

所別：生產系統工程與管理組 組別：研發科技組 身分別：一般生  
 科目：微積分 准考證號碼：□□□□□□□□ (考生自填)

考生注意事項：

一、考試時間 100 分鐘。

1. Evaluate each of the following limit if it exists: (Each 5 points, total 20 points)

(a)  $\lim_{x \rightarrow 0} \left( \frac{1}{1 - e^{-x}} - \frac{1}{x} \right)$  (5 points)

(b)  $\lim_{n \rightarrow 1} (2n - 1)^{\frac{2}{n-1}}$  (5 points)

(c)  $\lim_{x \rightarrow 0^+} (x^x)$  (5 points)

(d)  $\lim_{x \rightarrow 0^+} \left( 1 + \frac{1}{x} \right)^x$  (5 points)

2. Evaluate  $\int_0^{\infty} x e^{-\frac{x}{\beta}} dx$ . (10 points)

3. Let  $f(x) = \frac{\sigma}{\pi} \cdot \frac{1}{\sigma^2 + (x - \mu)^2}$ , where  $x \in \mathbb{R}$ ,  $\mu \in \mathbb{R}$ ,  $\sigma > 0$ . To show that

$$\int_{-\infty}^{\infty} f(x) dx = 1. \text{ (10 points)}$$

4. To show that  $\lim_{n \rightarrow \infty} \left\{ \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right\} = \frac{\pi}{4}$  (10 points)

5. Evaluate  $\int_0^1 \sin^{-1} x dx$ . (10 points)

6. Evaluate  $\int_{-\infty}^{\infty} e^{-x^2} dx$ . (10 points)

7. Evaluate  $\int_0^1 (\ln \frac{1}{x})^{0.5} dx$ . (10 points)

8. Find and classify all critical points of  $f(x, y) = x^3 + y^3 - 3x - 3y$  (10 points)

9. Use Lagrangian method to solve the following (10 points):

$$\text{Max } f(x, y) = 50x^{0.5}y^{0.25}$$

$$\text{Subject to } 150x + 100y = 25000$$