國立勤益科技大學九十八學年度研究所碩士班招生筆試試題卷

所別：機械所　　組別：各組

科目：工程數學

准考證號碼：□□□□□□□□（考生自填）

考生注意事項：
一、考試時間 100 分鐘、可用計算器作答。
二、請依序作答，並於每小題之答案劃雙底線。

1. Consider free, damped motion of a 1 kilogram mass element attached to a spring of force constant 500 N/m. Let the coefficient of damping be 20 N・sec/m.

(a) Find the frequency of the motion in rad/sec. ?

(b) Determine the free equation of motion ?

2. Use the Laplace transform to solve the initial value problem.

$y'+3y=f(t)$, $y(0)=0$, where $f(t)=\begin{cases}2 & 0\le t<1\\ -2 & 1\le t\end{cases}$

(a) Find Laplace transform $L[y(t)]$?

(b) Find the solution $y(t)$?

3. Solve the initial value problem $x'+(\tan t)x=\cos t$, $x(0)=\pi$

(a) Find the general solution?

(b) Find the particular solution?

4. A plane contains three points i.e. (3,1,4), (2,-2,0), and (1,0,-1).

(a) Find a vector normal to the plane ?

(b) Find an equation of the plane and graph the plane in the Cartesian coordinate?

5. A small projectile is fired vertically downward into a fluid medium with an initial velocity of 50 m/s. Due to the resistance of the fluid the particle experience a deceleration equal to a $a=-0.4v^2$, (m/s^2), where v is in m/s. (Hint : $a=dv/dt$, $\int\frac{dv}{-0.4v^2}=\int dt$, where t is time in second.)

(a) Determine the velocity function $v(t)$?

(b) Determine the projectile's velocity and position 4 second after it is fired?

6. Find the half-range expansions of the force function $f(t)=t,\ 0<t<\pi$.

(a) Expand $f(t)$ in a sine series?

(b) Plot the series function curve?

7. Determine the Laplace transforms

(a) $L[t^3e^t]$ (b) $L[\int_0^t \tau \sin\tau\, d\tau]$

8. Let S be the region bounded by the cone $S: x^2+y^2-z^2=0,\ 0\le z\le 1,$ and the plane $z=1$. Use divergence theorem, $\iint(\mathbf{F}\cdot\mathbf{n})dS=\iiint \mathrm{div}\mathbf{F}dV$,if the velocity is $\mathbf{F}=2x\mathbf{i}+3y\mathbf{j}+4z\mathbf{k}$.

(a) Evaluate $\iint_S(\mathbf{F}\cdot\mathbf{n})dS$? (b) Plot the S region?

9. An equation which express that the velocity vector is tangent to a streamline is $\mathbf{v}\times(\mathrm{dx}\mathbf{i}+\mathrm{dy}\mathbf{j})=\mathbf{0}$. The velocity is given by $\mathbf{v}=2x\mathbf{i}-4y\mathbf{j},(\mathrm{m/s})$, where x and y are in meters. Determine the properties at (2,-1)

(a) Find the streamline ?

(b) Find a unit vector normal to the streamline?

10. $\mathbf{Ax}=\mathbf{b}$ is the system of the equations as follow.

$x_1-x_2+x_3=1$

$2x_1+x_2+2x_3=2$

$3x_1+2x_2-x_3=-3$

(a) Find the inverse of **A**?

(b) Use the inverse matrix to solve the equations?