考生注意事項：

一、考試時間 100 分鐘

二、不得帶字典、翻譯機、計算機等

三、請先核對試題，答案卷（試卷）與准考證上之所組別與考試科目是否相符。

四、請依照題目順序在答案卷作答。

1. Formulate a boundary value problem model of heat conduction in a slender bar of length L if the left end is insulated and the right end is kept at temperature T_0. The initial temperature in the cross section at x is $f(x)$. (5%)

2. An elastic prismatic rod is vibrated in longitudinal direction by the wave equation as the following. If at time zero, the rod is stretched by a differential deformation and released from the static state.

$$c^2 u_{xx} - u_{tt} = 0, \qquad \text{for } t > 0,\ 0 < x < L$$

$$u_x(0, t) = 0, \qquad u_x(L, t) = 0, \quad \text{for } t \geq 0$$

$$u(x, 0) = (1 + \alpha)x, \quad u_t(x, 0) = 0, \quad \text{for } 0 \leq x \leq L$$

   (a) Give a word statement of the constant c. (3%)

   (b) Find a series solution. (15%)

3. The state of stress in a plate lying in the x-y plane is given $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$. Using Hooke's law, the strain stress relation is $\{\varepsilon\} = [S]\{\sigma\}$, where $[S]$ is the compliance matrix.

   (a) Calculate the inverse matrix of $[S]$. (7%)

   (b) When the strain are $\varepsilon_{xx} = 1$, $\varepsilon_{yy} = 2$, $\varepsilon_{xy} = 3$, and the Poisson ratio is ν, find the plane stresses. (5%)

$$[S] = \begin{bmatrix} \dfrac{1}{E} & -\dfrac{\nu}{E} & 0 \\[2mm] -\dfrac{\nu}{E} & \dfrac{1}{E} & 0 \\[2mm] 0 & 0 & \dfrac{1+\nu}{E} \end{bmatrix}$$

4. The stress state at a point a machine element with respect to a Cartesian coordinate system is $\sigma_{xy} = 1$, $\sigma_{yz} = 1$, $\sigma_{xz} = -2$, $\sigma_{xx} = 3$, $\sigma_{yy} = 0$, $\sigma_{zz} = 3$ (MPa).

   (a) Write the stress tensor in matrix form. (5%)

背面還有試題

(b) Compute the principle stresses and corresponding directions. (6%)

5. Find the general solution of the equation: $y' = \dfrac{y}{x} + \dfrac{x}{2y}$. (8%)

6. Analyze the quadratic form $f(x_1, x_2) = 3x_1^2 + 3x_2^2 + 2x_1 x_2$.
   (a) Find the standard form of $f(x_1, x_2)$. (5%)
   (b) If $f(x_1, x_2) = 4$, draft it in the principle axes. (7%)

7. Find the standard form of the quadratic form: $-2x_1 x_2 + 2x_3^2$. (6%)

8. Find the general solution of $x^2 y'' + xy' + 4y = f(x)$,
   (a) if $f(x) = 0$; (4%)　(b) if $f(x) = \sin(2\ln x)$. (8%)

9. Solve the differential equation (8%)
   $y''' + y' = 0$, with $y(0) = 0$, $y'(\pi) = 0$, $y''(\frac{\pi}{2}) = -1$.

10. Solve the differential equation
   $$y' + y = \begin{cases} 0, & 0 \le t \le \pi \\ 3\cos t, & \pi \le t \end{cases} \quad \text{with } y(0) = 0. \text{ (8%)}$$