1. Solve $(xy^2 + 2e^x)dx + (x^2y + 3e^y)dy = 0$. (10%)

2. What are the frequencies of vibration of a mass 2 kg as shown in Fig. 1. (15%)
   (a) on a spring with spring rate k_1 = 800Nt/m.
   (b) on a spring with spring rate k_2 = 1800Nt/m.
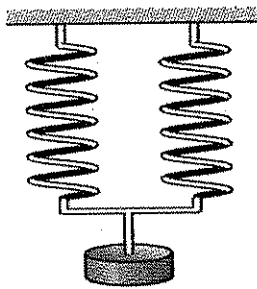   (c) on the two springs in parallel.
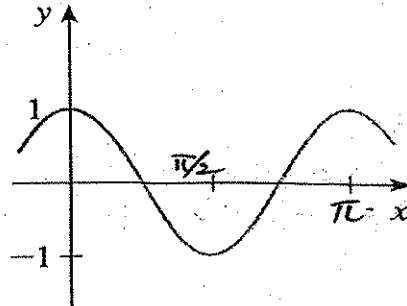

Fig. 1 The spring mass system.　　　Fig. 2 The Cosine function.

3. Determine the real domain cosine function $y(x)$ and find the Laplace transform. The function is shown as Fig. 2. (10%)

4. Use the Laplace transform to solve the given initial value problem.
   $$y'' + 4y = -3\sin(t-\pi)[u(t-\pi)], \quad y(0) = 1, y'(0) = 0.$$
   Where $u(t-\pi)$ is an unit step function. (20%)
   (a) Determine the $Y(s) = L[y(t)]$?
   (b) Find the solution y(t)?

5. If $A = \begin{bmatrix} -2 & -2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, please find the eigenvalues and eigenvectors of A. (10%)

6. If the mechanical work done by a force along the curve traced by
   $r(t) = \cos t \ \mathbf{i} + \sin t \ \mathbf{j}$ from t = 0 to t = π, (10%)

(a) determine the work done by $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j}$?

(b) determine the work done by $\mathbf{F} = -50\,\mathbf{i} + 10\,\mathbf{j}$?

7. If $f(x) = \begin{cases} \sin x, & -\pi < x < 0 \\ \cos x, & 0 < x < \pi \end{cases}$ is a periodic function, please find it's Fourier series.
(15%)

8. Determine the complex number as followings. (10%)

    (a) i^i ?   (Where $i = \sqrt{-1}$ is the imaginary unit.)

    (b) $(1-i)^4$?