所別:精密機械與製造科技研究所　　　　　組別:甲、乙

身分別:一般生

科目:工程數學　　　　　　　准考證號碼: ☐☐☐☐☐☐☐☐

(考生自填)

考生注意事項:

一、考試時間100分鐘

二、應考人可攜帶不具儲存程式功能之計算機

三、無論是否使用計算機,作答時均須詳列解答過程。

四、請依照題目順序在答案卷作答。

五、共兩頁

1. [15%] Consider the driven mechanical oscillator governed by the differential equation

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

where $m = 0.4kg, c = 5N - s/m, k = 1400N/m$, and let $F(t)$ is a periodic excitation force

$$F(t) = \begin{cases} 2 & 0 < t < \pi \\ -2 & \pi < t < 2\pi \end{cases} \quad F(t + 2\pi) = F(t)$$

Find the steady-state oscillation?

2.[15%] (a) Draw a labeled sketch of the graph of the function $f(t)$, where $H(t)$ is a Heaviside function (unit step function).

(b) Find the Laplace transform of $f(t)$? $f(t) = H(t - \pi)cos(t)$

3. [10%] Solve y(t) in the equation

$$y = t - 9 \int_0^t y(\tau)(t - \tau)d\tau$$

4.[15%] The one dimensional heat equation subject to the given conditions

$$u(0, t) = 0 \qquad u(L, t) = 0$$

$$u(x, t) = \begin{cases} \pi/2 & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases}$$

(a) Write out the differential equation (b) Solve the temperature function $u(x, t)$?

5.[10%] Solve the differential equation

$$\frac{d^2u}{dx^2} + \pi^2 u - 1 = 0 \quad 0 < x < 1$$

with boundary conditions

$$u(0) = 0 \quad u(1) = 0$$

6.[10%] Determine a formula for the $k - th$ power of the matrix

$$\mathbf{A} = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix}$$

7.[15%] Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{2x + y}{2y - x}$$

8.[10%] Find the directional derivative of $W = x^2y + 2xy^2 - z^3$ at $(1, 2, 3)$ in the direction $\mathbf{d} = 1\hat{i} - 2\hat{j} + 2\hat{k}$