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Fuzzy multiple goal programming applied to TFT-LCD supplier selection by downstream manufacturers

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ABSTRACT

In today's highly competitive environment, a good supply chain relationship is essential for a company to survive and to acquire reasonable profit. While a few large companies may be able to vertically integrate from the design stage to the final distribution of the entire supply chain, most companies can only focus on their specialized functions and to cooperate with upstream or downstream companies. Supplier selection, as a result, is very important for maintaining strategic alliances. The objective of this paper is to develop a fuzzy multiple goal programming (FMGP) model to help downstream companies to select thin film transistor liquid crystal display (TFT-LCD) suppliers for cooperation. Fuzzy analytic hierarchy process (FAHP) is applied first to analyze the importance of multiple factors by incorporating experts' opinion, and these factors include cost, yield and number of suppliers. Multi-choice goal programming is used next to consider the limits of various resources and to formulate the constraints. From the experimental design and examination, we can testify that the proposed model not only can consider multi-choice goals, decision-making behavior and limit of resources, it can also allocate the purchase among the selected supplier(s).

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1. Introduction

The spread of flat panel displays is inescapable in the digital era and will become an important human machine interface in the future. Because of their low weight, slender profile, low power consumption, high resolution, high brightness and low radiance advantages, the use of flat panel displays has been expanding from portable appliances to notebook and desktop monitors and even to large screen digital televisions. The two most important trends in flat panel display technology is larger display size along with higher resolution, and TFT-LCD is able to gain the greatest attention from both suppliers and consumers.

Outsourcing has become an important business approach since a competitive advantage may be gained by the cooperation with suppliers to provide products/services more effectively and efficiently (McCarthy & Anagnostou, 2004). Companies in a TFT-LCD supply chain can focus on only one or two steps in the supply chain while outsourcing the rest of steps to other companies. For instance, a TFT-LCD manufacturing company may receive orders from a notebook manufacturing company, which specifies the specification of the panels, and manufacture TFT-LCD modules

according to the design. It also needs to find upstream companies to obtain the required equipment, material and components. On the other hand, for a notebook manufacturing company, it also needs to find one or several suitable TFT-LCD manufacturing companies to obtain the required TFT-LCD module for producing notebook computers. In consequence, the selection of the right companies for cooperation is important for maintaining a competitive edge. In addition, how to distribute the amount of purchases to the selected manufacturers is also a problem faced by the purchasing companies.

The rest of this paper is organized as follows. Section 2 introduces the TFT-LCD industry. Section 3 reviews some recent researches on supplier selection. FAHP and goal programming are discussed in Section 4. Section 5 proposes a fuzzy MCGP model applied to select TFT-LCD companies for downstream manufacturers. Section 6 presents a case study of a notebook manufacturer in Taiwan to verify the practicality of the model. Some concluding remarks are made in the last section.

2. The TFT-LCD industry

In this section, we review the manufacturing process of TFT-LCD, the supply chain of TFT-LCD and the TFT-LCD industry in Taiwan.

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2.1. Manufacturing process of TFT-LCD

TFT-LCD has a sandwich-like structure consisting of two glass substrates with a layer of liquid crystal inside. In fact, a TFT-LCD module consists of a TFT panel (with TFT-array substrate, liquid crystal and color filter substrate), a driving-circuit unit (with LCD driver IC (LDI) chips, multi-layer PCBs and driving-circuits) and a backlight and chassis unit (with backlight, lamp, light-guide panel (LGP) and chassis).

The manufacturing of TFT-LCD, as depicted in Fig. 1, can be categorized into four main processes: TFT array fabrication, color filter (BM) fabrication, color filter (RGB) fabrication, cell assembly and module assembly.

2.2. Supply chain of TFT-LCD

A supply chain is defined as "an integrated process wherein a number of various business entities (i.e., suppliers, manufacturers, distributors and retailers) work together in an effort to: (1) acquire raw materials/components, (2) convert these raw materials/components into specified final products, and (3) deliver these final products to retailers" (Beamon, 1998). Supply chain management is becoming more and more important in industries due to shortened product life cycles, rising manufacturing costs and globalizing market economies.

The TFT-LCD supply chain involves the domains of optics, semiconductor, electrical engineering, chemical engineering, mechanical engineering and material. The upstream of the TFT-LCD supply chain includes the equipment (e.g. photo/etch equipment) and the material and components (e.g. glass substrate, backlight module and driver IC). The midstream processes the material provided by the material/components suppliers, by using the equipment provided by equipment suppliers. Panels are first manufactured and then assembled into TFT-LCD modules. The modules are used by downstream manufacturers to make final products such as notebook computer, LCD monitors, etc.

To summarize, in order to gain competitive advantage, a company in the TFT-LCD supply chain, as in many other industries, needs to keep good cooperation with other firms in the supply chain to provide products more effectively and efficiently.

2.3. TFT-LCD and notebook industries in Taiwan

One of the most brilliant industries in Taiwan is in the TFT-LCD industry. With the success in the information and semiconductor industry over the past decade, Taiwan has a strong background and foundation for developing the TFT-LCD industry. Most of the TFT-LCD panels in the world are supplied by Taiwan, Korea and Japan. Japan is the technology leader, but it has changed its strategy from manufacturing to R&D in the TFT-LCD industry, especially for large size TFT-LCD, due to its high manufacturing costs. With the transfer of Japan's TFT-LCD technology to Taiwan and the competitive advantages of Taiwan in abundant capital, numerous downstream clients and a complete supply chain, the TFT-LCD industry is flourishing in Taiwan (Chang, 2005). Taiwan is the re-

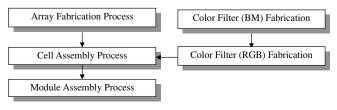


Fig. 1. TFT-LCD manufacturing process.

gion with the second largest capacity in small-to-medium size TFT-LCD, and the largest in large size TFT-LCD. As panel manufacturers continue to build up new capacity with new-generation fabs, a larger capacity is becoming a fact soon.

The case study in this paper is based on an anonymous notebook manufacturer in Taiwan in selecting the most suitable TFT-LCD manufacturer(s) for outsourcing. In recent years, demand for notebook computers continues to grow at a faster rate than that for desktop computers due to the portable characteristic of notebooks. Taiwan has two diversified businesses in notebooks: contract (OEM) manufacturing for the world's top brands such as Dell, HP and Toshiba, and own brand manufacturing such as Asus and Acer (People's Daily Online, 2004). It is an important base for the global production of notebook computers, and the global market share of notebooks manufactured by Taiwan firms was 73% in 2005 (DigiTimes Publication, 2006). Even though Taiwan firms have the majority of market share in notebook manufacturing. the profitability of the industry has been shrinking. To improve profit margins and to decrease labor and production costs, Taiwan manufacturers have been quick to relocate assembly lines offshore, and over 90% of Taiwan's manufacturers have set up production plants in China. However, the profit margin in the industry is expected to continue declining due to capacity expansion.

In conclusion, a notebook manufacturing company, in order to increase profit margin and to obtain satisfactory TFT-LCD modules, needs to find one or several suitable TFT-LCD manufacturing companies for cooperation. In addition, the distribution of the amount of purchases to the selected manufacturers should also be examined in order to keep competitive advantage in the global market.

3. Supplier selection problem

In the current business environment, global competition is an unpreventable fact, and customer demands are diversified. The result is progressively increased costs and sharply decreased profit. In consequence, purchasing has become a crucial job in establishing value-added contents of products and a vital determinant to ensure the profitability and survival of a company. Many companies are trying to reduce their costs while satisfying customer needs by strengthening their core competencies and outsourcing other functions. Selecting the right suppliers which can maintain a continuous supply relationship requires a careful assessment because suppliers have varied strengths and weaknesses.

The research on supplier selection is abundant, First publications can be traced back to the 1960s, and Weber, Current, and Benton (1991) and Ghodsypour and O'Brien (1998) did a comprehensive review on the past research. de Boer, Labro, and Morlacchi (2001) identified four research subjects within the research field of supplier selection: problem definition, formulation of criteria, prequalification and final selection. The latter two are mostly studied. The pre-qualification step is the process of reducing the set of suppliers to a smaller number of acceptable suppliers, and the methods that are often applied can be categorized into four kinds: categorical methods, data envelopment analysis (DEA), clustering analysis (CA), and case-based reasoning (CBR) system (de Boer et al., 2001). The final selection step is usually solved by five types of methods: linear weighting, total cost of ownership, mathematical programming (MP), statistics, and artificial intelligence (AI) (de Boer et al., 2001; Hong, Park, Jang, & Rho, 2005).

The simplest supplier selection method is the categorical method, by which each supplier characteristic is assigned good, satisfactory, neutral and unsatisfactory and then the total score for each supplier is summed up (Ghodsypour & O'Brien, 1998). Linear weighing method is one of the most common methods, and the concept is to give different weights to a number of criteria and to

select the supplier with the best weighted total score (Roodhooft & Konings, 1997). While most proposed methods belong to linear weighting and MP models (Hong et al., 2005), MP models are proved more effective than the linear weighting methods because they can optimize the explicitly stated objective (Kumar, Vrat, & Shankar, 2004). Muralidharan, Anantharaman, and Deshmukh (2002) did a comparison of various supplier rating methods and listed the advantages and limitations of the methods.

A MP model formulates the decision problem in terms of a mathematical objective function that needs to be maximized (e.g. profit) or minimized (e.g. cost) by varying the values of variables in the objective function (e.g. the amount ordered with a supplier) (Hong et al., 2005). MP models can be subdivided into linear programming, mixed integer programming, and goal programming/ multi-objective goal programming (MOP). Some researches by MP models are reviewed in Weber and Desai (1996). Weber. Current, and Desai (1998) and Muralidharan et al. (2002). Hong et al. (2005) proposed a mathematical programming model that considers the change in suppliers' supply capabilities and customer needs over a period in time, and the model not only can maximize revenue but also can satisfy customer needs. Multi-objective programming (MOP) also becomes a very popular tool since many criteria, not a single criterion, can be examined with different weights. Weber and Current (1993) introduced a MOP for selecting suppliers with order quantities in procurement environments characterized by multiple conflicting criteria. Weber (1996) applied DEA in supplier evaluation for an individual product and demonstrated the advantages of applying DEA.

In some works, two or more methodologies are combined to select suppliers. An integrated method was proposed by Ghodsypour and O'Brien (1998) to combine AHP and linear programming to choose the best supplier and to assign the optimum order quantity among selected suppliers. In the situation that selecting one supplier results in another being left out, Weber et al. (1998) combined MOP and DEA to deal with non-cooperative supplier negotiation strategies. To select and benchmark potential suppliers, Choy, Lee, and Lo (2003) designed an intelligent supplier relationship management system using hybrid case based reasoning and artificial neural networks techniques. By combining AHP and DEA, Liu and Hai (2005) constructed a voting analytic hierarchy process (AHP) method to select supplier by comparing the weighted sum of the selection number of rank vote, after determining the weights in a selected rank.

4. Review of mathematical methodologies

4.1. Fuzzy analytic hierarchy process (FAHP)

The AHP, a mathematically based MCDM tool, is very popular to academic researchers for data analysis and model verifications, and to provide critical information for managers to make business decisions (Lee, Kang, & Wang, 2005). Since its introduction by Saaty (1980) back to the early 1970s in response to the scarce resources allocation and planning needs for the military, AHP has been widely employed in decision-making analysis in various fields such as political, social, economic and management sciences (Lee et al., 2005). A complex problem is decomposed into several sub-problems in terms of hierarchical levels, and the factors of the same hierarchical level are compared relative to their impact on the solution of their higher level factor. Pairwise comparisons are employed among decision elements, and comparison matrices are formed. After the consistency of the matrices is examined, the relative weights of decision elements are estimated next. The relative weights are aggregated lastly to obtain an overall rating for the decision alternatives. Numerous applications of AHP have been published in the literature (Chan & Ip, 1995; Cheng, Li, & Ho, 2002; Padillo, Meyersdorf, & Reshef, 1997; Punniyamoorthy & Ragavan, 2003; Shim, 1989; Yang, Su, & Hsu, 2000; Yu & Li, 2001)

Fuzziness and vagueness are common characteristics in many decision-making problems, and a good decision-making model should be able to tolerate vagueness or ambiguity (Yu, 2002). In addition, decision makers very naturally provide uncertain answers rather than precise values, and it is difficult to transform qualitative preferences to point estimates (Lee et al., 2005). Therefore, pairwise comparison under traditional AHP may not be appropriate due to the necessity of selecting arbitrary values in the process, and a degree of uncertainty should be considered in some or all pairwise comparison values (Yu, 2002). In consequence, the incorporation of the fuzzy theory in AHP should be more appropriate and effective than conventional AHP. Many researches have been done on the development and the application of FAHP. and tremendous amount of FAHP methodologies are existed (Boender, de Graan, & Lootsma, 1989; Buckley, 1985; Chen, 1996; Cheng, 1999; Csutora & Buckley, 2001; Laarhoeven & Pedrycz, 1983; Lee et al., 2005; Lee, 2009; Murtaza, 2003).

A brief introduction of the fuzzy set theory is presented here. A triangular fuzzy number is very popular in fuzzy applications. As shown in Fig. 2, the triangular fuzzy number \widetilde{M} is represented by (a,b,c), and the membership function is defined as (Cheng, 1999; Lee et al., 2005):

$$\mu_{\widetilde{M}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leqslant x \leqslant b \\ \frac{c-x}{c-b}, & b \leqslant x \leqslant c \\ 0, & \text{otherwise} \end{cases}$$
 (1)

with $-\infty < a \le b \le c \le \infty$.

The strongest grade of membership is parameter b, that is, $f_M(b) = 1$, while a and c are the lower and upper bounds.

Many different methods have been devised to rank fuzzy numbers, and each method has its own advantages and disadvantages (Klir & Yan, 1995). A popular method is the intuition ranking method, which ranks triangular fuzzy numbers by drawing their membership function curves. A higher mean value and lower spread fuzzy number is preferred by human intuition (Lee & Li, 1988). Another popular fuzzy number ranking method is the α -cut method (Adamo, 1980). Centroid ranking method is also often used to rank fuzzy numbers (Yagar, 1978).

In this paper, the extent analysis method (EAM) is applied. Two triangular fuzzy number $M_1(m_1^-,m_1,m_1^+)$ and $M_2(m_2^-,m_2,m_2^+)$ shown in Fig. 3 are compared (Lee, 2009). When $m_1^-\geqslant m_2^-$, $m_1\geqslant m_2$, $m_1^+\geqslant m_2^+$, we define the degree of possibility $V(M_1\geqslant M_2)=1$. Otherwise, we can calculate the ordinate of the

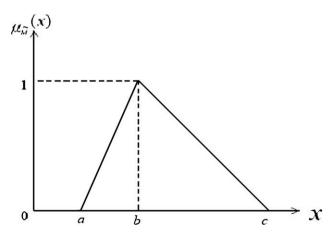


Fig. 2. Membership function of a triangular fuzzy number $\widetilde{M} = (a, b, c)$.

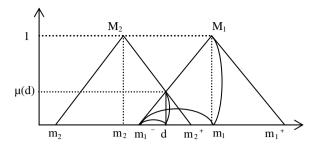


Fig. 3. Two triangular fuzzy numbers M_1 and M_2 (Lee, 2008).

highest intersection point (Chang, 1996; Lee, 2009; Zhu, Jing, & Chang, 1999)

$$V(M_2 \ge M_1) = hgt(M_1 \cap M_2) = \mu(d)$$

$$= \frac{m_1^- - m_2^+}{(m_2 - m_2^+) - (m_1 - m_1^-)}$$
(2)

We define the value of fuzzy synthetic extent with respect to factor *I* (Chang, 1996; Lee, 2009; Zhu et al., 1999)

$$F_{i} = \sum_{j=1}^{n} M_{ij} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} \right]^{-1}, \quad i = 1, 2, \dots, n$$
 (3)

$$\sum_{j=1}^{n} M_{ij} = \left(\sum_{j=1}^{n} m_{ij}^{-}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} m_{ij}^{+}\right), \quad i = 1, 2, \dots, n \tag{4}$$

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{n}M_{ij}\right]^{-1} = \left(1\left/\sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}^{+},1\right/\sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij},1\left/\sum_{i=1}^{n}\sum_{j=1}^{p}m_{ij}^{-}\right)\right.$$
(5)

A convex fuzzy number can be defined by

$$V(F \ge F_1, F_2, \dots, F_k) = \min V(F \ge F_i), \quad i = 1, 2, \dots, k$$
 (6)

$$d(F_i) = \min V(F_i \geqslant F_k) = w'_i, \quad k = 1, 2, \dots, n \quad \text{and} \quad k \neq i$$
 (7)

Based on the above procedure, the weights, w_i , of factors are

$$W' = (w'_1, w'_2, \dots, w'_n)^{\mathrm{T}}$$
(8)

After normalization, the priority weights are as follows:

$$W = (w_1, w_2, \dots, w_n)^{\mathrm{T}} \tag{9}$$

4.2. Goal programming (GP)

A goal programming (GP) model is useful in dealing with multicriteria decision problems where the goals cannot simultaneously be optimized. GP allows decision makers to consider several objectives together in finding a set of acceptable solutions and to obtain an optimal compromise. It was first introduced by Charnes and Cooper (1961), and further developed by Lee (1972), Ignizio (1985), and many others (Tamiz, Jones, & Romero, 1998; Chang, 2007). The purpose of GP is to minimize the deviations between the achievement of goals and their aspiration levels (Chang, 2007). Sharma, Benton, and Srivastava (1989) proposed a GP formulation for vendor selection to attain goals pertaining to price, quality and lead-time under demand and budget constraints. Buffa and Jackson (1983) also proposed the use of GP for price, quality and delivery objectives to evaluate vendors. An integrated AHP and preemptive goal programming based multi-criteria decisionmaking (MCDM) methodology is developed by Wang, Huang, and Dismukes (2004) to select the best set of multiple suppliers to satisfy capacity constraint.

Determining precisely the goal value of each objective is difficult for decision makers since possibly only partial information can be obtained (Chen & Tsai, 2001). Some approaches, such as probability distribution, penalty function fuzzy numbers and various types of thresholds, are used to reformulate the GP models in order to incorporate uncertainty and imprecision into the formulation (Chen & Tsai, 2001). Narasimhan (1980) was the first to propose fuzzy goal programming (FGP) by using the fuzzy set theory with preference-based membership function to GP. Since then, many achievements have been made in areas of preemptive FGP, weight additive model and stochastic model (Chang, 2007). Some researchers have investigated FGP regarding the problem formulation, the relative importance and the fuzzy priority of the fuzzy goals, and associated solution algorithms (Chen & Tsai, 2001). A review of the past researches on FGP is done by Chen and Tsai (2001) and Chang (2007).

Kim and Whang (1998) investigated the application of tolerance concepts to goal programming in a fuzzy environment by formulating a FGP problem with unequal weights as a single linear programming problem with the concept of tolerance. The model could reflect the decision maker's view on subjective fuzzy business goals based on his/her experience or intuition. Chen and Tsai (2001) formulated FGP by "incorporating different importance and preemptive priorities by using an additive model to maximize the sum of achievement degrees of all fuzzy goals." The approach allowed the decision maker to determine a desirable achievement degree for each fuzzy goal and to reflect explicitly the relative importance of these goals. Kumar et al. (2004) presented a fuzzy goal programming approach that considered multiple objectives and dealt with some of the parameters that were fuzzy in nature. A fuzzy mixed integer goal programming was formulated. Three primary goals are minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries, while the constraints are regarding buyer's demand, vendors' capacity, vendors' quota flexibility, purchase value of items, budget allocation to individual vendor, etc.

Chang (2007) proposed an MCGP approach to solve a multichoice aspiration level (MCAL) problem, in which decision makers can set more aspiration levels to each goal of the multiple objective decision-making problem to find more appropriate resources so as to reach a higher aspiration level in the initial stage of the solution process. The approach is applicable when there is a goal that can be achieved from some specific aspiration levels (i.e., one goal mapping many aspiration levels) (Chang, 2007).

The achievement function of MCGP is (Chang, 2007)

Min
$$\sum_{i=1}^{n} w_i (d_i^+ + d_i^-)$$
 (10)

s.t.
$$f_i(X) - d_i^+ + d_i^- = \sum_{j=1}^m g_{ij} S_{ij}(B), \quad i = 1, 2, \dots, n$$
 (11)

$$d_i^+, d_i^- \geqslant 0, \quad i = 1, 2, \dots, n$$
 (12)

$$S_{ii}(B) \in U_i(x), \quad i = 1, 2, ..., n$$
 (13)

$$X \in F(F \text{ is a feasible set})$$
 (14)

where d_i is the deviation from the target value g_i ; w_i represents the weight attached to the deviation; $d_i^+ = \max(0, f_i(X) - g_i)$ and $d_i^- = \max(0, g_i - f_i(X))$ are, respectively, over- and under-achievements of the ith goal; $S_{ij}(B)$ represents a function of binary serial number; and $U_i(x)$ is the function of resources limitations.

For something that is more/higher the better in the aspiration levels, the highest possible value of membership function is 1, based on the fuzzy theory (Charnes & Cooper, 1961). To achieve the maximization of g_{ij} $S_{ij}(B)$, the flexible membership function goal with aspiration level 1 (i.e., the highest possible value of membership function) is used as follows (Chang, 2007):

$$\frac{g_{ij}S_{ij}(B) - g_{\min}}{g_{\max} - g_{\min}} - d_i^+ + d_i^- = 1, \tag{15}$$

where g_{max} and g_{min} are, respectively, the upper and lower bound of the right-hand side (i.e., aspiration levels) of Eq. (11).

For easy calculation, the fractional form of Eq. (15) is

$$\frac{1}{L_i}g_{ij}S_{ij}(B) - \frac{1}{L_i}g_{\min} - d_i^+ + d_i^- = 1, \tag{16}$$

where $L_i = g_{\text{max}} - g_{\text{min}}$

For something that is less/lower the better in the aspiration levels, the similar idea of maximization of g_{ij} $S_{ij}(B)$ can be used to achieve the minimization of g_{ij} $S_{ij}(B)$. The flexible membership function goal with the aspiration level 1 (i.e., the lowest possible value of membership function) is as follows (Chang, 2007):

$$\frac{g_{\max} - g_{ij}S_{ij}(B)}{g_{\max} - g_{\min}} - d_i^+ + d_i^- = 1, \tag{17}$$

where g_{max} and g_{min} are, respectively, the upper and lower bound of the right-hand side (i.e., aspiration levels) of Eq. (11).

The fractional form of Eq. (17) can also be converted into a polynomial form

$$\frac{1}{L_i}g_{\max} - \frac{1}{L_i}g_{ij}S_{ij}(B) - d_i^+ + d_i^- = 1.$$
 (18)

5. Formulation of fuzzy multi-choice goal programming for TFT-LCD manufacturer selection

In this section, a MCGP model with the incorporation of FAHP is proposed for the selection of TFT-LCD manufacturers by a notebook (NB) manufacturer. The steps are summarized as follows:

- Step 1. Form a committee of experts in NB industry and define the TFT-LCD supplier selection problem. The selection of suitable TFT-LCD manufacturers for a NB company to purchase TFT-LCD modules is essential for the NB company to be successful. With a comprehensive review of the literature, consultation with domain experts and consideration of data accessibility, the factors for determining the performance of TFT-LCD companies can be organized.
- Step 2. Formulate a questionnaire to compare factors pairwisely in their contribution toward achieving the goal of selecting the best TFT-LCD supplier. With a fuzzy number, $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$, to represent the pairwise comparison value on the overall objective, and the opinions of experts are collected and combined into a fuzzy pairwise comparison matrix \tilde{A} , where the triangular fuzzy numbers $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ are defined as in Table 1.

$$\widetilde{A} = [\widetilde{a}_{ij}] = \begin{bmatrix}
\widetilde{a}_{11} & \widetilde{a}_{12} & \cdots & \widetilde{a}_{1n} \\
\widetilde{a}_{21} & \widetilde{a}_{22} & \cdots & \widetilde{a}_{2n} \\
\vdots & \vdots & \cdots & \vdots \\
\widetilde{a}_{n1} & \widetilde{a}_{n2} & \cdots & \widetilde{a}_{nn}
\end{bmatrix} \\
= \begin{bmatrix}
1 & \widetilde{a}_{12} & \cdots & \widetilde{a}_{1n} \\
\widetilde{a}_{21} & 1 & \cdots & \widetilde{a}_{2n} \\
\vdots & \vdots & 1 & \vdots \\
\widetilde{a}_{n1} & \widetilde{a}_{n2} & \cdots & 1
\end{bmatrix}, \text{ for } i, j = 1, 2, \dots n$$

where $\tilde{a}_{ij} = (x^-, x, x^+)$ and $a_{ij} \cdot a_{ji} \approx 1$.

Table 1Characteristic function of the fuzzy numbers from Lee et al. (2005), with kind permission of Springer Science + Business Media

Fuzzy number	Characteristic (membership) function
Ĩ	(1,1,3)
$ ilde{x}$	(x-2, x, x+2) for $x=3, 5, 7$
$ ilde{f 9}$	(7,9,9)

Step 3. Combine fuzzy matrices of experts into an integrated fuzzy matrix and check its consistency.

matrix and check its consistency.
$$\widetilde{B} = [\widetilde{b}_{ij}] = \begin{bmatrix}
\widetilde{b}_{11} & \widetilde{b}_{12} & \cdots & \widetilde{b}_{1n} \\
\widetilde{b}_{21} & \widetilde{b}_{22} & \cdots & \widetilde{b}_{2n} \\
\vdots & \vdots & \cdots & \vdots \\
\widetilde{b}_{n1} & \widetilde{b}_{n2} & \cdots & \widetilde{b}_{nn}
\end{bmatrix} \\
= \begin{bmatrix}
1 & \widetilde{b}_{12} & \cdots & \widetilde{b}_{1n} \\
\widetilde{b}_{21} & 1 & \cdots & \widetilde{b}_{2n} \\
\vdots & \vdots & 1 & \vdots \\
\widetilde{b}_{n1} & \widetilde{b}_{n2} & \cdots & 1
\end{bmatrix}, \text{ for } i, j = 1, 2, \dots n$$

where $\tilde{b}_{ij} = [\tilde{a}^1_{ij} \otimes \cdots \otimes \tilde{a}^k_{ij}]^{1/k} = (m^-_{ij}, m_{ij}, m^+_{ij})$ and $b_{ij} \cdot b_{ji} \approx 1$, m^-_{ij} is the geometric average of the smallest assigned value among the experts, m^+_{ij} is the geometric average of the largest assigned value among the experts, and m^-_{ij} is the geometric average of the middle values among the experts.Based on Buckley (1985) and Csutora and Buckley (2001), let $B = [b_{ij}]$ be a positive reciprocal matrix, and $\widetilde{B} = [\widetilde{b}_{ij}]$ be a fuzzy positive reciprocal matrix, if B is consistent, then \widetilde{B} is also consistent. If \widetilde{B} is not consistent, the questionnaire must be modified by the experts.

- Step 4. By adopting the extent analysis method (EAM) proposed by Chang (1996), calculate crisp relative importance weights (priority vectors) for factors. By using Eqs. (2)–(9), we can get the weights, w_i , of factors. The normalized weights of the factors are w_i , $i = 1, 2 \dots n$.
- Step 5. Set the GP model for the supplier selection. The objective is to maximize the satisfaction of the suppliers. The goals are G_1 , G_2 ,..., G_i ,..., G_n , and w_i from Step 4 are the weights for G_i

$$\text{Max } Z_0 = w_1 \times G_1 + w_2 \times G_2 + \dots + w_n \times G_n$$
 (21)

Step 6. Set the MCGP model by adopting Eqs. 10,11,12,13,14,15, 16,17,18. An example is as follows:

Min
$$Z = \sum_{i=1}^{n} \frac{w_i}{L_i} (d_{i1}^+ + d_{i1}^- + L_i (d_{i2}^+ + d_{i2}^-))$$
 (22)

s.t.
$$f_i(X) - d_i^+ + d_i^- = \sum_{i=1}^m g_{ij} S_{ij}(B), \quad i = 1, 2, \dots, n$$
 (23)

$$f_{i}(X) - d_{i1}^{+} + d_{i1}^{-} = g_{i}^{\max} z_{i} + g_{i}^{\min} (1 - z_{i}),$$

$$i = 1, 2, \dots, n$$
(24)

$$\frac{1}{L_i}(g_i^{max}z_i + g_i^{min}(1-z_i)) - d_{i2}^+ + d_{i2}^-$$

$$= \frac{1}{L_{i}} (g_{i}^{max} \text{ or } g_{i}^{min}), \quad i = 1, 2, \dots, n \tag{25} \label{eq:25}$$

$$d_i^+, d_i^-, d_{i1}^+, d_{i1}^-, d_{i2}^+, d_{i2}^- \geqslant 0, \quad i = 1, 2, \dots, n$$
 (26)

$$X \in B(B \text{ is a feasible set})$$
 (27)

$$z_i \in \{0, 1\} \tag{28}$$

6. A case study for evaluating TFT-LCD companies

To examine the practicality and the effectiveness of the proposed MCGP model for supplier evaluation, we use an anonymous notebook (NB) manufacturing company in Taiwan in the selection of TFT-LCD company(s) as an example. Depending on the factor used, one TFT-LCD company may perform better than the others. Therefore, experts are interviewed first to decide the factors for selecting suppliers. The procedures and results of the proposed model in the case study are as follows.

Purchasing managers and related experts in the anonymous company are invited to define the TFT-LCD supplier selection problem and to prepare a supplier candidates list. With a comprehensive

(32)

Table 2 The integrated fuzzy matrix

	Unit purchase cost	Yield rate	Number of suppliers	w'	w
Unit purchase cost	(1,1,1)	(1.25,2.14,4.36)	(1.72,3.94,6.01)	1.00	0.50
Yield rate	$(4.36^{-1},2.14^{-1},1.25^{-1})$	(1,1,1)	(1.55,3.68,5.72)	0.84	0.42
Number of supplier	$(6.01^{-1},3.94^{-1},1.72^{-1})$	(5.72 ⁻¹ ,3.68 ⁻¹ ,1.55 ⁻¹)	(1,1,1)	0.18	0.09

review of the literature, consultation with domain experts and consideration of data accessibility, the major factors for selecting TFT-LCD companies are unit purchase cost (C), yield rate and number of suppliers. A questionnaire is prepared for the decision makers to compare factors pairwisely in their contribution toward achieving the goal of selecting the best TFT-LCD supplier. The integrated fuzzy matrix is calculated by Eq. (20) and is shown in Table 2. The consistency of the integrated fuzzy matrix is examined.

Then, by applying Eq. (3), we have

$$F_1 = (0.19, 0.51, 1.41),$$

$$F_2 = (0.13, 0.37, 0.93),$$

$$F_3 = (0.06, 0.11, 0.28).$$

Finally, by using Eqs. (6) and (7), we obtain

$$d(F_1) = \min(1, 1) = 1,$$

$$d(F_2) = \min(0.84, 1) = 0.84,$$

$$d(F_3) = \min(0.18, 0.35) = 0.18.$$

Thus, the importance weights (w') for unit purchase cost, yield rate and number of suppliers are 1, 0.84 and 0.18, respectively. As shown in Table 2, the normalized weights (w) are 0.50, 0.42 and 0.09, respectively.

Five potential TFT-LCD companies in Taiwan are selected for evaluation. Because the NB company is located in Taiwan, which has many well-known and larger-scale TFT-LCD manufacturers, it is in the best interest of the NB company to simply select the suppliers in Taiwan for cooperation. The NB company needs to purchase 27,000 units of TFT-LCD modules, and the on-time delivery quantity must be at least 24,000 units. The budget ranges from \$3,500,000 to \$4,900,000. The supplier profiles shown in Table 3 represent the data set for the unit purchase cost (C_i in \$), product yield rate, capacity constraint, budget allocation and on-time delivery rates.

The GP model for the supplier selection is set next. The objective is to maximize the satisfaction of the suppliers, and the goals are purchase $cost(G_1)$, product yield rate (G_2) , and number of suppliers (G_3) .

Max
$$Z_0 = [G_1, G_2, G_3]$$

where

 G_1 is the Min $[\sum_{q=1}^5 C_q X_q] \leqslant g_1^{\max} \times z_1 + g_1^{\min} \times (1-z_1)$, and is to minimize the total purchase cost;

 G_2 is the Max $[\sum_{q=1}^5 Y_q X_q] \geqslant g_2^{max} \times Z_2 + g_2^{min} \times (1-Z_2)$, and is to maximize the product yield rate; and

 G_3 is the $\sum_{q=1}^5 R_q = g_{31} \times z_3 \times z_4 + g_{32} \times z_3 \times (1-z_4) + g_{33} \times (1-z_3) \times z_4$, and is to set the number of suppliers to a desired number(s).

 C_q is the purchase cost from supplier q, Y_q is the product yield rate from supplier q, and R_q is a binary number (1 = supplier q is selected for cooperation).

The MCGP model is as follows:

Min
$$Z = 0.50 \times (1/140,000)$$

 $\times (d_{11}^+ + d_{12}^- + 140,000 \times (d_{13}^+ + d_{14}^-))$
 $+ 0.42 \times (1/3000)$
 $\times (d_{21}^+ + d_{22}^- + 3000 \times (d_{23}^+ + d_{24}^-))$
 $+ 0.09 \times (1/2) \times (d_3^+ + d_3^-)$ (29)

s.t.
$$175X_1 + 180X_2 + 176X_3$$

 $+ 172X_4 + 171X_5 - d_{11}^+ + d_{12}^- = 4,900,000$
 $\times z_1 + 3,500,000 \times (1 - z_1)$ (30)

$$(1/140,000) \times (4,900,000 \times z_1 + 3,500,000)$$

$$\times (1 - z_1)) - d_{13}^+ + d_{14}^- = 2.5$$
 (31)

$$0.975X_1 + 0.995X_2 + 0.980X_3 + 0.985X_4$$

$$+0.915X_5 - d_{21}^+ + d_{22}^- = 27,000$$

 $\times z_2 + 24,000 \times (1 - z_2)$

$$(1/3000) \times (27,000 \times z_2 + 24,000)$$

$$\times (1 - z_2)) - d_{23}^+ + d_{24}^- = 9 \tag{33}$$

$$R_1 + R_2 + R_3 + R_4 + R_5 - d_3^+ + d_3^-$$

= $2 \times z_3 \times z_4 + 3 \times z_3 \times (1 - z_4)$

$$+4\times(1-z_3)\times z_4\tag{34}$$

$$X_1 + X_2 + X_3 + X_4 + X_5 \leqslant 27,000 \tag{35}$$

$$X_1 \le 20.000$$
 (36)

$$X_2 \leqslant 25,000$$
 (37)

$$X_3 \leqslant 8000 \tag{38}$$

$$X_4 \leqslant 7000 \tag{39}$$

$$X_5 \leqslant 5000 \tag{40}$$

$$175X_1 \leqslant 3,000,000 \tag{41}$$

$$180X_2 \leqslant 4,000,000 \tag{42}$$

$$176X_3 \leqslant 2,000,000 \tag{43}$$

$$172X_4 \leqslant 1,000,000 \tag{44}$$

$$171X_5 \leqslant 1,000,000 \tag{45}$$

 $0.90X_1 + 0.98X_2 + 0.97X_3 + 0.85X_4$

$$+0.83X_5 \geqslant 24,000 \tag{46}$$

$$X_q = \sum_{\nu=1}^{16} 2^{\nu-1} y_{\nu} \tag{47}$$

$$y_{\nu} \in \{0, 1\}, \nu = 1, 2, \dots, 16$$
 (48)

$$z_p \in \{0,1\}$$
 , $p = 1,2,3,4$ (49)

$$R_q \in \{0,1\}$$
 , $q = 1,2,3,4,5$ (50)

where all variables are nonnegative.

The objective is to minimize Z based on the goals selected and the weights obtained from FAHP. The constraints are explained as follows. Constraint (30) is the total purchase cost of TFT-LCD modules for the NB company, and constraint (31) is to minimize the cost. Constraint (32) calculates the yield rate, and constraint (33) is to maximize the yield rate. Constraint (34) calculates the number of suppliers, and the number of suppliers is 2, 3 or 4. Constraint (35) makes sure that the total demand (D) is met by the summation of the supply of each supplier, X_i . Constraint (36)–(40) states the capacity of each supplier available to the company. Constraint (41)-(45) put restrictions on the budget amount allocated to each supplier for supplying the modules. Constraint (46) makes sure that the amount of on-time delivery must be greater than or equal to a specified amount, and the constants show the ontime delivery rates of suppliers. Constraint (47) makes the purchase quantity from each supplier an integer. Constraint (48) and (49) are respectively to let y_v and z_p be a binary number. Constraint (50) is to let R_q be a binary number, and the value

Table 3Supplier source data of the case study

Supplier number	Unit purchase cost (C_i) (\$)	Product yield rate (%)	Capacity constraint (units)	Budget allocation (\$)	On-time delivery rate (%)
1	175	0.975	20,000	3,000,000	0.90
2	180	0.995	25,000	4,500,000	0.98
3	176	0.980	8,000	2,000,000	0.97
4	172	0.985	7,000	1,000,000	0.85
5	171	0.915	5,000	1,000,000	0.83

Table 4Nature of quota allocation at the optimal solution

Supplier number	Allocation of modules (units)	% of capacity used	% of budget allocated	Ranking by cost	Ranking by yield
1	0	0	0	3	4
2	21,187	85	76	5	1
3	0	0	0	4	3
4	5,813	83	100	2	2
5	0	0	0	1	5

is 1 if the TFT-LCD modules are purchased from supplier i, and vice versa.

The MCGP model is solved using LINGO (1999), and the solutions are Z = 0.4961439, $X_1 = 0$, $X_2 = 21,187$, $X_3 = 0$, $X_4 = 5,813$, and X_5 = 0. Therefore, in the best interest of the NB company, it should purchase 21,187 units from supplier 2 and 5813 units from supplier 4. As shown in Table 4, supplier 4 ranks the first, and receives the full allocation of budget. Supplier 2 ranks the second and receives the rest amount of the orders. Since the capacity assigned to supplier 4 is not used up, more modules can be allocated to the supplier if its budget constraint is elevated. An analysis shows that if the budget constraint of supplier 4 is removed, the bonding constraint becomes the capacity constraint and a total of 7000 modules are assigned to supplier 4. The rest 20,000 modules are assigned to supplier 2, and the objective value is Z = 0.4944143. Since the objective is to minimize Z, the objective value is improved. However, even if the capacity constraint is also removed, not all the modules will be assigned to the supplier. A sensitivity analysis shows that if both the budget and capacity constraints are not considered for all suppliers, the results are: Z = 0.4770408, $X_1 = 0$, $X_2 = 8077$, $X_3 = 0$, $X_4 = 18,923$, and $X_5 = 0$. As a result, 18,923 units are assigned to supplier 4, and 8077 units are assigned to supplier 2.

Note that the optimal solution is not obtained simply based on the cost or yield alone. Even though supplier 5 has the lowest unit cost, it is not selected. In addition, supplier 2 ranks the first in yield, but it is selected after supplier 4, which ranks the second in yield. In consequence, the MCGP model can indeed solve the supplier selection problem by considering various factors and constraints.

7. Conclusions

Supplier selection and evaluation process is very complicated involving interrelationship among two or more organizations in a supply chain, and the process is multi-objective in nature. The selection of one (or several) TFT-LCD manufacturers for subcontracting is essential for a notebook company and any other company that requires TFT-LCD modules. In this research, an MCGP model is proposed to evaluate the performance of TFT-LCD manufacturers and to allocate the purchase amount to the selected companies, while the number of suppliers that should be selected can be set as preferred. Fuzzy AHP is applied first to obtain the weights of the criteria, and an MCGP approach is used to find the optimal solution of module allocation to suppliers. The proposed model is

effective for handling real situations, not only in TFT-LCD industry but also in supgeneral.

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