sliding surface $s = 0$ is an attracting surface. It guarantees the robust stability of the SMC (Slotine & Li, 1991).

3.2. SFC

An SFC can also be employed for the system (8). Design a state feedback control gain

$$K_f(x) = [k_{f1}\ k_{f2}\ k_{f3}] = \left[\frac{-10 - f_{31}}{b_3}\ \frac{-11 - f_{32}}{b_3}\ \frac{-11 - f_{33}}{b_3}\right],$$

Such that the control signal is given by

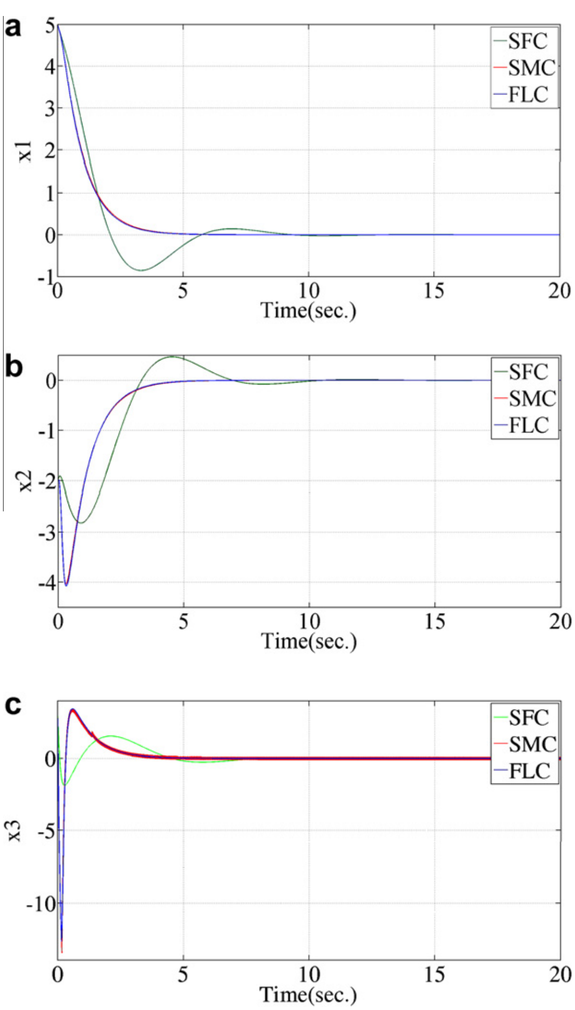

Fig. 5. Time responses of x_1,x_2 and x_3 with SFC, SMC and FLC.

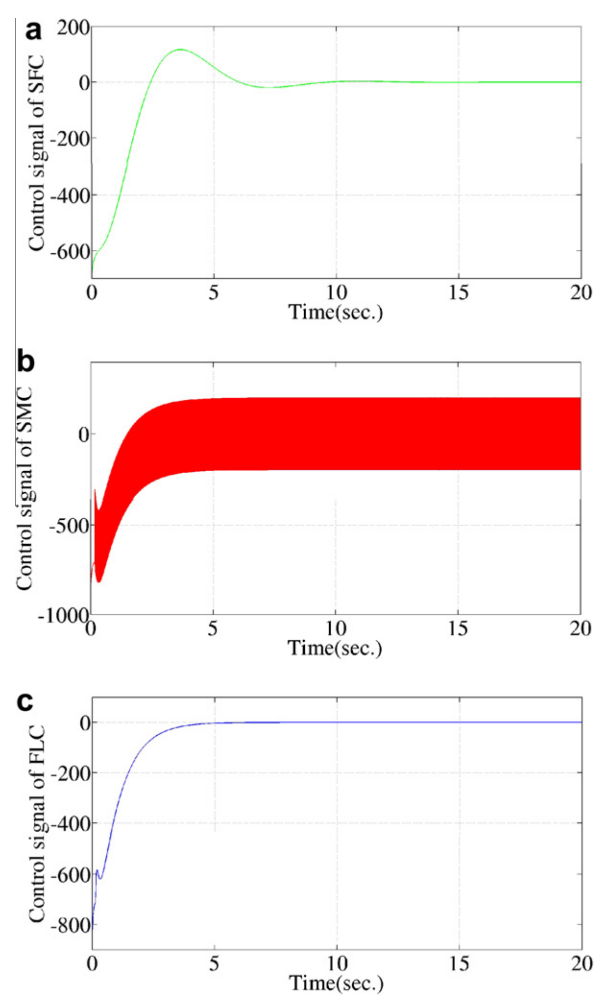

Fig. 6. The responses of the control input: (a) SFC; (b) SMC; (c) FLC.

$$u_2 = K_f(x) \cdot x. \tag{14}$$

By using the same Lyapunov function V of (12), if the control law of (14) is employed, we have

$$\begin{aligned}
\dot{V} &= s\dot{s} = x^T c^T c\dot{x} = x^T c^T c(f(x) + b(x)K_f(x)x) \\
&= x^T \begin{bmatrix} 1 \\ 1 \\ 0.1 \end{bmatrix} [1\ 1\ 0.1] \left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \right. \\
&\quad \left. + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \begin{bmatrix} \frac{-10 - f_{31}}{b_3} & \frac{-11 - f_{32}}{b_3} & \frac{-11 - f_{33}}{b_3} \end{bmatrix} \right) x \\
&= -x^T \begin{bmatrix} 1 & 1 & 0.1 \\ 1 & 1 & 0.1 \\ 0.1 & 0.1 & 0.01 \end{bmatrix} x \leqslant 0.
\end{aligned} \tag{15}$$

3.3. FLC

Define the rules of an FLC as follows:

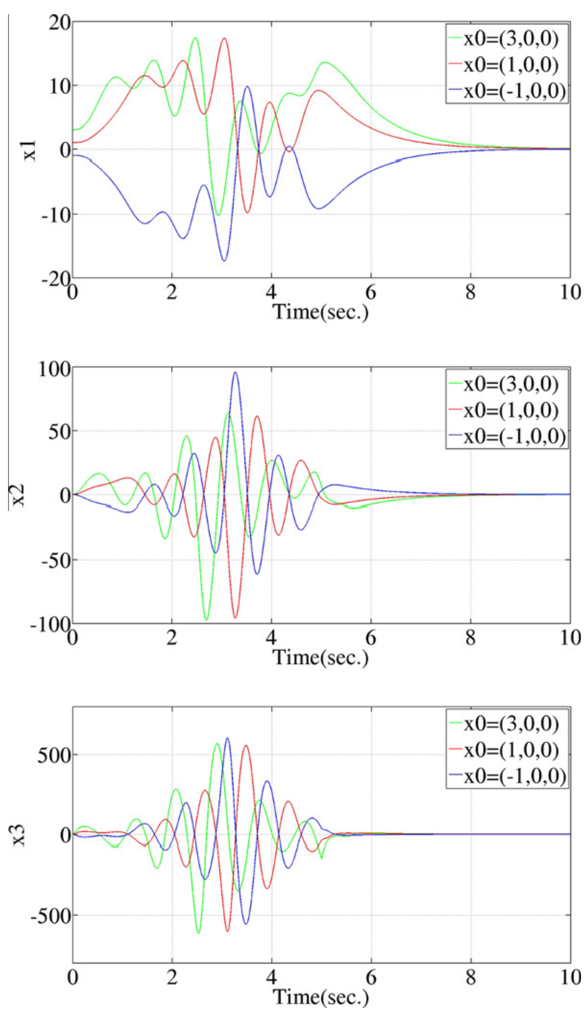

Fig. 7. System responses of x_1, x_2 and x_3 for different initial conditions: Green lines for $(x_1(0), x_2(0), x_3(0)) = (3,0,0)$, red lines for $(x_1(0), x_2(0), x_3(0)) = (1,0,0)$ and blue lines for $(x_1(0), x_2(0), x_3(0)) = (-1,0,0)$. The control is activated at time $t = 5$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

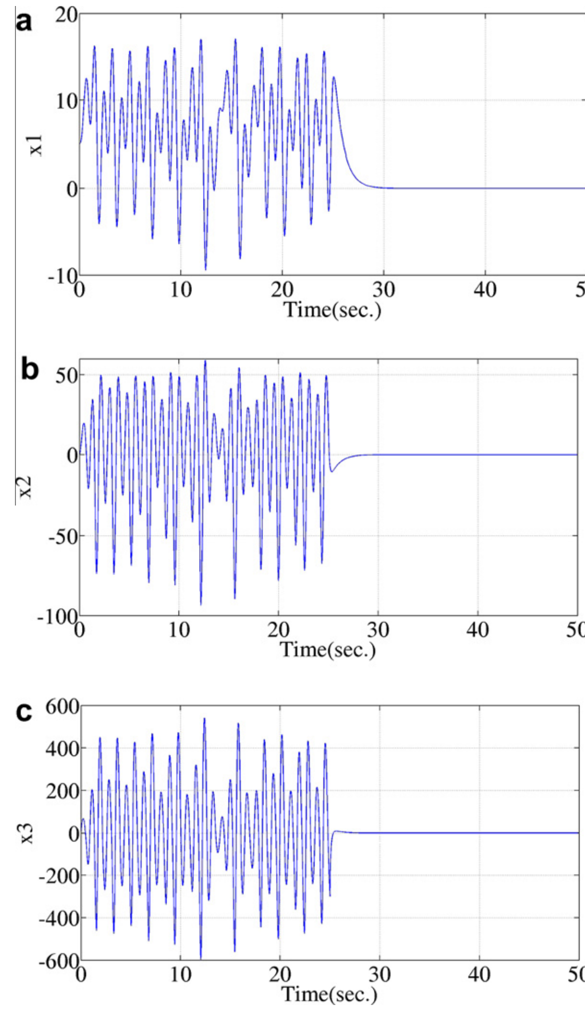

Fig. 8. Time response of the states x_1, x_2, x_3 for the controlled system under the parameter value μ changes from $\mu = -130$ to $\mu = -146$ at $t = 30$.

$$\text{Rule 1: If } s \text{ is PE THEN } u_2 = -(cb(x))^{-1}(cf(x) + k), \tag{16.a}$$
$$\text{Rule 2: If } s \text{ is ZE THEN } u_2 = K_f(x) \cdot x, \tag{16.b}$$
$$\text{Rule 3: If } s \text{ is NE THEN } u_2 = -(cb(x))^{-1}(cf(x) - k). \tag{16.c}$$

Where PE, ZE and NE are fuzzy levels of s of which the membership functions are shown in Fig. 4.

To prove the stability of the fuzzy-controlled chaotic rod-type plasma torch system, we need to ensure x tend to zero for all operation conditions. Although the Lyapunov function of (12) is a function of s only, but not x, thanks to the stability of the sliding plane, x tends to o if $s = 0$. Hence, from the work by Wong et al. (Wong, Leung, & Tam, 1998), the proof of system stability is reduced to proving that s tends to zero on applying each rule to the chaotic rod-type plasma torch system. This cannot be reached immediately, because the conditions V is positive define and $\dot{V} \leqslant 0$ may imply that s tends to a finite constant instead of zero. To verify that s must finally go to zero we need to prove that $\dot{V} < 0$ at $V \neq 0$ for all operation conditions. This proof is given in the Appendix of the work done by Wong et al. (Wong et al., 1998). Hence, the closed-loop system under the control of the proposed FLC is stable.

4. Simulation results

In this section, simulation results are presented to demonstrate and verify the performance of the present design. The parameter value $\mu = -130$ is selected for that (0,0,0) is an unstable equilibrium point. The initial state is $(x_1(0),x_2(0),x_3(0)) = (5,-2,3)$ and the value $k = 10$ is selected. For comparison, the responses on applying the SMC alone and the SFC alone are also taken. The states response of x_1,x_2 and x_3 are shown in Fig. 5. It can be seen that the settling time of x_1 on applying the FLC is better than that on applying the SFC only. The states responses on using the FLC and the SMC are similar, but no chattering exists in x_3 if the FLC is used. The time responses of control inputs with different control tools are shown in Fig. 6. It can be seen that the control signal displays a serious chattering on applying SMC, but it is chattering free on applying FLC.

It is known that chaotic system is very sensitive to initial conditions. Fig. 7 shows the system state responses under initial states $(x_1(0),x_2(0),x_3(0)) = (3,0,0)$, $(x_1(0),x_2(0),x_3(0)) = (1,0,0)$ and $(x_1(0),x_2(0),x_3(0)) = (-1,0,0)$ with $\mu = -130$. It demonstrates that the FLC can overcome the chaotic behavior in rod-type plasma torch system under different initial conditions after the control input is activated at $t = 5$. To test the robustness of the FLC, simulations under abrupt change of system parameter value are conducted. The state responses of x_1,x_2 and x_3 on applying the FLC and the parameter value μ changes from $\mu = -130$ to $\mu = -146$ at $t = 30$ are shown in Fig. 8. From Fig. 8, it can be seen that the system states of controlled system are regulated to the desired zero state after the control input is activated at $t = 25$. No matter how the parameter μ changes, it is kept up with zero state all the time. From these responses, we can see that the FLC designed is robust to parameter variations.

5. Conclusion

In this paper we focused on the study of nonlinear behavior in arc plasma dynamics. It is clear from this study that the chaotic behavior occurring in the arc plasma system is induced by period-doubling bifurcation. In order to overcome the undesired chaotic behavior, an FLC combining an SMC and an SFC to chaos suppression control of rod-type plasma torch system is designed. It has been clearly shown that a good transient responses, as well as robustness to parameter variations, can be obtained due to the SMC. However, chattering does not take place, due to the effect of the SFC near the equilibrium point. The combined controller then inherits the merits of the two conventional controllers in chaos control. The simulation results show that the proposed control enables to regulate the chaotic rod-type plasma torch system to the unstable equilibrium point $(x_1,x_2,x_3) = (0,0,0)$ asymptotically in spit of parameter variation. This study provides a strategy for controlled design to suppress the occurrence of fluctuation and operation performance for practical design of arc plasma torch. The main feature of this approach is that it gives the flexibility to construct a control law so that the control strategy can be easily extended to any chaotic systems.

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