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Inventory replenishment model using fuzzy multiple objective programming: A case study of a high-tech company in Taiwan

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ABSTRACT

The progress in high technology has led to the wide use of thin film transistor-liquid crystal display (TFT-LCD). The evolution of the manufacturing technology of TFT-LCD keeps increasing the size of TFT-LCD since a larger TFT-LCD allows a larger display application and an improved productivity. However, as the size of TFT-LCD increases, the size of TFT-array substrates and color filter substrates has to increase simultaneously. This leads to a more complicated inventory problem of large-sized substrates. Therefore, this paper considers a color filter replenishment problem in TFT-LCD manufacturing with the consideration of storage space, yield rate, quantity discounts and multiple suppliers. We first formulate the color filter replenishment problem as a fuzzy multiple objective programming, and then a fuzzy multiple objective programming with assigned weights for objectives based on experts' opinions is proposed. An example with four cases is given to illustrate the practicality for empirical investigation. The results demonstrate that both methods are effective tools for inventory management of color filters for multi-periods. In addition, the methods can be applied or modified for managing inventory in general.

1. Introduction

The spread of flat panel displays (FPDs) is inescapable in the digital era and is quickly becoming the preferred choice in many applications of human–machine interface. Because of their low weight, slender profile, low power consumption, high resolution, high brightness and low radiance advantages, the use of FPDs has been expanding from portable appliances to notebooks and desktop monitors and even to large screen digital televisions. Among the industry, thin film transistor-liquid crystal display (TFT-LCD) is the primary FPD technology and represents more than 80% of the FPD market. A statistics data indicated that global TFT-LCD market will increase to US$56 billion in 2009 [16].

In the fabrication of TFT-LCD panels, color filter substrates, one of the most expensive raw materials, are usually purchased from color filter manufacturers, and sufficient amount of them must be available in the plant to maintain a smooth production flow. In addition, the size and the unit cost of color filters increases as the generation of TFT-LCD increases through technology progress, and the storage of these large-sized color filters and the high holding cost become important issues that must be tackled. To summarize, in order to reduce cost and to ensure product availability, the inventory management of color filters is especially important in TFT-LCD manufacturing.

The purpose of this research is to construct a color filter's inventory model with the consideration of storage space, yield rate, quantity discounts and multiple suppliers. The objective of this model is to minimize total cost, maximize yield rate and fix the replenishments to a desired number over the planning horizon. A fuzzy multiple objective programming (FMOP) model is first proposed. Next, we extend the model by considering the opinions of experts on the importance of various objectives by developing a fuzzy multiple objective programming with assigned weights for objectives (FMOP-W). To determine the importance of various objectives, fuzzy analytic hierarchy process (FAHP) and extent analysis method (EAM) is applied to incorporate experts' opinions. The results of the two methods can both satisfy the decision-makers' desirable achievement level of multiple objectives in a fuzzy environment.

The remaining of this paper is organized as follows. Section 2 introduces the manufacturing process of TFT-LCD and reviews some recent related works. In the subsequent section, the problem under consideration and the assumptions are described. The construction of the algorithm is presented next. A numerical example with four cases is presented to examine the practicality of the proposed algorithms. The results of the FMOP and FMOP-W methods are compared for different cases. Some concluding remarks are made in the last section.

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2. Color filter inventory problem and related methodology research

In this section, the manufacturing process of TFT-LCD is introduced first, and some recent research in inventory models and fuzzy multiple objective programming are reviewed. The basic concept of the extent analysis of fuzzy analytic hierarchy process (FAHP) is presented.

2.1. Manufacturing process of TFT-LCD and color filter inventory problem

TFT-LCD has a sandwich-like structure consisting of two glass substrates with a layer of liquid crystal inside. The top substrate is fitted with a color filter that contains the black matrix and resin film containing three primary-color (red, green and blue) dyes or pigments. The bottom substrate is TFT-array that contains the TFTs, storage capacitors, pixel electrodes and interconnect wiring. The two glass substrates are assembled with a sealant, and spacers are used to maintain the gap between the substrates [3]. Liquid crystal material is injected between two substrates. The outer face of each glass substrate has a sheet of polarizer film. Each end of the gate has a set of bonding pads and data-signal bus-lines to attach LCD driver IC (LDI) chips [3].

The manufacturing of TFT-LCD, can be categorized into five major processes: TFT-array fabrication, color filter (BM) fabrication, color filter (RGB) fabrication, cell assembly and module assembly. A TFT-LCD manufacturer usually has different plants for TFT-array fabrication, cell assembly and module assembly. On the other hand, color filters are usually purchased from color filter manufacturers, even though there is a trend for vertical integration between color filter manufacturers and TFT-LCD manufacturers or a certain degree of alliance between the two.

The TFT-LCD industry is becoming extremely competitive and cost-sensitive, and cost control efforts should stress on the increase in the size of the substrates, the decrease in the number of process steps, the simplification of the processes, the decrease in cycle time, the improvement in yields, and especially the reduction of the price of materials [33]. The newer fabs use G7.5 (1950 mm × 2250 mm) and G7 (1870 mm × 2200 mm) glass, and the transition to G8 LCD glass (2160 mm × 2400 mm) started in 2007 [33]. However, substrates of more than 3000 mm will face practical limits due to handling and transportation issues and fab- and operational-related height constraints [33]. The number of panels that can be cut from an assembled substrate is variable, depending on the size of the substrate and the size of the final panel. The size of substrates has increased from 320 mm × 400 mm for the first generation, which was started in the early 1990s, to the current 1500 mm × 1800 mm for the sixth generation and 1870 mm × 2200 mm for the seventh generation [36]. The number of panels that can be cut from a sixth generation substrate can range from 2 to 24 for 54-in. panels to 17-in. panels, respectively. However, to have the most economic efficiency, a G6 substrate should be cut up to twelve pieces of 26-in. panels or six pieces of 37-in. panels to achieve an efficiency rate of 87% and 86%, respectively [36].

The front-end processes, i.e., array fabrication and cell assembly, are highly automated and comprise the major portion, e.g. 90%, of the total investment. Even though the front-end processes have heavy investment on the equipment, the equipment investment of TFT-LCD manufacturers only accounts for 15% of the total manufacturing cost [15]. Raw materials, on the other hand, account for as high as 79% of the total manufacturing cost [13]. Therefore, the focuses of TFT-LCD manufacturers, in addition to the good utilization of processes, should be the reduction in the cost of materials, the increase in the utilization rate of materials and the reduction in the failure rate of processes [15]. In the cell plant, a monthly (medium-term) production plan based on the aggregate demand forecast provided by the sales department is developed first, and is disaggregated into daily (short-term) production plans [43]. Because a TFT-array substrate and a color filter substrate must be paired into the assembly station, adequate amount of color filters must be stored in the cell plant to avoid any shortages and the stoppage of production flow.

Color filter, among all materials, has a very critical role in TFT-LCD manufacturing. First, the cost of color filters is 25% of raw material cost or 16% of total manufacturing cost, exceeding that for all other materials except backlight units. For other major materials, the costs of backlight units and PCBs are respectively 27% and 14% of raw material cost [13]. Second, transportation and handling risk is high for newer generation color filters due to the continuous increases in the size of substrates. Third, with larger sizes of color filter substrates, a larger storage space is required in the cell plant, so is the capital cost for acquiring them. Fourth, because of the frequent upgrade in generations and the progress in manufacturing technology, the price of color filters of a same generation has a decreasing trend, and color filters of older generations may face an obsolete problem. Fifth, as stated before, a TFT-array substrate must be matched with a color filter substrate, and sufficient amount of color filters must be in the plant in order to fully utilize the capital-intensive equipment. Time for the acquisition of color filters is usually 7–10 days and can be even longer, depending on the location of suppliers and the priority level of orders, such as hot, rush or normal. Acquisition time must be considered when implementing the production plan to avoid equipment stoppage and unnecessary higher purchasing cost. To summarize, in order to achieve cost reduction and ensure product availability, the inventory management of color filters is especially important in TFT manufacturing.

Inventory management has always been a hot topic, and tremendous amount of mathematical models have been developed, including, and not limited to, linear programming, non-linear programming, dynamic programming, geometric programming, gradient-based non-linear programming and fuzzy geometric programming. Mixed integer programming is also a popular method, and some recent works are Tarim and Kingsman [37], Wang and Sarker [39], Wang and Sarker [40], Tarim and Kingsman [38]. Chang et al. [7], considering variable lead-time, price-quantity discount and resource constraints, developed a mixed integer approach for solving a single item multi-supplier problem. Kang [17] proposed a dynamic programming model and a mixed 0-1 linear programming model to solve a control wafers replenishment problem with inventory deterioration. da Silva et al. [10] used a multiple-criteria mixed-integer linear programming model to solve an aggregate production planning problem.

Quantity discounts and storage spaces are important issues that may need to be considered in inventory management. With quantity discounts, the purchase price from the suppliers is reduced if a large order is placed. The two major types of quantity discounts are all-units discount and incremental discount [5]. In the all-units discount, the discounted price is applied to all units beginning with the first unit, if the quantity purchased belongs to a specified quantity level predetermined by the supplier. With a number of price breaks, the unit discounted price decreases as the quantity level increases. In the incremental discount, the discounted price is only applied to those units inside the price break quantity; thus, different prices are applied to the units belonging to different price breaks. Constrained storage space is considered by Kanyalkar and Adil [18] and Mandal et al. [31] in their inventory models.

Because the size of color filters is relatively large and storage capacity in a TFT-LCD plant is limited, only a limited quantity of color filters can be stored in the plant. With the special characteristics of color filters, we will formulate the color filter inventory replenishment problem, with the consideration of yield rate, quantity discounts, storage space and multiple suppliers.

2.2. Fuzzy multiple objective programming (FMOP)

Multiple objective programming (MOP) is one of the popular methods for decision making in a complex environment. In an MOP, decision-makers try to optimize two or more objectives simultaneously under various constraints. A complete optimal solution seldom exists, and a Pareto-optimal solution is usually used [42]. There are a few methods, such as the weighting methods, which assign priorities to the objectives and set aspiration levels for the objectives, are used to derive a compromise solution [35]. When formulating a MOP, various factors need to be included in the description of the objective functions and the constraints; however, they are often imprecisely or ambiguously known. Therefore, fuzzy multiple objective programming (FMOP) may be more appropriate since the parameters can be represented by fuzzy numbers.

Different kinds of FMOP models have been proposed to solve different decision-making problems that involve fuzzy values in objective function parameters, constraints parameters, or objectives [44]. Another research direction is the transformation of a FMOP problem into a crisp programming [23]. Some most recent research related to FMOP is reviewed here. Karsak and Kuzgunkaya [20] proposed a FMOP approach to facilitate the selection of flexible manufacturing system (FMS), and the objectives are assigned weights indicating their importance using linguistic variables. Karsak [19] further used FMOP in the quality function deployment (QFD) planning process to prioritize design requirements. Kumar et al. [21] formulated a vendor selection problem using fuzzy mixed integer goal programming (GP), in which some of the parameters are fuzzy in nature, and the three primary goals are minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries. Kumar et al. [22] developed another fuzzy multi-objective integer programming approach for vendor selection problem in a supply chain, and the three goals are cost-minimization, quality-maximization and maximization of on-time-delivery. Amid et al. [1] also solved a supplier selection problem in a supply chain by establishing a fuzzy multi-objective linear model by applying an asymmetric fuzzy-decision-making technique. Lam and Tang [24] proposed a multi-objective linear programming model that took into account the cost objective and other important criteria in a multiechelon supply chain ranging from the upstream suppliers' quality to end customers' satisfaction level and considered multiple time-phased demands. El-Wahed and Lee [11] presented an interactive FGP approach to determine the preferred compromise solution for the multi-objective transportation problem. Fan et al. [12] proposed a GP approach to solve group decision-making problems based on multiplicative preference relations and fuzzy preference relations. Li et al. [29] presented a two-phase max–min fuzzy compromise approach to solve fuzzy multiple objective linear programming (FMOLP) problems by automatically computing proper membership thresholds. Wu et al. [44] developed a new approximate algorithm for solving FMOLP problems involving fuzzy parameters in any form of membership functions in both objective functions and constraints. Mishra et al. [32] proposed a fuzzy goal-programming model with multiple objectives and constraints to solve the machine–tool selection and operation allocation problem. Pramanik and Roy [34] presented a procedure for solving multilevel programming problems in a large hierarchical decentralized organization through FGP, which achieved the highest degree of each membership goal by minimizing negative deviational variables. Yaghoobi and Tamiz [45] constructed a model based on MINMAX approach for solving FGP problems. Hu et al. [14] proposed a generalized varying-domain optimization method for FGP approach to solve multi-objective optimization problem with multiple priorities, so that a higher priority expected by decision maker can achieve a higher satisfactory degree. Chang [6] presented a method to program binary FGP model so that it can be solved using the integer programming method. Araz et al. [2] developed an outsourcer evaluation and management system with two phases. PROMETHEE is used in the first phase to evaluate the suppliers, and fuzzy goal programming with multiple objectives is applied in the second phase to select the most appropriate outsourcers and simultaneously allocate the ordered quantities to these outsourcers. Wang and Yang [41] applied AHP and fuzzy compromise programming to obtain a compromise solution for allocating order quantities to each supplier with quantity discounts. Wee et al. [42] proposed an inverse weight fuzzy non-linear programming to formulate a multi-objective joint replenishment inventory problem with deteriorating items. Even though there are works of FMOP on inventory replenishment problems [1,2,21,22,24], none has tackled the problem by considering storage space, yield rate, quantity discounts and multiple suppliers simultaneously.

In solving a fuzzy multiple objective programming model, linear membership functions are usually considered for fuzzy parameters. A linear membership function has a continuously increasing or decreasing value over the range of parameter, and the lower and upper acceptable values of parameter are defined. Fuzzy objectives for minimization, maximization and target are, respectively:

$$\mu_{G_1}(f_{G_1}(x)) = \begin{cases} 1, & \text{if } f_{G_1}(x) \le L_{G_1} \\ \dfrac{U_{G_1} - f_{G_1}(x)}{U_{G_1} - L_{G_1}}, & \text{if } L_{G_1} \le f_{G_1}(x) \le U_{G_1}, \quad \text{for minimization objective} \\ 0, & \text{if } f_{G_1}(x) \ge U_{G_1} \end{cases} \tag{1}$$

$$\mu_{G_2}(f_{G_2}(x)) = \begin{cases} 1, & \text{if } f_{G_2}(x) \ge U_{G_2} \\ \dfrac{f_{G_2}(x) - L_{G_2}}{U_{G_2} - L_{G_2}}, & \text{if } L_{G_2} \le f_{G_2}(x) \le U_{G_2}, \quad \text{for maximization objective} \\ 0, & \text{if } f_{G_2}(x) \le L_{G_2} \end{cases} \tag{2}$$

$$\mu_{G_3}(f_{G_3}(x)) = \begin{cases} 0, & \text{if } f_{G_3}(x) \le L_{G_3} \\ \dfrac{f_{G_3}(x) - L_{G_3}}{T_{G_3} - L_{G_3}}, & \text{if } L_{G_3} \le f_{G_3}(x) \le T_{G_3} \\ 1, & \text{if } f_{G_3}(x) = T_{G_3}, \quad \text{for target objective} \\ \dfrac{U_{G_3} - f_{G_3}(x)}{U_{G_3} - T_{G_3}}, & \text{if } T_{G_3} \le f_{G_3}(x) \le U_{G_3} \\ 0, & \text{if } f_{G_3}(x) \ge U_{G_3} \end{cases} \tag{3}$$

where U_G, L_G and T_G are, respectively, the upper bound, lower bound and target of the fuzzy objective.

The fuzzy solution for fuzzy multiple objectives can be given as:

$$\mu_D(x) = \left\{ \bigcap_{for\ all\ g} \mu_{G_g}(x) \right\} \tag{4}$$

where $\mu_D(x)$ and $\mu_{G_g}(x)$ represent the membership functions of solution and objective functions.

The optimal solution (x^*) is:

$$\mu_D(x^*) = \max \mu_D(x) = \max \left[\min_{for\ all\ g} \mu_{G_g}(x) \right] \tag{5}$$

In finding the optimal solution (x^*) in the above fuzzy model, it is equivalent to solving the following crisp model [47]:

$$\text{Maximize } \lambda \tag{6}$$

s.t.

$$\lambda \le \mu_G(x) \tag{7}$$

$$g(x) \le b \tag{8}$$

$$x \ge 0 \tag{9}$$

$$\lambda \in [0, 1] \tag{10}$$

where λ, $\mu_G(x)$, $g(x)$ and b represent the membership function, objective functions, constraints and right-hand sides.

2.3. Fuzzy analytic hierarchy process (FAHP)

Analytic hierarchy process (AHP) is a well-known mathematically based multiple-criteria decision-making (MCDM) tool. While fuzziness and vagueness are common characteristics in many decision-making problems, fuzzy set theory has been incorporated in the AHP, so-called fuzzy AHP (FAHP), to tolerate vagueness or ambiguity in the decision process.

The FAHP is used in the FMOP-W method to determine the weights of different objectives in this research. The extent analysis method (EAM), proposed first by Chang [8], is applied. A brief review of the method is given here. Two triangular fuzzy number $M_1(m_1^-, m_1, m_1^+)$ and $M_2(m_2^-, m_2, m_2^+)$ shown in Fig. 1 are compared. When $m_1^- \ge m_2^-, m_1 \ge m_2, m_1^+ \ge m_2^+$, the degree of possibility is defined as $V(M_1 \ge M_2) = 1$. Otherwise, the ordinate of the highest intersection point can be calculated [8,27,28,46]:

$$V(M_2 \ge M_1) = hgt(M_1 \cap M_2) = \mu(d) = \frac{m_1^- - m_2^+}{(m_2 - m_2^+) - (m_1 - m_1^-)} \tag{11}$$

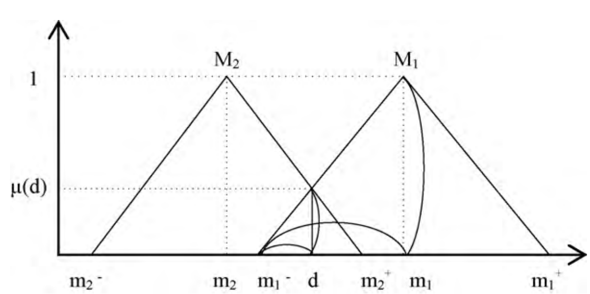

Fig. 1. Two triangular fuzzy numbers M_1 and M_2 (Lee [27]).

The value of fuzzy synthetic extent with respect to factor l is:

$$F_l = \sum_{j=1}^{g} M_{lj} \otimes \left[\sum_{l=1}^{g}\sum_{j=1}^{g} M_{lj} \right]^{-1}, \quad l = 1, 2, \ldots, g \tag{12}$$

$$\sum_{j=1}^{g} M_{lj} = \left(\sum_{j=1}^{g} m_{lj}^-, \ \sum_{j=1}^{g} m_{lj}, \ \sum_{j=1}^{g} m_{lj}^+ \right), \quad l = 1, 2, \ldots, g \tag{13}$$

$$\left[\sum_{l=1}^{g}\sum_{j=1}^{g} M_{lj} \right]^{-1} = \left(\frac{1}{\sum_{l=1}^{g}\sum_{j=1}^{g} m_{lj}^+}, \frac{1}{\sum_{l=1}^{g}\sum_{j=1}^{g} m_{lj}}, \frac{1}{\sum_{l=1}^{g}\sum_{j=1}^{g} m_{lj}^-} \right) \tag{14}$$

A convex fuzzy number is defined by:

$$V\left(F \ge F_1, F_2, \ldots, F_g\right) = \min V\left(F \ge F_j\right), \quad j = 1, 2, \ldots, g \tag{15}$$

$$d(F_l) = \min V\left(F_l \ge F_j\right) = w_l',$$
$$l = 1, 2, \ldots, g, \ \ j = 1, 2, \ldots, g \text{ and } l \ne j \tag{16}$$

The weights, w_l', of factors are:

$$W' = \left(w_1', w_2', \ldots, w_g'\right)^T \tag{17}$$

After normalization, the priority weights are:

$$W = \left(w_1, w_2, \ldots, w_g\right)^T \tag{18}$$

3. Problem description and assumptions

Some of the notations and assumptions used in this paper are similar to that of Kang [17] and Lee and Kang [26]. In order to simplify the complexity of the environment, we shall restrict the investigation with the following assumptions:

- Because the cell plant is make-to-stock (MTS), the demand rate for color filters is reasonably constant in a period. However, it can be different in different periods.
- Each period can only place at most one order from each supplier.
- The replenishment lead-time is of known duration, and the entire order quantity is delivered at the same time in the beginning of a period.
- Shortages are not allowed.
- The price of each unit is dependent on the order quantity. All-units discount schedule is considered.
- Storage space is limited.
- The inventory holding cost for each unit is known and constant, independent of the price of each unit.
- Planning horizon is finite and known. In the planning horizon, there are T periods, and the duration of each period is the same.
- The initial inventory level (X_1) is zero.
- The order lead-time is zero.
- All the required notations in this paper are defined as below.

Notations

Indices

i	supplier ($i = 1, 2, \ldots, I$)
k	price break ($k = 1, 2, \ldots, K$)
t	planning period ($t = 1, 2, \ldots, T$)
v	integer number for calculating purchase quantity ($v = 1, 2, \ldots, V$)

Parameters

d_t	demand in period t
e_i	yield rate of supplier i

r_1 minimum yield rate of color filters in a planning horizon
r_2 maximum yield rate of color filters in a planning horizon
c_1 minimum total cost of color filters in a planning horizon
c_2 maximum total cost of color filters in a planning horizon
n_1 minimum number of replenishments in a planning horizon
n_2 maximum number of replenishments in a planning horizon
n_T the target number of replenishments in a planning horizon
h inventory holding cost, per unit per period
M a large number
o_i ordering cost per replenishment from supplier i
p_{ik} unit purchase cost from supplier i with price break k
q_{ik} the upper bound quantity of supplier i with price break k
s storage space (volume) available at the plant

Decision variables
λ the degree of membership function, i.e., the overall satisfactory level of compromise
λ_c the degree of satisfaction for cost
λ_r the degree of satisfaction for yield rate
λ_n the degree of satisfaction for number of replenishments
w_c the normalized importance weight for cost
w_r the normalized importance weight for yield rate
w_n the normalized importance weight for number of replenishments
C total cost of color filters in a planning horizon
N number of replenishments in a planning horizon
R average yield rate of color filters in a planning horizon
$P(Q_{it})$ purchase cost for one unit based on the discount schedule of supplier i with order quantity Q_{it} in period t
Q_{it} purchase quantity from supplier i in period t
X_t beginning inventory level in period t
U_{itk} a binary variable, set equal to 1 if color filters are purchased from supplier i with price break k in period t, and 0 if no purchase is made from supplier i with price break k in period t
Z_{it} a binary variable, set equal to 1 if color filters are purchased from supplier i in period t, and 0 if no purchase is made from supplier i in period t
Y_t beginning usable inventory level in period t, and $Y_t = X_t + \sum_{i=1}^{I} Z_{it} \times Q_{it}$
β_{itv} a binary variable for calculating purchase quantity from supplier i in period t

Fig. 2 is the graphical representation of multi-period inventory system for color filters. The beginning inventory level in period $t+1$ (X_{t+1}) is equal to the beginning inventory level in period t (X_t) plus the purchase amount from supplier i in period t ($\sum_{i=1}^{I} Z_{it} \times Q_{it}$) and minus the demand in period t (D_t), where Z_{it} represents whether color filters are purchased from supplier i in period t (1 if a purchase is made, and 0 if no purchase is made). The beginning usable inventory level in period $t+1$ (Y_{t+1}) is equal to the beginning inventory level in period $t+1$ (X_{t+1}) plus the purchase amount in period $t+1$ ($\sum_{i=1}^{I} Z_{it+1} \times Q_{it+1}$).

The total cost of color filters includes ordering cost, holding cost and purchase cost in a planning horizon, and it is:

$$\text{Total cost} = \text{Total ordering cost} + \text{Total holding cost} + \text{Total purchase cost} \tag{19}$$

Eq. (20) calculates the total ordering cost for the system, where o_i is the ordering cost per time from supplier i and Z_{it} represents whether color filters are purchased from supplier i in period t (1 if a purchase is made, and 0 if no purchase is made).

$$\text{Total ordering cost} = \sum_{t=1}^{T} \sum_{i=1}^{I} o_i \times Z_{it} \tag{20}$$

The beginning inventory in a period is equal to the beginning inventory level in the previous period plus the purchase quantity in the previous period minus the demand in the previous period as shown in Eq. (21). Thus, the holding cost in period t is equal to the holding cost per unit times the average inventory in period t. The average inventory in period t is the sum of the beginning usable inventory in period t (the beginning inventory for period t, X_t, plus the purchase quantity in period t) and the ending inventory in period t (the beginning inventory level in period $t+1$), and divide by 2. The total holding cost for a planning horizon is the summation of the holding cost for each period, as in Eq. (22).

$$\begin{aligned}\text{Beginning inventory in this period} &= \text{Beginning inventory in the previous period} \\ &+ \text{Purchase quantity in the previous period} \\ &- \text{Demand in the previous period}\end{aligned} \tag{21}$$

$$\begin{aligned}\text{Total holding cost} &= \sum_{t=1}^{T} \frac{h}{2}(Y_t + X_{t+1}) \\ &= \sum_{t=1}^{T} \frac{h}{2}\left(X_t + \sum_{i=1}^{I} Z_{it} \times Q_{it} + X_{t+1}\right)\end{aligned} \tag{22}$$

The total purchase cost is obtained by Eq. (23), where $P(Q_{it})$ is the unit purchase cost based on the discount schedule with the order quantity Q_{it}, and Z_{it} represents whether color filters are purchased from supplier i in period t (1 if a purchase is made, and 0 if no purchase is made).

$$\text{Total purchase cost} = \sum_{t=1}^{T} \sum_{i=1}^{I} (P(Q_{it}) \times Q_{it} \times Z_{it}) \tag{23}$$

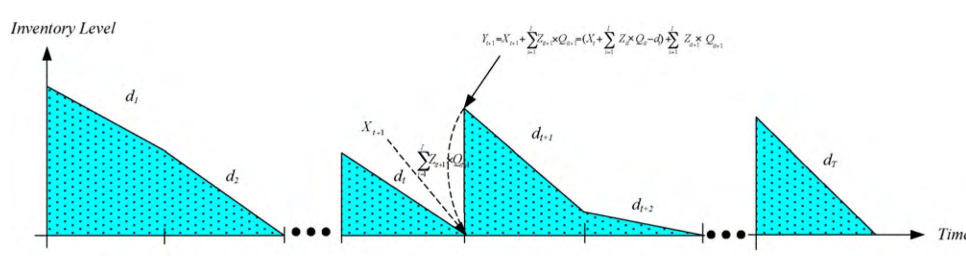

Fig. 2. Graphical representation of color filter inventory system.

In order to incorporate all-units quantity discount into the model, the following four equations are required:

$$minP(Q_{it}) \tag{24}$$

$$\text{s.t.} \quad P(Q_{it}) = \sum\nolimits_{k=1}^{K} p_{ik} \times U_{itk}, \quad i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \tag{25}$$

$$q_{ik-1} + M \times (U_{itk} - 1) \leq Q_{it} < q_{ik} + M \times (1 - U_{itk}),$$
$$i = 1, 2, \ldots, I, \;\; t = 1, 2, \ldots, T \text{ and } k = 1, 2, \ldots, K \tag{26}$$

$$\sum\nolimits_{k=1}^{K} U_{itk} = 1, \quad i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \tag{27}$$

where U_{itk} is a binary variable, M is a big positive value, and q_{ik} is the kth discount level from supplier i.

The objective function (24) is to minimize the purchase cost per unit from supplier i in period t. Eq. (25) determines the purchase cost per unit, $P(Q_{it})$, under the discount schedule based on the quantity purchased from supplier i in period t. Eq. (26) sets the purchase quantity between a lower bound quantity q_{ik-1} and an upper bound quantity q_{ik} in a price break k for supplier i, where M is a large number. Eq. (27) makes sure that color filters can only be purchased with one single price break k from supplier i in period t.

In addition, the products purchased from supplier i in period t must be an integer. Eq. (28) sets the purchase quantity to be an integer, and Eq. (29) is a binary variable for calculating the purchase quantity from supplier i in period t.

$$Q_{it} = \sum_{v=1}^{V} 2^{v-1} \beta_{itv}, \quad i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \tag{28}$$

$$\beta_{itv} \in \left\{0, 1\right\}, \quad v = 1, 2, \ldots, V \tag{29}$$

4. Formulation of the color filter inventory problem

Color filter has a very critical role in TFT-LCD manufacturing due to its high cost, large size for storage and prohibited shortage, etc. A good inventory management thus is necessary. The overall objective of the model is to maximize the satisfaction of the color filter replenishment management, and the multiple objectives are minimizing total cost (C), maximizing yield rate (R), and setting suitable number of replenishments (N). Two models are proposed to maximize degree membership function of color filters in the system and to determine an appropriate inventory level of color filter for each period in a planning horizon.

The first model is a fuzzy multiple objective programming (FMOP) model, and the second one is a fuzzy multiple objective programming with assigned weights for different objectives (FMOP-W). The major difference between the two models is the weights of the objectives. For FMOP, there is no weight setting for the objectives, and the aim is to maximize or minimize each objective. For FMOP-W, the importance of each objective is first determined by the experts, and the aim is to maximize the overall satisfaction of the weighted objectives.

4.1. Fuzzy multiple objective programming model (FMOP)

In this section, we formulate the color filter inventory problem into a FMOP model to solve the multi-period color filter problem and to determine an appropriate replenishment policy of color filters for each period. In this paper, we assume that a production planner's objective is to determine the optimal purchase amount of color filters in each period while considering the objectives of minimizing total cost, maximizing yield rate and fixing the replenishments to a desired number in a planning horizon. In each period, sufficient color filters must be supplied for use in time, and shortages are not allowed.

The proposed FMOP model is formulated as follows:

$$\text{Maximize } \lambda \tag{30}$$

$$\text{s.t.}\, \lambda \leq \frac{c_2 - C}{c_2 - c_1} \tag{31}$$

$$\lambda \leq \frac{R - r_1}{r_1 - r_2} \tag{32}$$

$$\lambda \leq \frac{N - n_1}{n_T - n_1} \tag{33}$$

$$\lambda \leq \frac{N - n_2}{n_T - n_2} \tag{34}$$

$$C = \sum_{t=1}^{T} \left(\sum_{i=1}^{I} o_i \times Z_{it} + \frac{h}{2} \times \left(X_t + \sum_{i=1}^{I} Z_{it} \times Q_{it} + X_{t+1} \right) + \sum_{i=1}^{I} P(Q_{it}) \times Q_{it} \times Z_{it} \right) \tag{35}$$

$$N = \sum_{t=1}^{T} \sum_{i=1}^{I} Z_{it} \tag{36}$$

$$Y_t = X_t + \sum_{i=1}^{I} Z_{it} \times Q_{it}, \quad \textit{for all } t \tag{37}$$

$$X_{t+1} = Y_t - d_t, \quad \textit{for all } t \tag{38}$$

$$X_t + \sum_{i=1}^{I} Z_{it} \times Q_{it} \leq s, \quad \textit{for all } t \tag{39}$$

$$Q_{it} = \sum_{v=1}^{V} 2^{v_{it}-1} \beta_{itv}, \quad \textit{for all } i, t \tag{40}$$

$$P(Q_{it}) = \sum_{k=1}^{K} p_{ik} \times U_{itk}, \quad \textit{for all } i, t \tag{41}$$

$$q_{ik-1} + M \times (U_{itk} - 1) \leq Q_{it} < q_{tk} + M \times (1 - U_{itk}), \quad \textit{for all } i, t, k \tag{42}$$

$$\sum_{k=1}^{K} U_{itk} = 1, \quad \textit{for all } i, t \tag{43}$$

$$R = \left(\sum_{t=1}^{T} \sum_{i=1}^{I} Z_{it} \times Q_{it} \times e_i \right) / \sum_{t=1}^{T} \sum_{i=1}^{I} Z_{it} \times Q_{it} \tag{44}$$

$$Z_{it} \in \left\{0, 1\right\}, \quad \textit{for all } i, t \tag{45}$$

$$\beta_{itv} \in \left\{0, 1\right\}, \quad \textit{for all } i, t, v \tag{46}$$

$$U_{itk} \in \left\{0, 1\right\}, \quad \textit{for all } i, t, k \tag{47}$$

and all variables are nonnegative. (48)

The objective function (30) is to maximize the satisfaction of inventory management. The operative constraints are as follows.

Table 1
Characteristic function of the fuzzy numbers (Lee et al. [25]).

Fuzzy number	Characteristic (membership) function
$\tilde{1}$	(1, 1, 3)
$\tilde{x}$	$(x-2, x, x+2)$ for $x=3, 5, 7$
$\tilde{9}$	(7,9,9)

Constraint (31) is to minimize the total cost of color filters in a planning horizon, where c_1 is the total cost and c_2 is the maximum total cost in a planning horizon. Constraint (32) is to maximize the yield rate of color filters in a planning horizon, where R_1 is the minimum yield rate and R_2 is the maximum yield rate in a planning horizon. Constraints (33) and (34) are to set the number of replenishments in a planning horizon to a desired number, where n_1 is the minimum replenishment number(s), n_2 is the maximum replenishment number(s), and n_T is the desired replenishment number(s) in a planning horizon. Constraint (35) calculates the total cost, which includes the ordering cost, holding cost and purchase cost in a planning horizon. These costs are explained before in Eqs. (20)–(22). Constraint (36) calculates the number of replenishments in a planning horizon, N, by summing up the number of replenishments over the periods in a planning horizon. In constraint (37), the beginning inventory after the replenishment in period t, Y_t, is equal to the beginning inventory level in the period, X_t, plus the purchase quantity in the period, $\sum_{i=1}^{I} Z_{it} \times Q_{it}$. In constraint (38), the beginning inventory of period $t+1$, X_{t+1}, is equal to the beginning inventory after the replenishment in period t, Y_t, minus the demand in period t, d_t. Constraint (39) ensures that the beginning usable inventory must be less than or equal to the storage space s. Constraint (40) ensures that the purchase quantity is an integer. Constraints (41)–(43), as explained in Eqs. (21)–(23), consider the quantity discount of color filters. Constraint (44) calculates the overall yield rate. Constraint (45) lets color filters be either purchased or not purchased in each period. Constraint (46) is a binary variable for calculating the purchase quantity from supplier i in period t. Constraint (47) is a binary variable for determining the price break k applied to the purchase quantity for supplier i in period t.

4.2. *Fuzzy multiple objective programming-weighted model (FMOP-W)*

In this model, the weights of the objectives must be determined first. By adopting the FAHP and extent analysis method (EAM) proposed by Chang [8] and Lee et al. [28], the relative importance weights for objectives are calculated. With the incorporation of the weights for objectives, FMOP-W can be constructed. The steps of the FMOP-W model are as follows:

Step 1. Form a committee of experts in the TFT-LCD industry and define the color filter replenishment problem.

Step 2. Formulate a questionnaire to compare objectives pairwisely in their contribution toward achieving the maximum satisfaction of color filter replenishment management in TFT-LCD manufacturing. Use a fuzzy number, $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$, as defined in Table 1, to represent the pairwise comparison value between every two objectives. The opinions of each expert are collected and combined into a fuzzy pairwise comparison matrix $\tilde{A}^f$.

$$\tilde{A}^f = \left[\tilde{a}_{lj}^f\right] = \begin{bmatrix} \tilde{a}_{11}^f & \tilde{a}_{12}^f & \cdots & \tilde{a}_{1g}^f \\ \tilde{a}_{21}^f & \tilde{a}_{22}^f & \cdots & \tilde{a}_{2g}^f \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{a}_{g1}^f & \tilde{a}_{g2}^f & \cdots & \tilde{a}_{gg}^f \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12}^f & \cdots & \tilde{a}_{1g}^f \\ \tilde{a}_{21}^f & 1 & \cdots & \tilde{a}_{2g}^f \\ \vdots & \vdots & 1 & \vdots \\ \tilde{a}_{g1}^f & \tilde{a}_{g2}^f & \cdots & 1 \end{bmatrix},$$

$$\text{for } l = 1, 2, \ldots, g,\ j = 1, 2, \ldots, g \text{ and } f = 1, 2, \ldots, F \tag{49}$$

where $\tilde{a}_{lj}^f = (x^-, x, x^+)$ and $a_{lj}^f \cdot a_{jl}^f \approx 1$.

Step 3. Combine fuzzy matrices of experts into an integrated fuzzy matrix and check its consistency.

$$\tilde{B} = \left[\tilde{b}_{lj}\right] = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \cdots & \tilde{b}_{1g} \\ \tilde{b}_{21} & \tilde{b}_{22} & \cdots & \tilde{b}_{2g} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{b}_{g1} & \tilde{b}_{g2} & \cdots & \tilde{b}_{gg} \end{bmatrix} = \begin{bmatrix} 1 & \tilde{b}_{12} & \cdots & \tilde{b}_{1g} \\ \tilde{b}_{21} & 1 & \cdots & \tilde{b}_{2g} \\ \vdots & \vdots & 1 & \vdots \\ \tilde{b}_{g1} & \tilde{b}_{g2} & \cdots & 1 \end{bmatrix},$$

$$\text{for } l, j = 1, 2, \ldots, g \tag{50}$$

where $\tilde{b}_{lj} = \left(\tilde{a}_{lj}^l \oplus \cdots \oplus \tilde{a}_{lj}^f\right)/F = \left(m_{lj}^-, m_{lj}, m_{lj}^+\right)$ and $b_{lj} \cdot b_{jl} \approx 1$, m_{lj}^-: the arithmetic average of the smallest assigned value among the experts, m_{lj}^+: the arithmetic average of the largest assigned value among the experts, and m_{lj}: the arithmetic average of the middle values among the experts.

Based on Buckley [4] and Csutora and Buckley [9], let $B = [b_{lj}]$ be a positive reciprocal matrix, and $\tilde{B} = [\tilde{b}_{lj}]$ be a fuzzy positive reciprocal matrix. If B is consistent, then $\tilde{B}$ is also consistent. If $\tilde{B}$ is not consistent, the questionnaire must be modified by the experts.

Step 4. Calculate crisp relative importance weights (priority vector) for objectives by adopting the extent analysis method (EAM) proposed by Chang [8]. By using Eqs. (11)–(18), we can get the weights, w'_g, of the objectives.

Step 5. Formulate the FMOP-W for the color filter replenishment problem. The overall objective is to maximize the satisfaction of the model. The degrees of satisfaction for the objectives are $\lambda_1, \lambda_2, \ldots \lambda_g$, and w_g from Step 4 are the weights for λ_g. The proposed FWOP-W model can be formulated as follows:

$$\text{Maximize } \lambda = w_1 \times \lambda_1 + w_2 \times \lambda_2 + \cdots + w_g \times \lambda_g \tag{51}$$

$$\text{s.t. } \lambda_g \le \mu_g(x), \quad \textit{for all } g \tag{52}$$

$$\lambda_g \in [0, 1], \quad \textit{for all } g \tag{53}$$

$$\text{Constraints} \tag{54}$$

where λ_g, $\mu_g(x)$ and w_g represent the degrees of satisfaction, membership functions and weights for objectives.

With the objectives of minimizing total cost, maximizing yield rate and fixing the replenishments to a desired number in a planning horizon, the formulation of the color filter replenishment problem is as follows:

$$\text{Maximize } \lambda = w_c \times \lambda_c + w_r \times \lambda_r + w_n \times \lambda_n \tag{55}$$

$$\text{s.t. constraints (31)–(48)} \tag{56}$$

where w_c, w_r and w_n are the normalized importance weights for cost, yield rate and number of replenishments, respectively; λ_c, λ_r and λ_n are the degrees of satisfaction for cost, yield rate and number of replenishments, respectively.

Special cases will be studied for the two proposed models, with combinations of limited or unlimited storage space, multiple suppliers and different discount schedules for the suppliers.

5. Numerical example

In order to illustrate the effectiveness of the proposed FMOP model and FMOP-W model, a numerical example is presented, and four special cases are analyzed. The software LINGO [30] is used to implement the proposed models on the cases. The results of the FMOP and FMOP-W models are compared for different cases.

5.1. *Basic input information*

Actual data is taken from an anonymous TFT-LCD manufacturer located on the Science-Based Industrial Park in Hsinchu, Taiwan. The manufacturer has different plants for TFT-array fabrication,

Table 2
Demand of each period in a planning horizon.

t	1	2	3	4	5	6	7	8	9	10
d_t	634	368	428	1200	1822	691	1618	1100	393	271

Table 3
Discount schedule for supplier A.

Price break (k)	Purchase quantity (Q)	Price per unit ($P(Q)$)
1	0–999	40
2	1000–1999	39.5
3	2000–2999	39
4	3000 or more	38.6

cell assembly and module assembly. After the TFT-array fabrication, TFT-array substrates are moved to the cell plant to assemble with color filter substrates, which are purchased from color filter manufacturers. Therefore, adequate number of color filters must be purchased and stored in the plant. The manufacturer currently has one supplier, supplier A, and is considering cooperating with another supplier, supplier B. The objectives of the model are to minimize total cost, maximize yield rate and fix the replenishments to a desired number, and in turn, to determine the optimal purchase amount of color filters from each supplier in each period.

Based on an interview with the management of the TFT-LCD fab, the following assumptions are made. In practice, the lead-time of each replenishment is 1 day (1 period), and the planning horizon contains 7 periods. Therefore, this study sets the planning horizon to be 10 periods to be comprehensive. The ordering cost of supplier A (o_1) and supplier B (o_2) per replenishment is set to be \$120 and \$110, respectively. In addition, we set unit holding cost per period (h), which includes the handling cost, storage cost and capital cost, to be \$1. In order to calculate λ, we need to set the minimum total cost (c_1) and maximum total cost (c_2) in the planning horizon. The minimum total cost (c_1), set to be \$332,065, is calculated by only considering the purchase cost and assuming that color filters are purchased in the lowest unit price from supplier A, the current supplier for the firm. The maximum total cost (c_2), set to be \$346,463, is calculated by considering all the costs, including purchase cost, order cost and holding cost, and assuming that color filters are purchased in the highest unit price.

Since the cell plant is make-to-stock and each planning horizon contains 10 days, the demand of color filters in each period is assumed to be deterministic. Table 2 shows the demand d_t in each period t. The minimum number (n_1) and maximum number (n_2) of replenishments in the planning horizon are set to be 1 and 8, respectively. The target number (n_T) of replenishments in the planning horizon is set to be 5.

Table 3 shows the discount schedule for supplier A under different purchase quantities. For instance, if the purchase quantity in a period is between 2000 and 2999 units, the price for each unit, starting from the first unit, is \$39. Table 4 shows the discount schedule for supplier B under different purchase quantity. The minimum yield rate (r_1) and maximum yield rate (r_2) of purchase quantities are set to be 0.91 and 1.00, respectively. In addition, the yield rate of color filters from supplier A is 0.95, and that from supplier B is 0.98.

Four special cases are examined here, as shown in Table 5. Each case may be varied in its storage space (limited or unlimited), supplier (A, B or A and B) and purchase price (by discount schedule).

Table 4
Discount schedule for supplier B.

Price break (k)	Purchase quantity (Q)	Price per unit ($P(Q)$)
1	0–1199	40
2	1200–2399	39.3
3	2400–3599	38.9
4	3600 or more	37.8

Table 5
Data for the four cases.

Case	Storage space (s)	Supplier
I	3000	A
II	3000	B
III	3000	A and B
IV	Infinite	A and B

Under case I, the maximum storage space is 3000 units of color filters, which are purchased solely from supplier A. Discounted price is given to all units based on the total quantity purchased at a time using the discount schedule for supplier A in Table 3. Under case II, the maximum storage space is 3000 units, and color filters are only purchased from supplier B using the discount schedule in Table 4. Under case III, the maximum storage space is 3000 units, and color filters can be purchased from supplier A and/or B using the discount schedules in Tables 3 and 4, respectively. Under case IV, there is no limit on the storage space since storage space is infinite. Color filters can be purchased from supplier A and/or B using the discount schedules in Tables 3 and 4, respectively.

For calculating the weights of objectives under the FMOP-W, a questionnaire is prepared to ask decision-makers to compare multiple objectives pairwisely in their contribution toward achieving the overall satisfaction of color filter replenishment. The multiple objectives are minimizing total cost, maximizing yield rate and fixing the replenishments to a desired number. The integrated fuzzy matrix is calculated by steps 1–3 in Section 4.2 and is shown in Table 6. The consistency of the integrated fuzzy matrix is examined.

Then, by applying Eq. (12), we have

$$F_1 = (0.16,\ 0.56,\ 1.54),$$

$$F_2 = (0.12,\ 0.27,\ 0.85),$$

$$F_3 = (0.08,\ 0.17,\ 0.45).$$

Finally, by using Eqs. (15) and (16), we obtain

$$d(F_1) = \min(1, 1) = 1,$$

$$d(F_2) = \min(0.71, 1) = 0.71,$$

$$d(F_3) = \min(0.43, 0.76) = 0.43.$$

By applying Eq. (17), the importance weights ($\boldsymbol{w}'$) for the objectives of minimizing total cost, maximizing yield rate and fixing the replenishments to a desired number are 1, 0.71 and 0.43, respectively. By applying Eq. (18), the normalized weights ($\boldsymbol{w}$), as shown in Table 6, are 0.47, 0.33 and 0.20, respectively. The weights for the objectives are then used for further analysis in FMOP-W.

Table 6
The integrated fuzzy matrix.

	Total cost	Yield rate	Number of replenishments	$\boldsymbol{w}'$	$\boldsymbol{w}$
Total cost	(1, 1, 1)	(1, 2.33, 4.33)	(1, 3, 5)	1.00	0.47
Yield rate	$(4.33^{-1}, 2.33^{-1}, 1^{-1})$	(1, 1, 1)	(1, 1.67, 3.67)	0.71	0.33
Number of replenishments	$(5^{-1}, 3^{-1}, 1^{-1})$	$(3.67^{-1}, 1.67^{-1}, 1^{-1})$	(1, 1, 1)	0.43	0.20

Table 7
Comparison of solutions for case I.

Model	t	1	2	3	4	5	6	7	8	9	10	C	R	N	λ
	X_t		368				691		382	393		$341,767	0.950	7	0.326
FMOP	Q_{1t}	1002		428	1200	2513		2000	1111		271				
	Q_{2t}														
	X_t		796	428			691		382	664	271	$342,576	0.950	5	0.473
FMOP-W	Q_{1t}	1430			1200	2513		2000	1382						
	Q_{2t}														

Table 8
Comparison of solutions for case II.

Model	t	1	2	3	4	5	6	7	8	9	10	C	R	N	λ
	X_t		566	198			691		1100		271	$341,314	0.980	6	0.358
FMOP	Q_{1t}														
	Q_{2t}	1200		230	1200	2513		2718		664					
	X_t		796	428			691		1100		271	$341,503	0.980	5	0.620
FMOP-W	Q_{1t}														
	Q_{2t}	1430			1200	2513		2718		664					

5.2. Experimental result and analysis

Based on the proposed models, LINGO (2006) is used on a PC/P4 with CPU 3.4 GHz. The results of the four cases under FMOP and FMOP-W are obtained and summarized in Tables 7–10.

5.2.1. Case I

The maximum storage space is 3000 units, and color filters can only be purchased from supplier A. The price is determined by the discount schedule in Table 3. The beginning inventory in period 1 (X_1) is zero. Under FMOP, seven purchases are made: 1002 units in period 1, 428 units in period 3, 1200 units in period 4, 2513 units in period 5, 2000 units in period 7, 1111 units in period 8, and 271 units in period 10. With purchase quantity discount, the total cost for a planning horizon is $341,767. The yield rate is 0.950. The degrees of membership function for the three objectives, total cost, yield rate, and number of replenishments are 0.326, 0.444 and 0.6, respectively. Therefore, the total satisfaction is 0.326, with 1.0 being the highest satisfaction.

Under FMOP-W, five purchases are made: 1430 units in period 1, 1200 units in period 4, 2513 units in period 5, 2000 units in period 7, and 1382 units in period 8. The total cost for a planning horizon is $342,576, and the yield rate is 0.950. The degrees of membership function for total cost, yield rate, and number of replenishments are 0.269, 0.444 and 1, respectively. With different weights for objectives, the total satisfaction is 0.473.

5.2.2. Case II

The maximum storage space is 3000 units, and color filters can only be purchased from supplier B. The price is determined by the discount schedule in Table 4. Due to the different discount schedules for suppliers A and B, the replenishment decisions are different from that in case I. Under FMOP, there are six purchases during the planning horizon. The total cost for a planning horizon is $341,314, and the yield rate is 0.980. The degrees of membership function for total cost, yield rate, and number of replenishments are 0.358, 0.782 and 0.8, respectively. The total satisfaction is 0.358.

Under FMOP-W, five purchases are made. The total cost for a planning horizon is $341,503, and the yield rate is 0.980. The degrees of membership function for the objectives are 0.344, 0.782 and 1, respectively. The total satisfaction is 0.620.

5.2.3. Case III

The condition under case III is better than those under cases I and II. While case I or II has only one supplier, case III has two suppliers. Under FMOP, there are six purchases: 1002 units from supplier A in period 1, 428 units from supplier B in period 3, 1200 units from supplier B in period 4, 2513 units from supplier B in

Table 9
Comparison of solutions for case III.

Model	t	1	2	3	4	5	6	7	8	9	10	C	R	N	λ
	X_t		368				691		1382	228	271	$340,631	0.976	6	0.405
FMOP	Q_{1t}	1002													
	Q_{2t}			428	1200	2513		3000		382					
	X_t		796	428			691		1382	228	271	$341,757	0.980	5	0.612
FMOP-W	Q_{1t}														
	Q_{2t}	1430			1200	2513		3000		382					

Table 10
Comparison of solutions for case IV.

Model	t	1	2	3	4	5	6	7	8	9	10	C	R	N	λ
	X_t		368			2513	691		1493	393		$338,266	0.966	5	0.569
FMOP	Q_{1t}	1002						3111							
	Q_{2t}			428	3713						271				
	X_t		796	428		2513	691		1100			$339,181	0.980	5	0.695
FMOP-W	Q_{1t}														
	Q_{2t}	1430			3713			2718		393	271				

period 5, 3000 units from supplier A in period 7, and 382 units from supplier B in period 9. With a looser constraint, case III performs better than cases I and II, with a total cost of $340,631 and degrees of membership function for the objectives of 0.405, 0.733 and 0.8, respectively. Thus, the total satisfaction is 0.405.

Under FMOP-W, five purchases are made, and all are purchased from supplier B. The total cost for a planning horizon is $341,757, and the yield rate is 0.980. The degrees of membership function for the objectives are 0.328, 0.782 and 1, respectively. The total satisfaction is 0.612.

5.2.4. Case IV

With an unlimited storage space, two suppliers and different discount schedules for suppliers, this case performs the best among all four cases under both FMOP and FMOP-W. Under FMOP, there are five purchases, and a total cost of $338,266 is incurred. The degrees of membership function for total cost, yield rate, and number of replenishments are 0.569, 0.622 and 1, respectively. The total satisfaction is 0.569, a rather large improvement from that in case III. Under FMOP-W, five purchases are made, and again, all purchases are from supplier B. The reasons behind are because supplier B has a higher yield rate than supplier A does and supplier B provides a lower unit cost when a purchase of a larger quantity is placed. The total cost for a planning horizon is $339,181, and the yield rate is 0.980. The degrees of membership function for the objectives are 0.505, 0.782 and 1, respectively. The total satisfaction is 0.695, the highest satisfaction among all cases under two models.

The performance of the FMOP model and the FMOP-W model are compared in Table 11. While the FMOP only deals with one overall objective function, the FMOP-W can incorporate multiple objective functions, with different objective weights. For the FMOP, the degree of satisfaction of a model is the minimization of the maximum degrees of satisfaction among multiple objectives. On the other hand, the FMOP-W calculates a weighted degree of satisfaction based on the weights of different objectives. In conclusion, when it is not necessary to set objective weights, FMOP can effectively plan the inventory replenishment for each period. On the other hand, FMOP-W can be applied to take into account the real production requirement by incorporating experts' opinions.

Two recent works by Wang and Yang [41] and Wee et al. [42], which also applied fuzzy multiple objective programming on the inventory problem, are compared with the proposed models, as shown in Table 12. Among the compared items, Wang and Yang [41] considered six of them, while Wee et al. [42] considered four items. On the other hand, the proposed FMOP and FMOP-W considered eight and nine items, respectively. Therefore, the two proposed models are relatively outstanding overall. In a future study, deterioration can be integrated into the proposed models.

It is worth noting that the an increase in the number of suppliers (i), periods (t) or quantity discounts (k) will increase the problem size and may make the problem become a NP-hard problem and computationally prohibitive. Nevertheless, FMOP and FMOP-W proposed in this research successfully formulated the color filter replenishment problem in TFT-LCD manufacturing with the consideration of storage space, yield rate, quantity discounts and multiple suppliers. Under the current stated environment, FMOP and FMOP-W could transform all objectives and constraints into linear functions. A practical case study illustrated the effectiveness of the proposed method, which is readily available for applied applications.

Table 11
Comparison of the two models.

Factor	FMOP model	FMOP-W model
Objective function(s)	Single	Multiple
Objective weight(s)	Not presented	Presented
Constraints	Fuzzy	Fuzzy
Degree of satisfaction	Max–min	Pool
Planning horizon	Multi-period	Multi-period
Demand	Certain	Certain
Total cost	Presented	Presented
Yield rate	Presented	Presented
Number of replenishments	Presented	Presented

Table 12
Comparisons among similar approaches.

Compared items	Wang and Yang [41]	Wee et al. [42]	Proposed FMOP	Proposed FMOP-W
Algorithm	Exact	Heuristic	Exact	Exact
Fuzzy multiple objective programming	Yes	Yes	Yes	Yes
Supplier selection	Yes	No	Yes	Yes
Quantity discount	Yes	No	Yes	Yes
Replenishment	No	Yes	Yes	Yes
Deterioration	No	Yes	No	No
Multi-periods	No	Yes	Yes	Yes
Solved by common software packages	Yes	No	Yes	Yes
Solve binary behavior	Yes	No	Yes	Yes
Consider qualitative judgment	Yes	No	No	Yes

6. Conclusion

This paper proposes two models to determine the replenishment quantity of color filters from multiple suppliers for multiple periods with the consideration of storage space, quantity discounts and multiple suppliers. FMOP can effectively plan the inventory replenishment for each period without pre-determining the weights of objectives. On the other hand, FMOP-W can consider the opinions of experts in setting the weights of objectives. The case study demonstrates the practicality of the proposed models in achieving the best satisfaction under multiple goals, which are minimizing total cost, maximizing yield rate and fixing the replenishments to a desired number. To the best of our knowledge, a model, which considers storage space, quantity discounts and multiple suppliers to simultaneously minimize total cost, maximize yield rate and fix the replenishments to a desired number, is non-existent. The analysis provided in this study is very useful for managers in designing a replenishment policy for TFT-LCD manufacturers to deal with color filters which have the characteristics of large size, multiple suppliers and different discount schedules for suppliers.

The proposed models can be tailored and applied to other inventory management problems. For future research, we can consider a more complete case for supply chain management in TFT-LCD manufacturing. A model that takes into account fuzzy demand, variable lead-time, safety stock, and different priority of orders can be established. A dynamic production environment may be considered with more issues, for example, product deterioration, non-linear purchasing (production) cost, and non-linear setup cost. If these are concerns, then the assumptions need to be relaxed by the modification of the objectives and constraints. There are some fuzzy inventory models to deal with fuzzy backorders, with fuzzy demand, with fuzzy lead-time, with fuzzy order quantity, with fuzzy capacity, with signed distance of fuzzy sets, and so on. Metaheuristic algorithms, such as tabu search, ant colony optimization and evolutionary algorithms, have also generated considerable interest in the replenishment problem. These can be the future research directions.

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