1. Slove the following problems.

(a) $y' = (y - 2x)^2 - 7, y(0) = 0.$ (15%)

(b) $y'' + 9y = 9\csc 3x.$

(c) $y' + y = 3\cos t \cdot U(t - \pi), y(0) = 5,$ where U is the unit step function.

2. For matrix $\mathbf{A} = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix},$ (a) find the eigenvalues and the corresponding eigenvectors of $\mathbf{A},$ (b) find the matrix $\mathbf{P}$ that diagonalizes $\mathbf{A}$ and the diagonal (15%) matrix $\mathbf{D}$ such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{DP}.$

3. Expand $f(t) = t^2, 0 < t < L,$ in a (a)cosine series, (b)sine series, (c) Fourier series. (15%)

4. Evaluate $\oint_C \dfrac{z^2 - 4z + 4}{z + 1} dz,$ where C is the circle $|z| = 2.$ (10%)

5. Please calculate the area enclosed by the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$ (15%)

6. Compute the line integral $\oint_C (2xy - x^2)dx + (x + y^2)dy,$ where C is the (15%) boundary of the area enclosed by parabolic $y = x^2$ and straight line $y = x.$

7. If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ matrices, and $\mathbf{A}^2 = \mathbf{A}, \mathbf{B}^2 = \mathbf{B}$ prove that

(a) If $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A} + \mathbf{B}$ then $\mathbf{AB} = \mathbf{0},$ (15%)

(b) If $\mathbf{AB} = \mathbf{BA}$ then $(\mathbf{A} + \mathbf{B} - \mathbf{AB})^2 = \mathbf{A} + \mathbf{B} - \mathbf{AB}.$