Analysis on Delay-Partitioning-Based Asymptotic Stability for Large-Scale Systems with Neutral Delay and Switched-Type

延遲分割應用於大型中立延遲切換型系統的漸近穩定度分析

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Abstract

This paper deals with the problem of delay-partitioning-based asymptotic stability analysis for large-scale systems with neutral delay and switched-type. By applying weighting-delay-parameter approach, introducing both singular model transformation technique and Finsler's lemma, and constructing an augmented Lyapunov-Krasovskii functional combined with free matrices, a novel delay-partitioning-dependent stability criterion is derived to guarantee the asymptotic stability of above systems. The obtained criterion is formulated in terms of matrix inequalities, which can be efficiently solved via standard numerical software. Two numerical examples are included to show that the proposed method is effective and can provide less conservative results.

Keywords: Large-scale systems, neutral delay, asymptotic stability, weighting-delay-parameter approach, singular model transformation.

摘要

本文旨在處理大型中立延遲切換型系統之延遲分割相關漸近穩定度問題。藉由加權延遲參數方法、奇異模型轉換技巧、芬斯勒補助定理及具有自由矩陣之擴展型李亞普諾-克羅斯威斯基泛函數，針對上述系統，提出新延遲分割相關穩定準則。本文所提之準則表示為矩陣不等式形式，可便於軟體模擬求解。舉例證實本研究方法明顯改善文獻結果。

關鍵字詞：大型系統，中立延遲，漸近穩定度，加權延遲參數方法，奇異模型轉換。

1. Introduction

It is well known that a wide class of physical systems in power systems, chemical procedure control systems, navigation systems, automobile speed change system, etc. may be appropriately described by the switched model. Switched systems are a special class of hybrid dynamical systems, which consist of a family of subsystems and a switching law specifying the switching between the subsystems. Recently, there has been increasing interest in the stability problem of switched systems with time delay due to their significance both in theory and applications. To the best of our knowledge, it seems that few people have studied the asymptotic stability problem for large-scale switched-type systems with time-varying neutral delay. This has motivated our research.

In this paper, we will give preliminary knowledge for our main result. First of all, consider the following large-scale switched-type system with time-varying neutral delay

$$\dot{x}_i(t)=\sum_{k=1}^{r}\alpha^k(t)\{A_i^k x_i(t)+\bar{A}_i^k x_i(t-d(t))+C_i^k\dot{x}_i(t-d(t)) + \sum_{\substack{j=1\\ j\neq i}}^{N}[B_{ij}^k x_j(t)+\bar{B}_{ij}^k x_j(t-d(t))]\} \quad (1a)$$

$$\sum_{k=1}^{r}\alpha^k(t)=1,\quad x_i(t)=\varphi_i(t),\qquad t\in[-h,\quad 0] \quad (1b)$$

$$\alpha^k(t)=\begin{cases}1, & \text{when the switched system is described by the } k\text{th mode}\\ 0, & \text{otherwise}\end{cases} \quad (1c)$$

where $x_i(t)\in R^{n_i}$ is the state vector of the ith subsystem, A_i^k, $\bar{A}_i^k$, B_{ij}^k, $\bar{B}_{ij}^k$, C_i^k are known constant matrices with appropriate dimensions, $i=1,2,\cdots,N$, $k=1,2,\cdots,r$. The delay $d(t)$ is a time-varying continuous function satisfying $0\le d(t)\le h$ and $\dot{d}(t)\le\mu$. $\varphi_i(t)$ is a given continuous vector-valued initial function.

The following notations will be used throughout this paper. The notation $F>G$ ($F\ge G$) means that the matrix $F-G$ is positive definite (positive semi-definite) for two symmetric matrices F, G. I_i is an identity matrix of appropriate dimensions.

Assumption 1[1]: All the eigenvalues of matrices C_i^k, $i=1,2,\cdots,N$, are inside the unit circle.

Lemma 1[2]: For any real vectors κ_1, κ_2 and any matrix $M>0$ with appropriate dimensions, it follows that

$$2\kappa_1^{\mathrm{T}}\kappa_2 \le \kappa_1^{\mathrm{T}}M^{-1}\kappa_1 + \kappa_2^{\mathrm{T}}M\kappa_2 \qquad (2)$$

Lemma 2[3]: For any symmetric positive definite matrix P and scalars $\lambda>0$, $\delta>1$, the following inequality holds

$$-\int_0^{\lambda}\delta e^{\mathrm{T}}(s)Pe(s)ds \le -\int_0^{\lambda} e^{\mathrm{T}}(s)P\,e(s)ds - \frac{(\delta-1)}{\lambda}\Big(\int_0^{\lambda}e(s)ds\Big)^{\mathrm{T}} \times P\Big(\int_0^{\lambda}e(s)ds\Big) \qquad (3)$$

Lemma 3[4]: For any symmetric positive definite matrix Q and scalars $0 \le b_1 < b_2$, the following inequality holds

$$-\int_{t-b_2}^{t-b_1}\dot{x}^{\mathrm{T}}(\theta)Q\dot{x}(\theta)d\theta \le -\frac{1}{b_2-b_1}[x(t-b_1)-x(t-b_2)]^{\mathrm{T}}Q \times [x(t-b_1)-x(t-b_2)] \qquad (4)$$

Lemma 4(Finsler's lemma)[5]: Consider a vector $\zeta \in R^n$, a symmetric positive definite matrix $S \in R^{n\times n}$ and a matrix $D \in R^{m\times n}$, such that $\mathrm{rank}(D) < n$. The following conditions are equivalent:

(i) $\zeta^{\mathrm{T}}S\zeta < 0$, $\forall\zeta$ such that $D\zeta = 0$, $\zeta \ne 0$ (5a)

(ii) $(D^{\perp})^{\mathrm{T}}SD^{\perp} < 0$ (5b)

2. Main Result

In the following theorem, a novel delay-partitioning-dependent criterion for asymptotic stability of large-scale neutral-delay switched-type system (1) is proposed in terms of matrix inequalities.

Theorem 1: Under Assumption 1, the large-scale neutral-delay switched-type system (1) is asymptotically stable for $i=1,2,\cdots,N$ and $k=1,2,\cdots,r$, if there exist positive definite matrices L_{11i}, L_{22i}, L_{33i}, Y_{1i}, Y_{2i}, Z_{11i}, Z_{22i}, Z_{33i}, Z_{44i}, Z_{55i}, Z_{66i}, Z_{77}, P_i, Q_i, R_i, U_i, W_{1i}, W_{2i}, W_{3i}, W_{4i}, X_{1i}, X_{2i}, M_{ij}, $\overline{M}_{ij}$, $\hat{M}_{ij}$, $\widetilde{M}_{ij}$, real matrices H_i, K_i, L_{12i}, L_{13i}, L_{23i}, Z_{12i}, Z_{13i}, Z_{14i}, Z_{15i}, Z_{16i}, Z_{17i}, Z_{23i}, Z_{24i}, Z_{25i}, Z_{26i}, Z_{27i}, Z_{34i}, Z_{35i}, Z_{36i}, Z_{37i}, Z_{45i}, Z_{46i}, Z_{47i}, Z_{56i}, Z_{57i}, Z_{67i}, and scalars $0<\rho<1$, $\delta_i>1$ such that the following conditions hold

$$\begin{bmatrix} Y_{1i} & H_i \\ H_i^{\mathrm{T}} & Y_{2i} \end{bmatrix} > 0 \qquad (6a)$$

$$\begin{bmatrix} L_{11i} & L_{12i} & L_{13i} \\ L_{12i}^{\mathrm{T}} & L_{22i} & L_{23i} \\ L_{13i}^{\mathrm{T}} & L_{23i}^{\mathrm{T}} & L_{33i} \end{bmatrix} > 0 \qquad (6b)$$

$$Z_i = \begin{bmatrix} Z_{11i} & Z_{12i} & Z_{13i} & Z_{14i} & Z_{15i} & Z_{16i} & Z_{17i} \\ Z_{12i}^{\mathrm{T}} & Z_{22i} & Z_{23i} & Z_{24i} & Z_{25i} & Z_{26i} & Z_{27i} \\ Z_{13i}^{\mathrm{T}} & Z_{23i}^{\mathrm{T}} & Z_{33i} & Z_{34i} & Z_{35i} & Z_{36i} & Z_{37i} \\ Z_{14i}^{\mathrm{T}} & Z_{24i}^{\mathrm{T}} & Z_{34i}^{\mathrm{T}} & Z_{44i} & Z_{45i} & Z_{46i} & Z_{47i} \\ Z_{15i}^{\mathrm{T}} & Z_{25i}^{\mathrm{T}} & Z_{35i}^{\mathrm{T}} & Z_{45i}^{\mathrm{T}} & Z_{55i} & Z_{56i} & Z_{57i} \\ Z_{16i}^{\mathrm{T}} & Z_{26i}^{\mathrm{T}} & Z_{36i}^{\mathrm{T}} & Z_{46i}^{\mathrm{T}} & Z_{56i}^{\mathrm{T}} & Z_{66i} & Z_{67i} \\ Z_{17i}^{\mathrm{T}} & Z_{27i}^{\mathrm{T}} & Z_{37i}^{\mathrm{T}} & Z_{47i}^{\mathrm{T}} & Z_{57i}^{\mathrm{T}} & Z_{67i}^{\mathrm{T}} & Z_{77i} \end{bmatrix} > 0 \qquad (6c)$$

$$(1-\mu)X_{1i} - Z_{77i} > 0 \qquad (6d)$$

$$(D_i^{\perp})^{\mathrm{T}}\Pi_i D_i^{\perp} < 0 \qquad (6e)$$

$$\begin{bmatrix} (1-\rho\mu)U_i & K_i \\ K_i^{\mathrm{T}} & X_{2i} \end{bmatrix} \ge 0 \qquad (6f)$$

where

$$D_i^{\perp} = \begin{bmatrix} I_i & I_i & 0 & 0 & I_i \\ 0 & I_i & 0 & 0 & 0 \\ I_i & 0 & 0 & 0 & 0 \\ I_i & I_i & 0 & 0 & 0 \\ 0 & 0 & I_i & 0 & 0 \\ 0 & 0 & 0 & I_i & 0 \\ 0 & 0 & 0 & 0 & I_i \end{bmatrix} \qquad (7a)$$

$$\Pi_i = \begin{bmatrix} \Pi_{11i} & \Pi_{12i} & \Pi_{13i} & \Pi_{14i} & \Pi_{15i} & \Pi_{16i} & \Pi_{17i} \\ \Pi_{12i}^{\mathrm{T}} & \Pi_{22i} & \Pi_{23i} & \Pi_{24i} & \Pi_{25i} & \Pi_{26i} & \Pi_{27i} \\ \Pi_{13i}^{\mathrm{T}} & \Pi_{23i}^{\mathrm{T}} & \Pi_{33i} & \Pi_{34i} & \Pi_{35i} & \Pi_{36i} & \Pi_{37i} \\ \Pi_{14i}^{\mathrm{T}} & \Pi_{24i}^{\mathrm{T}} & \Pi_{34i}^{\mathrm{T}} & \Pi_{44i} & \Pi_{45i} & \Pi_{46i} & \Pi_{47i} \\ \Pi_{15i}^{\mathrm{T}} & \Pi_{25i}^{\mathrm{T}} & \Pi_{35i}^{\mathrm{T}} & \Pi_{45i}^{\mathrm{T}} & \Pi_{55i} & \Pi_{56i} & \Pi_{57i} \\ \Pi_{16i}^{\mathrm{T}} & \Pi_{26i}^{\mathrm{T}} & \Pi_{36i}^{\mathrm{T}} & \Pi_{46i}^{\mathrm{T}} & \Pi_{56i}^{\mathrm{T}} & \Pi_{66i} & \Pi_{67i} \\ \Pi_{17i}^{\mathrm{T}} & \Pi_{27i}^{\mathrm{T}} & \Pi_{37i}^{\mathrm{T}} & \Pi_{47i}^{\mathrm{T}} & \Pi_{57i}^{\mathrm{T}} & \Pi_{67i}^{\mathrm{T}} & \Pi_{77i} \end{bmatrix} \qquad (7b)$$

$$\Pi_{11i} = R_iA_i^k + (A_i^k)^{\mathrm{T}}R_i - \frac{1}{\rho h}[(1-\mu)X_{1i} - Z_{66i}] + W_{1i} + W_{2i} + K_i + L_{11i} + W_{4i} + Y_{1i} + \rho h\,(U_i + Z_{11i}) + \sum_{\substack{j=1 \\ j\ne i}}^{N}\{R_i[B_{ij}^kM_{ij}(B_{ij}^k)^{\mathrm{T}} + \overline{B}_{ij}^k\overline{M}_{ij}(\overline{B}_{ij}^k)^{\mathrm{T}}]R_i + M_{ji}^{-1} + \hat{M}_{ji}^{-1}\} \qquad (7c)$$

$$\Pi_{12i} = P_i - R_i + (A_i^k)^{\mathrm{T}}Q_i + H_i + \rho hZ_{12i},\ \Pi_{13i} = R_i\overline{A}_i^k + \rho hZ_{13i} \qquad (7d)$$

$$\Pi_{14i} = \rho hZ_{14i} + \frac{1}{\rho h}[(1-\mu)X_{1i} - Z_{77i}] \qquad (7e)$$

$$\Pi_{15i} = R_iC_i^k + \rho hZ_{15i},\ \Pi_{16i} = \rho hZ_{16i} + L_{12i},\ \Pi_{17i} = Z_{17i} + L_{13i} \qquad (7f)$$

$$\Pi_{22i} = W_{3i} + Y_{2i} + \rho hZ_{22i} - 2Q_i + \sum_{\substack{j=1 \\ j\ne i}}^{N}Q_i[B_{ij}^k\hat{M}_{ij}(B_{ij}^k)^{\mathrm{T}} + \overline{B}_{ij}^k\widetilde{M}_{ij}(\overline{B}_{ij}^k)^{\mathrm{T}}]Q_i \qquad (7g)$$

$$\Pi_{23i}=Q_i\bar{A}_i^k+\rho hZ_{23i},\ \Pi_{24i}=\rho hZ_{24i} \tag{7h}$$

$$\Pi_{25i}=Q_iC_i^k+\rho hZ_{25i},\ \Pi_{26i}=\rho hZ_{26i},\Pi_{27i}=Z_{27i} \tag{7i}$$

$$\Pi_{33i}=\rho hZ_{33i}-(1-\mu)(W_{1i}+Y_{1i})+\sum_{\substack{j=1\\ j\neq i}}^{N}(\bar{M}_{ji}^{-1}+\tilde{M}_{ji}^{-1}) \tag{7j}$$

$$\Pi_{34i}=\rho hZ_{34i},\Pi_{35i}=-(1-\mu)H_i+\rho hZ_{35i} \tag{7k}$$

$$\Pi_{36i}=\rho hZ_{36i},\ \Pi_{37i}=Z_{37i} \tag{7l}$$

$$\Pi_{44i}=-K_i-\frac{1}{\rho h}[(1-\mu)X_{1i}-Z_{77i}]-\frac{\delta_i}{h}X_{2i}$$
$$-(1-\rho\mu)W_{2i}+\rho hZ_{44i} \tag{7m}$$

$$\Pi_{45i}=\rho hZ_{45i},\ \Pi_{46i}=\rho hZ_{46i}+\frac{\delta_i}{h}X_{2i},\ \Pi_{47i}=Z_{47i} \tag{7n}$$

$$\Pi_{55i}=\rho hZ_{55i}-(1-\mu)(W_{3i}+Y_{2i}),\Pi_{56i}=\rho hZ_{56i},\ \Pi_{57i}=Z_{57i} \tag{7o}$$

$$\Pi_{66i}=\rho hZ_{66i}+L_{22i}-\frac{\delta_i}{h}X_{2i},\ \Pi_{67i}=Z_{67i}+L_{23i} \tag{7p}$$

$$\Pi_{77i}=L_{33i}-\frac{(\delta_i-1)}{\rho h}X_{2i} \tag{7q}$$

Proof: Based on singular model transformation [6], system (1) can be written as

$$\dot{x}_i(t)=y_i(t) \tag{8a}$$

$$0=\sum_{k=1}^{r}\alpha^k(t)\{-y_i(t)+C_i^k y_i(t-d(t))+A_i^k x_i(t)+\bar{A}_i^k x_i(t-d(t))$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N}[B_{ij}^k x_j(t)+\bar{B}_{ij}^k x_j(t-d(t))]\} \tag{8b}$$

By means of the idea of [7] and [8], we use the following Lyapunov-Krasovskii functional to derive the stability criterion

$$V(t)=\sum_{i=1}^{N}[V_{1i}(t)+V_{2i}(t)+V_{3i}(t)+V_{4i}(t)+V_{5i}(t)+V_{6i}(t)$$
$$+V_{7i}(t)+V_{8i}(t)+V_{9i}(t)+V_{10i}(t)+V_{11i}(t)] \tag{9}$$

where

$$V_{1i}(t)=[x_i^{\mathrm T}(t)\quad y_i^{\mathrm T}(t)]\begin{bmatrix}I_i&0\\0&0\end{bmatrix}\begin{bmatrix}P_i&0\\R_i&Q_i\end{bmatrix}\begin{bmatrix}x_i(t)\\y_i(t)\end{bmatrix} \tag{10a}$$

$$V_{2i}(t)=\int_{t-d(t)}^{t}x_i^{\mathrm T}(s)W_{1i}x_i(s)ds \tag{10b}$$

$$V_{3i}(t)=\int_{t-\rho d(t)}^{t}x_i^{\mathrm T}(s)W_{2i}x_i(s)ds \tag{10c}$$

$$V_{4i}(t)=\int_{t-d(t)}^{t}y_i^{\mathrm T}(s)W_{3i}y_i(s)ds \tag{10d}$$

$$V_{5i}(t)=\int_{t-h}^{t}x_i^{\mathrm T}(s)W_{4i}x_i(s)ds \tag{10e}$$

$$V_{6i}(t)=\int_{-\rho d(t)}^{0}\int_{t+\theta}^{t}x_i^{\mathrm T}(s)U_i x_i(s)dsd\theta \tag{10f}$$

$$V_{7i}(t)=\int_{t-d(t)}^{t}\begin{bmatrix}x(s)\\y(s)\end{bmatrix}^{\mathrm T}\begin{bmatrix}Y_{1i}&H_i\\H_i^{\mathrm T}&Y_{2i}\end{bmatrix}\begin{bmatrix}x(s)\\y(s)\end{bmatrix}ds \tag{10g}$$

$$V_{8i}(t)=\int_{-d(t)}^{0}\int_{t+\theta}^{t}y_i^{\mathrm T}(s)X_{1i}y_i(s)dsd\theta \tag{10h}$$

$$V_{9i}(t)=\int_{-h}^{0}\int_{t+\theta}^{t}\delta_i y_i^{\mathrm T}(s)X_{2i}y_i(s)dsd\theta \tag{10i}$$

$$V_{10i}(t)=\int_{0}^{t}\int_{\theta-\rho d(\theta)}^{\theta}e_i^{\mathrm T}(\theta,s)Z_i e_i(\theta,s)dsd\theta \tag{10j}$$

$$V_{11i}(t)=\int_0^t\begin{bmatrix}x_i(\theta)\\x_i(\theta-h)\\\int_{\theta-\rho d(\theta)}^{\theta}y_i(s)ds\end{bmatrix}^{\mathrm T}\begin{bmatrix}L_{11i}&L_{12i}&L_{13i}\\L_{12i}^{\mathrm T}&L_{22i}&L_{23i}\\L_{13i}^{\mathrm T}&L_{23i}^{\mathrm T}&L_{33i}\end{bmatrix}\begin{bmatrix}x_i(\theta)\\x_i(\theta-h)\\\int_{\theta-\rho d(\theta)}^{\theta}y_i(s)ds\end{bmatrix}d\theta \tag{10k}$$

where $e_i(\theta,s)=[x_i^{\mathrm T}(\theta)\ y_i^{\mathrm T}(\theta)\ x_i^{\mathrm T}(\theta-d(\theta))\ x_i^{\mathrm T}(\theta-\rho d(\theta))\ y_i^{\mathrm T}(\theta-d(\theta))\ x_i^{\mathrm T}(\theta-h)\ y_i^{\mathrm T}(s)]^{\mathrm T}$ and matrix Z_i is defined in (6c).

Taking the time derivative of $V(t)$ along the trajectories of system (1) and noting that $0\le d(t)\le h$ and $\dot{d}(t)\le\mu$, it yields

$$\dot V(t)=\sum_{i=1}^{N}[\dot V_{1i}(t)+\dot V_{2i}(t)+\dot V_{3i}(t)+\dot V_{4i}(t)+\dot V_{5i}(t)+\dot V_{6i}(t)$$
$$+\dot V_{7i}(t)+\dot V_{8i}(t)+\dot V_{9i}(t)+\dot V_{10i}(t)+\dot V_{101}(t)] \tag{11}$$

where

$$\dot V_{1i}(t)=2[x_i^{\mathrm T}(t)\quad y_i^{\mathrm T}(t)]\begin{bmatrix}P_i&R_i\\0&Q_i\end{bmatrix}$$
$$\times\begin{bmatrix}y_i(t)\\\left(\begin{aligned}&\sum_{k=1}^{r}\alpha^k(t)\{-y_i(t)+C_i^k y_i(t-d(t))\\&+A_i^k x_i(t)+\bar{A}_i^k x_i(t-d(t))\\&+\sum_{\substack{j=1\\ j\neq i}}^{N}[B_{ij}^k x_j(t)+\bar{B}_{ij}^k x_j(t-d(t))]\}\end{aligned}\right)\end{bmatrix} \tag{12a}$$

$$\dot V_{2i}(t)\le x_i^{\mathrm T}(t)W_{1i}x_i(t)$$
$$-(1-\mu)x_i^{\mathrm T}(t-d(t))W_{1i}x_i(t-d(t)) \tag{12b}$$

$$\dot V_{3i}(t)\le x_i^{\mathrm T}(t)W_{2i}x_i(t)$$
$$-(1-\rho\mu)x_i^{\mathrm T}(t-\rho d(t))W_{2i}x_i(t-\rho d(t)) \tag{12c}$$

$$\dot V_{4i}(t)\le y_i^{\mathrm T}(t)W_{3i}y_i(t)$$
$$-(1-\mu)y_i^{\mathrm T}(t-d(t))W_{3i}y_i(t-d(t)) \tag{12d}$$

$$\dot V_{5i}(t)\le x_i^{\mathrm T}(t)W_{4i}x_i(t)-x_i^{\mathrm T}(t-h)W_{4i}x_i(t-h) \tag{12e}$$

$$\dot V_{6i}(t)\le\rho h x_i^{\mathrm T}(t)U_i x_i(t)$$
$$-(1-\rho\mu)\int_{t-\rho d(t)}^{t}x_i^{\mathrm T}(s)U_i x_i(s)ds \tag{12f}$$

$$\dot V_{7i}(t)\le\begin{bmatrix}x(t)\\y(t)\end{bmatrix}^{\mathrm T}\begin{bmatrix}Y_{1i}&H_i\\H_i^{\mathrm T}&Y_{2i}\end{bmatrix}\begin{bmatrix}x(t)\\y(t)\end{bmatrix}$$
$$-(1-\mu)\begin{bmatrix}x(t-d(t))\\y(t-d(t))\end{bmatrix}^{\mathrm T}\begin{bmatrix}Y_{1i}&H_i\\H_i^{\mathrm T}&Y_{2i}\end{bmatrix}\begin{bmatrix}x(t-d(t))\\y(t-d(t))\end{bmatrix} \tag{12g}$$

$$\dot V_{8i}(t)\le hy_i^{\mathrm T}(t)X_{1i}y_i(t)$$
$$-(1-\mu)\int_{t-d(t)}^{t}y_i^{\mathrm T}(s)X_{1i}y_i(s)ds \tag{12h}$$

$$\dot{V}_{9i}(t)\le h\delta_i\, y_i^{\mathrm{T}}(t)X_{2i}y_i(t)-\int_{t-\rho d(t)}^{t}\delta_i\, y_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds-\int_{t-h}^{t-\rho d(t)}\delta_i\, y_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds \tag{12i}$$

$$\dot{V}_{10i}(t)=\rho d(t)\begin{bmatrix}x_i(t)\\ y_i(t)\\ x_i(t-d(t))\\ x_i(t-\rho d(t))\\ y_i(t-d(t))\\ x_i(t-h)\end{bmatrix}^{\mathrm{T}}\times\begin{bmatrix}Z_{11i}&Z_{12i}&Z_{13i}&Z_{14i}&Z_{15i}&Z_{16i}\\ Z_{12i}^{\mathrm{T}}&Z_{22i}&Z_{23i}&Z_{24i}&Z_{25i}&Z_{26i}\\ Z_{13i}^{\mathrm{T}}&Z_{23i}^{\mathrm{T}}&Z_{33i}&Z_{34i}&Z_{35i}&Z_{36i}\\ Z_{14i}^{\mathrm{T}}&Z_{24i}^{\mathrm{T}}&Z_{34i}^{\mathrm{T}}&Z_{44i}&Z_{45i}&Z_{46i}\\ Z_{15i}^{\mathrm{T}}&Z_{25i}^{\mathrm{T}}&Z_{35i}^{\mathrm{T}}&Z_{45i}^{\mathrm{T}}&Z_{55i}&Z_{56i}\\ Z_{16i}^{\mathrm{T}}&Z_{26i}^{\mathrm{T}}&Z_{36i}^{\mathrm{T}}&Z_{46i}^{\mathrm{T}}&Z_{56i}^{\mathrm{T}}&Z_{66i}\end{bmatrix}\begin{bmatrix}x_i(t)\\ y_i(t)\\ x_i(t-d(t))\\ x_i(t-\rho d(t))\\ y_i(t-d(t))\\ x_i(t-h)\end{bmatrix}$$
$$+\int_{t-\rho d(t)}^{t}2[x_i^{\mathrm{T}}(t)Z_{17i}+y_i^{\mathrm{T}}(t)Z_{27i}]y_i(s)ds$$
$$+\int_{t-\rho d(t)}^{t}2[x_i^{\mathrm{T}}(t-d(t))Z_{37i}+x_i^{\mathrm{T}}(t-\rho d(t))Z_{47i}]y_i(s)ds$$
$$+\int_{t-\rho d(t)}^{t}2[y_i^{\mathrm{T}}(t-d(t))Z_{57i}+x_i^{\mathrm{T}}(t-h)Z_{67i}]y_i(s)ds$$
$$+\int_{t-\rho d(t)}^{t}y_i^{\mathrm{T}}(s)Z_{77i}y_i(s)ds \tag{12j}$$

$$\dot{V}_{11i}(t)=\begin{bmatrix}x_i(t)\\ x_i(t-h)\\ \int_{t-\rho d(t)}^{t}y_i(s)ds\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}L_{11i}&L_{12i}&L_{13i}\\ L_{12i}^{\mathrm{T}}&L_{22i}&L_{23i}\\ L_{13i}^{\mathrm{T}}&L_{23i}^{\mathrm{T}}&L_{33i}\end{bmatrix}\begin{bmatrix}x_i(t)\\ x_i(t-h)\\ \int_{t-\rho d(t)}^{t}y_i(s)ds\end{bmatrix} \tag{12k}$$

Applying Lemma 1, we have

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}2x_i^{\mathrm{T}}(t)R_iB_{ij}^{k}x_j(t)$$
$$\le\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[x_i^{\mathrm{T}}(t)R_iB_{ij}^{k}M_{ij}(B_{ij}^{k})^{\mathrm{T}}R_ix_i(t)+x_j^{\mathrm{T}}(t)M_{ij}^{-1}x_j(t)]$$
$$=\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}x_i^{\mathrm{T}}(t)[R_iB_{ij}^{k}M_{ij}(B_{ij}^{k})^{\mathrm{T}}R_i+M_{ji}^{-1}]\,x_i(t) \tag{13a}$$

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}2y_i^{\mathrm{T}}(t)Q_iB_{ij}^{k}x_j(t)$$
$$\le\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[y_i^{\mathrm{T}}(t)Q_iB_{ij}^{k}\hat{M}_{ij}(B_{ij}^{k})^{\mathrm{T}}Q_iy_i(t)+x_j^{\mathrm{T}}(t)\hat{M}_{ij}^{-1}x_j(t)]$$
$$=\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[y_i^{\mathrm{T}}(t)Q_iB_{ij}^{k}\hat{M}_{ij}(B_{ij}^{k})^{\mathrm{T}}Q_iy_i(t)+x_i^{\mathrm{T}}(t)\hat{M}_{ji}^{-1}x_i(t)] \tag{13b}$$

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}2x_i^{\mathrm{T}}(t)R_i\bar{B}_{ij}^{k}x_j(t-d(t))$$
$$\le\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[x_i^{\mathrm{T}}(t)R_i\bar{B}_{ij}^{k}\bar{M}_{ij}(\bar{B}_{ij}^{k})^{\mathrm{T}}R_ix_i(t)+x_j^{\mathrm{T}}(t-d(t))\bar{M}_{ij}^{-1}x_j(t-d(t))]$$
$$=\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[x_i^{\mathrm{T}}(t)R_i\bar{B}_{ij}^{k}\bar{M}_{ij}(\bar{B}_{ij}^{k})^{\mathrm{T}}R_ix_i(t)+x_i^{\mathrm{T}}(t-d(t))\bar{M}_{ji}^{-1}x_i(t-d(t))] \tag{13c}$$

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}2y_i^{\mathrm{T}}(t)Q_i\bar{B}_{ij}^{k}x_j(t-d(t))$$
$$\le\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[y_i^{\mathrm{T}}(t)Q_i\bar{B}_{ij}^{k}\tilde{M}_{ij}(\bar{B}_{ij}^{k})^{\mathrm{T}}Q_iy_i(t)+x_j^{\mathrm{T}}(t-d(t))\bar{M}_{ij}^{-1}x_j(t-d(t))]$$
$$=\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}[y_i^{\mathrm{T}}(t)Q_i\bar{B}_{ij}^{k}\tilde{M}_{ij}(\bar{B}_{ij}^{k})^{\mathrm{T}}Q_iy_i(t)+x_i^{\mathrm{T}}(t-d(t))\tilde{M}_{ji}^{-1}x_i(t-d(t))] \tag{13d}$$

According to Lemma 2 and using the idea of [9], we get

$$-\int_{t-\rho d(t)}^{t}\delta_iy_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds\le-\frac{(\delta_i-1)}{\rho h}\left[\int_{t-\rho d(t)}^{t}y_i(s)ds\right]^{\mathrm{T}}X_{2i}\left[\int_{t-\rho d(t)}^{t}y_i(s)ds\right]-\int_{t-\rho d(t)}^{t}y_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds \tag{14a}$$

$$0=x_i^{\mathrm{T}}(t)K_ix_i(t)-x_i^{\mathrm{T}}(t-\rho d(t))K_ix_i(t-\rho d(t))-2\int_{t-\rho d(t)}^{t}x_i^{\mathrm{T}}(s)Ky_i(s)ds \tag{14b}$$

From (6d), (12h), (12j) and Lemma 3, we have

$$-\int_{t-\rho d(t)}^{t}y_i^{\mathrm{T}}(s)[(1-\mu)X_{1i}-Z_{77i}]y_i(s)ds\le-\frac{1}{\rho h}[x_i(t)-x_i(t-\rho d(t))]^{\mathrm{T}}[(1-\mu)X_{1i}-Z_{77i}]\times[x_i(t)-x_i(t-\rho d(t))] \tag{15a}$$

$$-\int_{t-h}^{t-\rho d(t)}\delta_iy_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds\le-\frac{\delta_i}{h}[x_i(t-\rho d(t))-x_i(t-h)]^{\mathrm{T}}X_{2i}\times[x_i(t-\rho d(t))-x_i(t-h)] \tag{15b}$$

From (11) − (15), we obtain

$$\dot{V}(t) \le \sum_{i=1}^{N}\sum_{k=1}^{r} \alpha^k(t)\{\omega_i^{\mathrm{T}}(t)\Pi_i\,\omega_i(t)$$

$$-\int_{t-\rho d(t)}^{t}\begin{bmatrix} x_i(s) \\ y_i(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}(1-\rho\mu)U_i & K_i \\ K_i^{\mathrm{T}} & X_{2i}\end{bmatrix}\begin{bmatrix} x_i(s) \\ y_i(s)\end{bmatrix}ds\} \quad (16)$$

where $\omega_i(t) = [x_i^{\mathrm{T}}(t)\ \ y_i^{\mathrm{T}}(t)\ \ x_i^{\mathrm{T}}(t-d(t))\ \ x_i^{\mathrm{T}}(t-\rho d(t))$ $y_i^{\mathrm{T}}(t-d(t))\ x_i^{\mathrm{T}}(t-h)\ (\int_{t-\rho d(t)}^{t} y_i(s)ds)^{\mathrm{T}}]^{\mathrm{T}}$ and matrix Π_i is defined in (7b).

Based on Leibniz-Newton formula, we get

$$x_i(t) - x_i(t-\rho d(t)) - \int_{t-\rho d(t)}^{t} y_i(s)ds = 0 \quad (17)$$

This means

$$D_i\omega_i(t) = 0 \quad (18)$$

where $D_i = [I_i \quad 0 \quad 0 \quad -I_i \quad 0 \quad 0 \quad -I_i]$.

From Lemma 4, it is seen that $\omega_i^{\mathrm{T}}(t)\Pi_i\omega_i(t) < 0$ is equivalent to inequality (6e). Obviously, if inequalities (6e) and (6f) hold, then $\dot{V}(t) < 0$, which ensures that system (8) is asymptotically stable [1]. It means that system (1) is asymptotically stable, too. The proof is completed.

3. Numerical Examples

In this section, two examples are given to show the benefits of our result.

Example 1: Consider the following large-scale switched time-varying-delay system composed of two individual switched systems:

Switched system 1 ($k = 1$):

$$\dot{x}_1(t) = \begin{bmatrix}-5.5 & 0 \\ 0 & -3.3\end{bmatrix}x_1(t) + \begin{bmatrix}-0.5 & 0.4 \\ 0.1 & -0.3\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}0.2 & 1 \\ 0.5 & 0.2\end{bmatrix}x_2(t) + \begin{bmatrix}0.5 & 0 \\ 0.1 & 0\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0 & -0.1 \\ -0.2 & 0\end{bmatrix}x_3(t) + \begin{bmatrix}-1 & 0 \\ 0.1 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_2(t) = \begin{bmatrix}-8.3 & 0 \\ 0 & -6.3\end{bmatrix}x_2(t) + \begin{bmatrix}0.7 & 0 \\ -0.5 & -1\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0.2 & 0.3 \\ 0.1 & 0.7\end{bmatrix}x_1(t) + \begin{bmatrix}1.1 & 0.2 \\ 0.3 & 0\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}1.1 & 0.1 \\ 0.3 & 0.2\end{bmatrix}x_3(t) + \begin{bmatrix}-1 & 0.5 \\ 0.7 & 0.1\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_3(t) = \begin{bmatrix}-9.2 & 0 \\ 0 & -7.2\end{bmatrix}x_3(t) + \begin{bmatrix}-1 & 1 \\ 0.5 & -3\end{bmatrix}x_3(t-d(t))$$
$$+\begin{bmatrix}0 & 0.4 \\ 1 & 0\end{bmatrix}x_2(t) + \begin{bmatrix}0.1 & 0 \\ 0.2 & 1\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0.5 \\ 0 & 0.1\end{bmatrix}x_1(t) + \begin{bmatrix}0 & 1 \\ 1 & 1\end{bmatrix}x_1(t-d(t)) \quad (19a)$$

Switched system 2 ($k = 2$):

$$\dot{x}_1(t) = \begin{bmatrix}-2.5 & 0 \\ 0 & -3.5\end{bmatrix}x_1(t) + \begin{bmatrix}-0.1 & 0 \\ 0.5 & -0.1\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0.2 \\ 0.1 & 0\end{bmatrix}x_2(t) + \begin{bmatrix}0 & 0 \\ 0.2 & 0.1\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix}x_3(t) + \begin{bmatrix}0 & 0.1 \\ 0.2 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_2(t) = \begin{bmatrix}-3.6 & 0 \\ 0 & -5\end{bmatrix}x_2(t) + \begin{bmatrix}-0.5 & 0 \\ 0 & -0.5\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0 & 0.1 \\ 0.5 & 0\end{bmatrix}x_1(t) + \begin{bmatrix}0 & 0.5 \\ 0.1 & 0\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}0.2 & 0 \\ 0 & 0.2\end{bmatrix}x_3(t) + \begin{bmatrix}0 & 0.1 \\ 0.1 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_3(t) = \begin{bmatrix}-7 & 0 \\ 0 & -2.6\end{bmatrix}x_3(t) + \begin{bmatrix}-0.5 & 0 \\ 0 & -0.5\end{bmatrix}x_3(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix}x_2(t) + \begin{bmatrix}0 & 0.1 \\ 0 & 0\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0 & 0 \\ 0.2 & 0\end{bmatrix}x_1(t) + \begin{bmatrix}0 & 0.3 \\ 0 & 0.2\end{bmatrix}x_1(t-d(t)) \quad (19b)$$

Our purpose in example 1 is to find the maximum allowed delay h of $d(t)$ satisfying $\dot{d}(t) \le \mu$, such that the switched system (19) is asymptotically stable. A comparison between our Theorem 1 and the method of [10] is shown in Table 1, which also displays the maximum allowed delay h and its time derivative μ for guaranteeing the asymptotic stability of system (19). Obviously, it can be seen that the delay-partitioning-dependent stability criterion in this paper is less conservative than one given by [10].

Table 1. Allowable delay bound *h* for different μ

μ	h ([10])	h (Our Theorem 1)
0.5	Fail	6.5731
1.0	Fail	5.3286
1.5	Fail	4.7217
2.0	Fail	4.1153
2.5	Fail	2.9025

Example 2: Consider the following switched systems with time-varying delay

Switched system 1:

$$\dot{x}(t)=\begin{bmatrix}-5.5 & 0\\ 0 & -1.5\end{bmatrix}x(t)+\begin{bmatrix}-0.1 & 0.2\\ 0.1 & 0.5\end{bmatrix}x(t-d(t)) \quad (20a)$$

Switched system 2:

$$\dot{x}(t)=\begin{bmatrix}-2.2 & 0\\ 0 & -7.7\end{bmatrix}x(t)+\begin{bmatrix}0.1 & 0.5\\ 0.5 & 0.1\end{bmatrix}x(t-d(t)) \quad (20b)$$

Our purpose in example 2 is to find the maximum allowed delay h of $d(t)$ satisfying $\dot{d}(t)\le\mu$, such that the switched system (20) is asymptotically stable. A comparison between our Theorem 1 and the methods of [11], [12] and [13] is shown in Table 2, which also displays the maximum allowed delay h and its time derivative μ for guaranteeing the asymptotic stability of system (20). It is clear that our new method produces better results than those in [11], [12] and [13].

Table 2. Allowable delay bound *h* for different *μ*

μ	h ([11])	h ([12])	h ([13])	h (Our Theorem 1)
0.1	1.5319	2.6381	3.7215	9.8128
0.3	0.9287	2.0236	2.8738	8.9236
0.7	0.6093	1.1153	1.5596	7.7531
0.9	0.5182	0.7928	1.2361	6.5762
1.1	0.3016	0.5527	0.7329	5.3573

4. Conclusion

A class of large-scale system with time-varying neutral delay and switched-type is studied in this paper. Based on weighting-delay-parameter approach, an augmented Lyapunov-Krasovskii functional form combined with free matrices, singular model transformation technique and Finsler's lemma, a new delay-partitioning-dependent stability criterion is derived in terms of matrix inequalities. Two numerical examples are given to show the effectiveness and benefits of the proposed criterion.

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