

國立勤益技術學院九十三年度研究所招生初試試題卷

所別：資電所
科目：工程數學

組別：

身分別：一般生或在職生

准考證號碼：□□□□□□□□ (考生自填)

考生注意事項：

- 一、考試時間 100 分鐘。
- 二、請以黑色或藍色原子筆作答。
- 三、不得攜帶字典、翻譯機、計算機等。

試題：(共 100 分)

1. Solve the following initial value problems. Indicate the method you are using and giving all steps in detail. (15%)

(1). $y' + (1/2)y = y^3$, $y(0) = 1$ (2). $y'' - 2y' + (4\pi^2 + 1)y = 0$, $y(0) = -2$,
 $y'(0) = 6\pi - 2$.

2. Solve the system of differential equations and initial conditions for the functions x and y using the Laplace transform. (15%)

$$x'' - 2x' + 3y' + 2y = 4,$$

$$2y' - x' + 3y = 0,$$

$$x(0) = x'(0) = y(0) = 0.$$

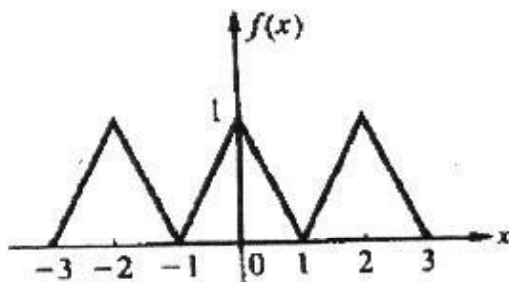
3. Find the eigenvalues of the matrix and, for each eigenvalue, a corresponding eigenvector. Check that eigenvectors associated with distinct eigenvalues are orthogonal. Find an orthogonal matrix that diagonalizes the matrix. (15%)

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

4. Solve the initial value problem of the following equation by diagonalization. (10%)

$$x_1' = x_1 + x_2 + 6e^{2t}, \quad x_2' = x_1 + x_2 + 2e^{2t}; \quad x_1(0) = 6, \quad x_2(0) = 0.$$

5. Find the Fourier series of the function f which has the graph shown in the following figure. (15%)



6. Let the 3x3 matrix be defined as

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 4 & 1 \\ -4 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$$

Find the inverse of the following matrices: (a). $\frac{1}{11}(\mathbf{B} + \mathbf{I})$; (b). $(\mathbf{A} - \mathbf{I})$. (20%)

7. Find the convolution of two finite discrete sequences $\mathbf{A}=[1,2,0,1,2]$, $\mathbf{B}=[3,2,1]$. (10%)