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Stochastic model reference predictive temperature control with integral action for an industrial oil-cooling process

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1. Introduction

The oil-cooling process has been extensively used for machine tools, such as cutting, milling and drilling machines [\(Tsai](#page-8-0) & [Huang,](#page-8-0) [2004](#page-8-0)). [Fig. 1](#page-1-0) illustrates the relationship between an industrial oilcooling process and a machine tool. As [Fig. 1](#page-1-0) shows, the oil coolant brings out the heat generated by the machine tool (cutter) and the work piece for the duration of manufacturing, thus maintaining the temperatures of the spindle and the platform at desired temperature set points. Hence, the oil coolant at the outlet of the oil-cooling machine must be controlled in order to avoid abnormal temperature rising for the working machine tool.

In recent years, a new kind of oil-cooling process with a variable-speed compressor and a feedback temperature controller has been proposed to control the temperature of the oil coolant with steady-state accuracy up to \pm 0.5 °C ([Tsai & Huang, 2004](#page-8-0)). As shown in [Fig. 2](#page-1-0), this kind of compressor can be driven by either a three-phase variable-frequency (VF) induction motor or a threephase and high-efficiency direct-current (DC) brushless motor. Unlike the convectional ON/OFF controller, the feedback temperature controller continuously adjusts the speed of the variablespeed compressor based on the control signals, thereby eliminating a great amount of frequent starting losses. Hence, such a process not only consumes less power, but also provides an improved temperature control performance for the oil-coolant at the outlet. Furthermore, this type of oil-cooling process is particularly useful for high-speed machine tools because of its

abstract

This paper presents a stochastic model reference predictive control (SMRPC) approach to achieving accurate temperature control for an industrial oil-cooling process, which is experimentally modeled as a simple first-order system model with given long time delay. Based on this model, the stochastic model reference predictive controller with control weighting and integral action is derived based on the minimization of an expected generalized predictive control (GPC) performance criteria. A real-time adaptive SMRPC algorithm is proposed and then implemented into a stand-alone digital signal processor (DSP). Experimental results show that the proposed control method is capable of giving accurate and satisfactory control performance under set-point changes, fixed load and load changes. & 2008 Elsevier Ltd. All rights reserved.

> abilities to precisely provide the oil coolant at desired temperatures in order to maintain the required manufacturing precision during operation and increase the lifetime of the high-speed machine tools.

> The complicated behavior caused by the variable-speed compressor and the heat exchange process indicates that the oil-cooling process is difficult to find its exact first-principle model and design of such a feedback temperature controller is nontrivial to accomplish accurate temperature control in the presence of modeling error and disturbances. To date, a simple process model from input–output experimental data has been successfully presented to approximate the oil-cooling process ([Tsai & Huang, 2004](#page-8-0)), and two kinds of feedback temperature controllers to achieve the design criterion have also been addressed in [Tsai and Huang \(2004\),](#page-8-0) [Tsai, Wang, Lin, and Teng](#page-8-0) [\(2006\)](#page-8-0). Researchers in [Tsai and Huang \(2004\)](#page-8-0) proposed a singlechip DSP-based model reference adaptive controller for a VF oil-cooling machine. Using the same first-order system model with time delay in [Tsai and Huang \(2004\),](#page-8-0) the authors in [Tsai et al.](#page-8-0) [\(2006\)](#page-8-0) proposed a direct self-tuning controller with integral action to control the machine. However, the model reference adaptive method proposed in [Tsai and Huang \(2004\)](#page-8-0) accomplished the temperature set-point regulation and tracking capability without integral action, namely that the method in [Tsai and Huang \(2004\)](#page-8-0) requires an exact mathematical model for the process and may provide poor control performance in case of occurrence of model errors, parameter variations and external disturbances. Although the direct self-tuner in [Tsai et al. \(2006\)](#page-8-0) includes integral control action, it cannot be applied to deal with industrial processes with long time delay, nonminimum-phase and even unstable properties.

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Model predictive control (MPC) approach has been considered as a powerful control method dealing with the temperature control problems ([Brdys, Grochowski, Gminski, Konarczak, &](#page-8-0) [Drewa, 2008](#page-8-0); [Haugwitz, Hagander,](#page-8-0) & Norén, 2007; [Huang](#page-8-0) [& Tsai,](#page-8-0) [2003;](#page-8-0) [Lu & Tsai, 2001](#page-8-0); [Nagy, Mahn, Franke,](#page-8-0) & Allgöwer, 2007; [Overloop, van Weijs,](#page-8-0) [& Dijkstra, 2008;](#page-8-0) [Pin, Falchetta, & Fenu,](#page-8-0) [2008;](#page-8-0) [Taur, Tao,](#page-8-0) & [Tsai, 1995;](#page-8-0) [Tsai & Lu, 1997, 1998a, 1998b\)](#page-8-0). MPC has, during the end of the 1970s, the 1980s and especially the 1990s, received a strong position when it comes to industrially implement advanced control methodologies ([Camacho & Bordons,](#page-8-0) [1999](#page-8-0); [Maciejowski, 2002](#page-8-0)). Comparisons between infinite-horizon linear quadratic (LQ) control and MPC have been well discussed in [Mosca \(1995\)](#page-8-0); the comparison results reveal that MPC can be studied within the framework of LQ control theory. One of popular implementations of MPC is generalized predictive control (GPC) which was proposed by Clarke. Like pole-placement control and infinite-horizon LQ control, GPC has been regarded as a powerful and useful model-based control method for a wide class of linear and nonlinear dynamic systems. The basic principle of the GPC is to generate a sequence of control signals at each sample interval that optimizes the GPC cost function in terms of the future trajectory tracking errors and the future control effort, in order to follow exactly the reference trajectory. From the viewpoint of controller design, the GPC can be synthesized using either the transfer function formulation [\(Camacho & Bordons, 1999;](#page-8-0) [Mosca,](#page-8-0) [1995](#page-8-0)) or the state-space formulation [\(Huang, Tan, & Lee, 2002;](#page-8-0) [Maciejowski, 2002](#page-8-0)). In the state-space formulation, the system under consideration is converted into a standard state-space form and then the standard GPC design procedure is applied to derive a control law. Hence, this paper takes the advantages of GPC in the

Fig. 1. Relationship between an oil-cooling machine and a machine tool.

transfer function formulation which has been shown capable of managing time-varying systems, systems with variable and unknown dead times. More importantly, in comparison with computation complexity of GPC control laws for the systems with long time delay, the transfer function formulation definitely gives a less computation requirement than the state-space formulation. With control weighting and long prediction horizon, GPC can further be applied to control systems with nonminimum-phase property ([Clarke, 1988;](#page-8-0) [Clarke, Mohtadi,](#page-8-0) & [Tuffs, 1987a, 1987b\)](#page-8-0) (i.e., systems whose transmission zeros are outside the unit circle), and even unstable systems.

Furthermore, adaptive predictive control has also attracted considerable attention in both academia and industry; in the context, the adaptability of this kind of controller can be achieved using either a self-tuning control method or a model reference approach ([Goodwin](#page-8-0) [& Sin, 1984;](#page-8-0) [Mosca, 1995\)](#page-8-0). For industrial applications, adaptive predictive control has been successfully applied to several industrial temperature control processes (Aström [& Wittenmark, 1995](#page-8-0); [Carati, Pinheiro, Pinheiro, Hey, &](#page-8-0) [Grundling, 2001](#page-8-0); [Cho, Edgar, & Lee, 2008](#page-8-0); [Lu](#page-8-0) [& Tsai, 2001](#page-8-0); [Salehi &](#page-8-0) [Shahrokhi, 2008](#page-8-0); [Tsai](#page-8-0) [& Lu, 1997\)](#page-8-0).

In comparison with the result in [Tsai and Huang \(2004\)](#page-8-0), this paper is written in two principal contributions. First, this paper develops a stochastic model reference adaptive predictive control with integral action and control weighting by integrating the model reference adaptive control ([Butler, 1992;](#page-8-0) [Narendra &](#page-8-0) [Annaswamy, 1989](#page-8-0)) and the well-known GPC. The developed control law is more generous than the previous one in which the control weighting has not been taken into account yet. Following the logics provided in the paper, one can easily extend the proposed method to second-order system models with time delay, and even higher-order system models with time delay. From theoretical viewpoint, the proposed control law is more useful than the one in [Tsai and Huang \(2004\)](#page-8-0) because it will be employed to cope with a much wider class of industrial processes and machines, including unstable systems, and systems with nonminimum-phase property. From the viewpoint of industrial applications, the proposed control provides a more pragmatic control effort to achieve temperature tracking goal, hereby resulting in energy saving. Second, the proposed control law together with relevant sensors and actuators has been implemented into a digital signal processor (DSP). The performance and merits of the proposed control method are exemplified by conducting several experiments based on the DSP-based controller. Although the DSP-based implementation scheme has been

Fig. 2. Illustrative diagram of the oil-cooling process with a variable-frequency driver.

shown in [Bielewicz, Debowski, and Lowiec \(1996\),](#page-8-0) [Carati et al.](#page-8-0) [\(2001\),](#page-8-0) [Lascu and Trzynadlowski \(2001\)](#page-8-0), [Lundquist and Carey](#page-8-0) [\(2001\),](#page-8-0) [Tsai et al. \(2006\)](#page-8-0), this paper does not pay attention to emphasize how to implement the controller, but focus on how to evaluate the effectiveness of the proposed DSP-based controller. Last but not least, the proposed method may be useful and effective in assisting engineers and practitioners to design a practical, low-cost but high-performance industrial oil-cooling machine for high-speed machine tools.

The remainder of the paper is organized as follows. Section 2 presents the multi-step predictive control with control weighting and integral control action for achieving high-precision temperature control. Section 3 proposes the real-time adaptive predictive control algorithm. In Section 4, the DSP-based oil-cooling temperature control system is described, and then several experimental results for controlling the oil-cooling process are shown. Section 5 concludes the paper.

2. Stochastic model reference predictive control with integral action and control weighting

This section aims at presenting a systematic way of combining GPC and model reference control (MRC) for obtaining a new type of stochastic model reference predictive control (SMRPC) with control weighting and integral action for the oil-cooling process. This new control method is based on the minimization of an expected GPC-like performance criterion. The feature of this method is to select a reference model with the desired system output, and to track the desired model output trajectory with least error by minimizing the cost function. To design the stochastic model reference predictive controller, an approximate process model for the oil-cooling process is required. Using the reaction curve method in [Tsai and Huang \(2004\),](#page-8-0) this process has been approximated by the following first-order autoregressive moving averaging with white noise (ARMAX) model:

$$
A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + \xi(k)
$$
\n(1)

where $A(q^{-1}) = 1 - aq^{-1}$, $B(q^{-1}) = b_0 + b_1 q^{-1}$, d is the time delay in the number of sampling periods, $u(k)$ is the control signal and $y(k) = y_0 - \bar{y}(k)$ where $\bar{y}(k)$ represents the current temperature output and y_0 denotes the initial temperature at $k = 0$. Further, q represents the shift operator meaning $q^{-1}{y(k)} = y(k-1)$, and $\xi(k)$ denotes a zero-mean discrete-time white Gaussian stochastic process with zero mean and variance σ^2 . Note that $B(q^{-1})|_{q=1} = b_0 + b_1 \neq 0$, and the process parameters a, b_0 and b_1 vary with temperature set points.

For designing the proposed control law with integral action, the oil-cooling process model (1) can be expressed by the following GPC model:

$$
A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + \xi(k)/\Delta
$$
 (2)

where $\Delta = 1 - q^{-1}$. Next, one makes the following assumptions about the reference model.

Assumption A1. The desired output $y^*(k)$ is generated by the following reference model:

$$
D(q^{-1})y^*(k) = q^{-d}N(q^{-1})u_c(k)
$$
\n(3)

which has the transfer function $G_m(q^{-1})=q^{-d}N(q^{-1})/D(q^{-1})$ with the reference input $u_c(k)$, where $N(q^{-1}) = n_0$ and $D(q^{-1}) =$ $1+d_1q^{-1}+d_2q^{-2}$. Note that the denominator polynomial $D(q^{-1})$ is asymptotically stable, namely that all the roots of the polynomial $D(q^{-1})$ lie within the unit circle in the z plane.

Assumption A2. The DC gain of the transfer function $G_m(q^{-1})$ is unity, i.e., $G_m(1) = 1$ and $n_0 = 1+d_1+d_2 \in R$.

Remark 1. A practical rule or guideline for choosing the model parameters d_1 and d_2 is simply described as follows. The model parameter n_0 in the numerator is determined such that the DC gain of the reference model equals unity, i.e., $n_0 = 1+d_1+d_2$. The model parameters d_1 and d_2 are chosen such that the two closedloop poles have desired corresponding analog damping ratio and natural frequency. The method relating the two closed-loop conjugate poles to the corresponding analog damping ratio and natural frequency can be found in [Phillips and Nagle \(1995\).](#page-8-0) For the industrial oil-cooling process, the model parameters d_1 , d_2 and n_0 are given by $d_1 = -1.715$, $d_2 = 0.733$ and $n_0 = 0.018$, thereby giving unity DC gain and two closed-loop poles of 0.9055 and 0.8095.

In what follows derives the proposed model reference predictive control law with integral action so as to minimize the expectation of the following GPC-like quadratic function with control weighting:

$$
J = E\left\{\sum_{j=d}^{H_p} |D(q^{-1})[y(k+j) - y^*(k+j)]|^2 + \sum_{j=0}^{H_u-1} \lambda_w(j)(\Delta u(k+j))^2\right\}
$$
(4)

where H_p means the maximum prediction horizon, H_u denotes the control horizon, and $\lambda_w(i)$, $i = 0,..., H_u-1$, represents the control weighting. Note that $H_p \ge H_u$ and $H_p-H_u \ge 0$. In order to establish the control law, the following Diophantine equation in (5) is used to solve for the polynomials $F_j(q^{-1})$, $\bar{G}_j(q^{-1})$ and $P_j(q^{-1})$ so that the optimal *j* step-ahead predictor of $y(k)$ can be found:

$$
D(q^{-1}) = \Delta F_j(q^{-1})A(q^{-1}) + q^{-j}\bar{G}_j(q^{-1})
$$

\n
$$
P_j(q^{-1}) = F_j(q^{-1})B(q^{-1})
$$
\n(5)

where

$$
F_j(q^{-1}) = 1 + f_{j,1}q^{-1} + f_{j,2}q^{-2} + \dots + f_{j,j-1}q^{-(j-1)}
$$

\n
$$
\bar{G}_j(q^{-1}) = g_{j,0} + g_{j,1}q^{-1}
$$

\n
$$
P_j(q^{-1}) = p_{j,0} + p_{j,1}q^{-1} + p_{j,2}q^{-2} + \dots + p_{j,j}q^{-j}
$$

Multiplying (1) by $\Delta F_j(q^{-1})q^j$ yields the following equation:

$$
F_j(q^{-1})A(q^{-1})\Delta y(k+j) = F_j(q^{-1})B(q^{-1})\Delta u(k+j-d) + F_j(q^{-1})q^j\xi(k)
$$
\n(6)

From (5), it follows that

$$
D(q^{-1})y(k+j) = F_j(q^{-1})A(q^{-1})\Delta y(k+j) + \bar{G}(q^{-1})y(k)
$$
\n(7)

Combining (6) and (7) gives

$$
D(q^{-1})y(k+j) = P_j(q^{-1})\Delta u(k+j-d) + \bar{G}_j(q^{-1})y(k) + F_j(q^{-1})\zeta(k+j)
$$
\n(8)

Thus, the optimal *j*-step-ahead output predictor of $y(k)$ is then given by

$$
D(q^{-1})\hat{y}(k+j) = P_j(q^{-1})\Delta u(k+j-d) + \bar{G}_j(q^{-1})y(k)
$$
\n(9)

With this optimal *j*-step-ahead predictor $\hat{y}(k + j)$ and the white Gaussian property of the noise $\xi(k)$, the minimization of expectation of the cost function J becomes

$$
J = E \Biggl\{ \sum_{j=d}^{H_p} \left| D(q^{-1}) \left[y(k+j) - y^*(k+j) \right] \right|^2 + \sum_{j=0}^{H_u - 1} \lambda_w(j) (\Delta u(k+j))^2 \Biggr\}
$$

=
$$
\sum_{j=d}^{H_p} \Biggl\{ \left| D(q^{-1}) \left[\hat{y}(k+j) - y^*(k+j) \right] \right|^2 \Biggl\{ + (1 + f_{j,1}^2 + f_{j,2}^2 + \dots + f_{j,j-1}^2) \sigma^2 \Biggr\}
$$

+
$$
\sum_{j=0}^{H_u - 1} \lambda_w(j) (\Delta u(k+j))^2 = \bar{J} + \sum_{j=d}^{H_p} (1 + f_{j,1}^2 + f_{j,2}^2 + \dots + f_{j,j-1}^2) \sigma^2
$$
(10)

where \bar{J} is given by

$$
\bar{J} = \sum_{j=d}^{H_p} \left| D(q^{-1}) \left[\hat{y}(k+j) - y^*(k+j) \right] \right|^2 + \sum_{j=0}^{H_u-1} \lambda_w(i) (\Delta u(k+j))^2
$$

Since the second term in *is independent of the future control* vector U, the minimum of J with respect to the future control vector U is completely identical to the minimum of \bar{J} with respect to the future control vector U, i.e., the finding of the optimal control vector U so as to minimize \overline{J} . Moreover, the new cost function \bar{J} can be rewritten by

$$
\bar{J} = PU + W + \bar{G}y(k) - R^{T}(PU + W + \bar{G}y(k) - R) + U^{T}AU
$$
 (11)
where

where

$$
P = \begin{bmatrix} p_{d,0} & 0 & \cdots & 0 \\ p_{d+1,1} & p_{d+1,0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{H_p,H_p-d} & p_{H_p,H_p-d+1} & \cdots & p_{H_p,H_p-d-H_u+1} \end{bmatrix} \in \mathbb{R}^{(H_p-d+1)\times H_u}
$$

$$
U = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \cdots & \Delta u(k+H_u-1) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{H_u \times 1}
$$

$$
W = \begin{bmatrix} \sum_{j=1}^{d} p_{dj} \Delta u(k-j) \\ \sum_{j=2}^{d+1} p_{d+1j} \Delta u(k-j+1) \\ \vdots \\ \sum_{j=H_p-d+1}^{H_p} p_{H_p,j} \Delta u(k-j+H_p-d) \end{bmatrix} \in \mathbb{R}^{(H_p-d+1)\times 1}
$$

$$
R = \begin{bmatrix} N(q^{-1})u_c(k) & N(q^{-1})u_c(k+1) & \cdots & N(q^{-1})u_c(k+H_p-d) \end{bmatrix}^T
$$

$$
\in \mathbb{R}^{(H_p-d+1)\times 1}
$$

$$
\bar{G} = \begin{bmatrix} g_{d,0} + g_{d,1}q^{-1} \\ g_{d+1,0} + g_{d+1,1}q^{-1} \\ \vdots \\ g_{H_p,0} + g_{H_p,1}q^{-1} \end{bmatrix} \in \mathbb{R}^{(H_p - d + 1) \times 1}
$$

$$
A = \begin{bmatrix} \lambda_w(0) & 0 & \cdots & 0 \\ 0 & \lambda_w(1) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_w(H_u - 1) \end{bmatrix} \in \mathbb{R}^{H_u \times H_u}
$$

Because the cost function \bar{J} is quadratic in U, an optimal solution for U is found easily by solving

$$
\frac{\partial \bar{J}}{\partial U}=0
$$

Therefore, the optimal control U^* without any constraints satisfies the following condition:

$$
P^{T}(PU^* + W + \bar{G}y(k) - R) + AU^* = 0
$$
\n(12)

which leads to obtain U^* as follows:

$$
U^* = (P^T P + A)^{-1} P^T (R - W - \bar{G}y(k))
$$
\n(13)

The present control increment $\Delta u(k)$ is the first entry of the vector U^* , and the present output signal $u(k)$ is given by

$$
u(k) = u(k-1) + \Delta u(k)
$$
 and $\Delta u(k) = [1 \ 0 \dots \ 0]U^*$ (14)

To reduce the computational complexity, one proposes the following recursive equations to find the parameters of the two polynomials $F_j(q^{-1})$ and $\bar{G}_j(q^{-1})$ by, for $j = 2$, $f_{2,1} = (a+1) + d_1, \quad g_{2,0} = d_2 + (a+1)f_{2,1} - a, \quad g_{2,1} = -af_{2,1}$ (15)

and for $j\geqslant3$

$$
f_{j,1} = d_1 + (a+1), \quad f_{j,2} = d_2 + (a+1)f_{j,1} - a
$$

\n
$$
f_{j,m} = (a+1)f_{j,m-1} - af_{j,m-2}, \quad m = 3, \dots, j-1
$$

\n
$$
g_{j,0} = (a+1)f_{j,j-1} - af_{j,j-2}, \quad g_{j,1} = -af_{j,j-1}
$$
\n(16)

Hence, the parameters of the polynomials, $P_j(q^{-1})$, $j \geq 2$, are recursively found as follows:

$$
p_{j,0} = b_0, \quad p_{j,1} = b_0 f_{j,1} + b_1
$$

\n
$$
p_{j,n} = b_0 f_{j,n} + b_1 f_{j,n-1}, \quad p_{j,j} = b_1 f_{j,j-1}, \quad n = 2, \dots, j \quad \text{for } j \ge 2 \quad (17)
$$

Before closing this section, the stability of the proposed predictive control with integral action and control weighting must be considered. According to (13) one obtains the following equation:

$$
\bar{C} = (P^{\mathrm{T}}P + \Lambda)^{-1}P^{\mathrm{T}} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,H_p-d+1} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,H_p-d+1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{H_u,1} & c_{H_u,2} & \cdots & c_{H_u,H_p-d+1} \end{bmatrix}
$$
(18)

The present control signal $u(k)$ can be calculated from (14) and then expressed as below:

$$
u(k) = u(k-1) + \sum_{i=0}^{H_p-d} c_{1,i+1} N(q^{-1}) u_c(k+i) - \sum_{i=0}^{H_p-d} c_{1,i+1} \sum_{j=i+1}^{i+d} q^{i-j} \Delta u(k)
$$

$$
- \sum_{i=0}^{H_p-d} c_{1,i+1} g_{d+i,0} y(k) - \sum_{i=0}^{H_p-d} c_{1,i+1} g_{d+i,1} y(k-1)
$$
(19)

which can be rewritten as the following general linear control form:

$$
\mathcal{R}(q^{-1})\Delta u(k) = T(q^{-1})u_c(k) - S(q^{-1})y(k)
$$
\n(20)

where

$$
\mathcal{R}(q^{-1}) = 1 + \sum_{i=0}^{H_p - d} c_{1,i+1} \sum_{j=i+1}^{i+d} q^{i-j}, \quad T(q^{-1}) = \sum_{i=0}^{H_p - d} q^i c_{1,i+1} N(q^{-1})
$$

$$
S(q^{-1}) = \sum_{i=0}^{H_p - d} c_{1,i+1} g_{d+i,0} + q^{-1} c_{1,i+1} g_{d+i,1}
$$

Note that, if the future reference temperature trajectory keeps constant, namely that $u_c(k+i)$ is equal to $u_c(k)$ for $i = 1,..., H_p-d$, then

$$
T(q^{-1}) = \sum_{i=0}^{H_p - d} c_{1,i+1} N(q^{-1})
$$
\n(21)

Applying control law (20) to system model (1) yields the overall closed-loop system:

$$
y(k) = \frac{q^{-d}B(q^{-1})T(q^{-1})}{C(q^{-1})}u_c(k) + \frac{\mathcal{R}(q^{-1})}{C(q^{-1})}\Delta\zeta(k)
$$
 (22)

where $C(q^{-1}) = A(q^{-1}) \mathcal{R}(q^{-1}) \Delta + q^{-d} B(q^{-1}) S(q^{-1})$. In general, the zeros of the closed-loop characteristic polynomial $C(q^{-1})$ can be made strictly inside the unit circle by choosing appropriate tuning parameters H_u , H_p and $\lambda_w(i)$, $i = 0, ..., H_u-1$ ([Clarke, 1988;](#page-8-0) [Clarke &](#page-8-0) [Mohtadi, 1989](#page-8-0)). Furthermore, if future reference temperature trajectory remains constant, then, from (5), (20), (21) and

Assumption A2, it follows that

$$
T(q^{-1})|_{q=1} = \sum_{i=0}^{H_p-d} c_{1,i+1} n_0 = S(q^{-1})|_{q=1}
$$

=
$$
\sum_{i=0}^{H_p-d} c_{1,i+1} (1+d_1+d_2)
$$

i.e., $T(1) = S(1)$, showing that the static gain of the transfer function (22) between output and reference is always one. The following theorem summarizes the main results.

Theorem 1. Suppose that Assumptions A1 and A2 hold, the closedloop characteristic polynomial C(q^{-1}) with the appropriate tuning parameters H_u , H_p and $\lambda_w(i)$, $i = 0, \ldots, H_u-1$, is asymptotically stable, and $B(q^{-1})|_{q=1} = b_0 + b_1 \neq 0$. Then the following two statements are true:

- (a) The DC gain of the overall transfer function $\bar{T}(q^{-1})=$ $[q^{-d}B(q^{-1})T(q^{-1})]/[C(q^{-1})]$ becomes 1, and the proposed control law (19) tracks any step input $u_c(k)$ without any steady-state error, i.e., $\lim_{k\to\infty} E\{y(k)\} = u_c(k) = c$ where c is the amplitude of the step input command, and $E{y(k)}$ represents the mean of the stochastic process $y(k)$.
- (b) If a constant disturbance D is exerted on the process such that $A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + D + \xi(k)$ and the proposed control (19) is applied, then for any step input $u_c(k)$, $\lim_{k\to\infty} E\{y(k)\}$ = $u_c(k) = c$ where c is the amplitude of the step input.

Proof. (a) To find the DC gain of the overall transfer function $\bar{T}(q^{-1})$, one obtains

$$
\bar{T}(q^{-1})|_{q=1} = \frac{q^{-d}B(q^{-1})T(q^{-1})}{C(q^{-1})}\Big|_{q=1}
$$

=
$$
\frac{q^{-d}B(q^{-1})T(q^{-1})}{A(q^{-1})\mathcal{R}(q^{-1})A + q^{-d}B(q^{-1})S(q^{-1})}\Big|_{q=1}
$$

=
$$
\frac{B(1)T(1)}{B(1)S(1)} = 1
$$

With the result, it is easy to show that

$$
\lim_{k \to \infty} E\{y(k)\} = \lim_{k \to \infty} E\left\{\bar{T}(q^{-1})u_c(k) + \frac{\Re(q^{-1})}{C(q^{-1})}\Delta \xi(k)\right\}
$$

$$
= \lim_{k \to \infty} E\{\bar{T}(q^{-1})c\} = \bar{T}(q^{-1})\Big|_{q=1} c = 1 \cdot c = c
$$

(b) Applying the proposed controller (19) to the process with the constant disturbance D gives

$$
y(k) = \overline{T}(q^{-1})u_c(k) + \frac{\mathcal{R}(q^{-1})}{C(q^{-1})}\Delta D + \frac{\mathcal{R}(q^{-1})}{C(q^{-1})}\Delta \xi(k)
$$

= $\overline{T}(q^{-1})u_c(k) + \frac{\mathcal{R}(q^{-1})}{C(q^{-1})}\Delta \xi(k)$

which leads to yield

$$
\lim_{k \to \infty} E\{y(k)\} = \lim_{k \to \infty} E\left\{\bar{T}(q^{-1})u_c(k) + \frac{\Re(q^{-1})}{C(q^{-1})}\Delta \xi(k)\right\}
$$

$$
= \lim_{k \to \infty} E\{\bar{T}(q^{-1})c\} = \bar{T}(q^{-1})\Big|_{q=1} c = 1 \cdot c = c
$$

This completes the proof of part (b). \Box

Remark 2. The proposed control (19) is indeed a combination of the discrete-time model reference adaptive control and the discrete-time GPC. In particular, if the maximum prediction horizon H_n is equal to the time delay of the system, d, and the control horizon H_u is set by unity, the proposed controller becomes to a weighted one-step-ahead controller with integral action ([Goodwin & Sin, 1984](#page-8-0)) or a generalized minimum-variance controller with integral action [\(Mosca, 1995](#page-8-0)). Like GPC, the proposed controller with a large control horizon H_u would be good to cope with system (1) with significant parameter variations, and the proposed controller with a long prediction horizon H_p would be robust against the plant uncertainties ([Banerjee](#page-8-0) & [Shan, 1995](#page-8-0)). Hence, the merits of the proposed controller hinge on its advantages of exact trajectory following capability with control weighting and robustness against plant variations. More importantly, this GPC-like control formulation gives the proposed controller with integral action which is useful and effective in eliminating steady-state errors caused by constant exogenous disturbances and modeling errors.

3. Real-time adaptive SMRPC algorithms

This section is devoted to proposing a real-time stochastic model reference predictive control algorithm with integral action and control weighting for the oil-cooling process. To construct the adaptive temperature controller, this proposed algorithm includes the following recursive least-squares (RLS) method with forgetting factor to estimate the process's parameters in real time:

$$
\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)(y(k) - \varphi^{T}(k)\hat{\theta}(k-1))
$$

\n
$$
K(k) = \Sigma(k-1)\varphi(k)(\lambda I_{3} + \varphi^{T}(k)\Sigma(k-1)\varphi(k))^{-1}
$$

\n
$$
\Sigma(k) = (I_{3} - K(k)\varphi^{T}(k))\Sigma(k-1)/\lambda
$$
\n(23)

where $\hat{\theta}(k - 1) = [\hat{b}_0 \quad \hat{b}_1 \quad \hat{a}]^T$; $\varphi(k) = [u(k - 4) \quad u(k - 5) \quad y(k)]^T$; I_3 is the third-order identity matrix, $\Sigma(k)$ is a 3×3 symmetric matrix, and $\Sigma(0) = \alpha I_3$; λ is the forgetting factor. In order to speed up convergence of the parameters in the startup phase of the algorithm, a pseudo random binary signal (PRBS) sequence designed in [Narendra and Annaswamy \(1989\),](#page-8-0) [Tsai and Huang](#page-8-0) [\(2004\)](#page-8-0) can be used as the best testing signal. Theoretically, the convergence rate of the RLS approach is inversely proportional to the number of the measured data (n) , i.e., $1/n$, if the measured noise is Gaussian and the inputs are persistently excited (Aström & [Wittenmark, 1995\)](#page-8-0). In addition, these three parameters can be successfully estimated if three measured data are linearly independently; however, the RLS method usually takes more data to achieve successful parameter estimation.

To reduce the computational delay, the proposed real-time algorithm employs the adaptive algorithm skeleton proposed in Å ström and Wittenmark (1995). [Fig. 3](#page-5-0) shows the block diagram of the proposed stochastic model reference adaptive predictive control system. The proposed real-time adaptive control algorithm is detailed in the following steps.

Step 1: Select sampling period $T(T = 10 s)$ and measure the initial temperature v_0 .

Step 2: Set the time delay $d (d = 4$ for the oil-cooling system), prediction horizon H_p , control horizon H_u and the control weighting quantity $\lambda_w(i)$, $i = 0, ..., H_u-1$.

Step 3: Select the parameters, d_1 , d_2 and n_0 , of the reference model.

Step 4: Input the control signal $u_c(k)$, measure the process output $\bar{y}(k)$ of the oil-cooling process, and then compute the quantity $y(k) = y_0 - \bar{y}(k)$.

Step 5: Generate the PRBS sequence if $k < k_0$ ($k_0 = 15$).

Step 6: Compute and output the following control signal $u(k)$, if $k\geq k_0$:

$$
\Delta u = \Delta u_{pre} + (P^{T}R - P^{T}G_{0})/(P^{T}P + \Lambda), \quad G_{0} = \begin{bmatrix} g_{d,0} \\ g_{d+1,0} \\ \vdots \\ g_{H_{p},0} \end{bmatrix} y(k)
$$

 $u(k) = u(k-1) + \Delta u$

Step 7: Estimate the system parameters \hat{a} , \hat{b}_0 and \hat{b}_1 with the forgetting factor λ using (23).

Step 8: Find the polynomials $F_j(q^{-1}), \bar{G}_j(q^{-1})$ and $P_j(q^{-1})$ based on (16) and (17).

Step 9: Update $k(= k+1)$.

Step 10: Compute Δu_{pre} where $\Delta u_{pre} = (-P^T W - P^T G_1)/(P^T P +$ Λ) and

$$
G_1 = \begin{bmatrix} g_{d,1} \\ g_{d+1,1} \\ \vdots \\ g_{H_p,1} \end{bmatrix} y(k-1)
$$

Step 11: Repeat Steps 4–11.

Remark 3. The selections of the sampling period T , the time delay d and the desired denominator polynomial $D(q^{-1})$ are crucial for stochastic model reference predictive control of the oil-cooling process. Indeed, the selections of these three parameters depend heavily upon a priori knowledge and operational experience of the oil-cooling process. To obtain the appropriate sampling period T and the correct time delay d in the temperature process, several open-loop experiments were conducted such that the open-loop reaction curve and the rise time were found; accordingly, the system time delay was estimated from the reaction curve and the sampling period T was chosen from the rise time using the method proposed by Åström and Wittenmark (1995). Once the sampling period T has been obtained, the time delay d will be calculated from the system time delay and the sampling period T. Note that the time delay $d (d = 4$ for the oil-cooling process) must be corrected estimated; a smaller time delay d will generate a fast desired output response that the real oil process cannot follow up, and a larger time delay d will cause a sluggish desired response to be tracked. The denominator polynomial $D(q^{-1})$ must also be carefully chosen before proceeding with the proposed control experiments. Computer simulations can be performed to select the polynomial $D(q^{-1})$ and examine the performance and efficacy of the proposed control controller for exactly following the desired output response. Worthy of mention is that since the oil-cooling process is a slow response system, the desired output response should be slow and not oscillatory so as to match the realistic response of the oil process.

Remark 4. Since the first-order system with time delay is used for approximating the dynamics of the oil-cooling process and then designing the stochastic model reference predictive controller, the adaptive controller's parameters change from one operation condition to another and they are estimated using a well-known RLS method with forgetting factor in real time. For successful estimation of those parameters in the oil-cooling process, the desired temperature trajectory commands and the noisy tem-

Fig. 3. Block diagram of the stochastic model reference adaptive predictive control system.

perature measurements shown in [Figs. 6, 8 and 10](#page-7-0) definitely meet the persistent excitation condition and then play a critical role on facilitating the convergence of those estimates, thus ensuring good performance of the RLS estimators at all operation conditions. On the other hand, the choices of the forgetting factor λ , prediction horizon H_p , control horizon H_u and the control weighting quantity λ_w are also very important. It has been shown that the choice of λ depends on the property of system changes and it is usually chosen in the range of $0.95 < \lambda < 1$. Since larger values of λ close to 1 result in slower forgetting and can be used for systems that change gradually, λ was chosen by 0.999 for this slowly time-varying temperature control process. Although a larger prediction horizon H_P often causes much robust control of the process, the controller requires more computations. The control horizon H_u usually equals the number of unstable or poorly damped poles, but for this simple oil-cooling process, $H_u = 1$ is good enough to provide good results. Finally, the smaller control weighting quantity λ_w will generate a fast output whereas the larger one will provide a more sluggish output.

4. Experimental results and discussion

4.1. Experimental setup

This proposed temperature control system is equipped with a three-phase, 220VAC rotary variable-speed compressor driven by a VF induction motor, a heat exchanger and a DSP-based controller with temperature sensing modules. [Fig. 4](#page-6-0) shows a recent picture and a block diagram of the experimental temperature control system. Such a machine is designed to have a cooling capability from 1200 kcal/h at 30 Hz to 2900 kcal/h at 90 Hz. Three independent heaters of 500, 1000 and 1000W are used to simulate the heat load generated by machine tools. The DSPbased controller consists of platinum temperature sensing modules with accuracy of ± 0.4 °C (R-V transducers) and a stand-alone DSP (TMS320F243 from Texas Instruments Co.) controller. Compared to a PC-based controller, this type of stand-alone single-chip DSP-based controller has much less volume and much lower cost. Furthermore, this DSP-based controller is especially suitable for executing high-performance but sophisticated real-time control algorithms. The linear PT100 temperature sensors with a signal processing circuit output voltage ranging from 0 to 5 V, corresponding temperatures from -19.9 to 82.3 °C. The accuracy and precision of the PT100 sensing signal processing unit are 0.5 and 0.1 \degree C, respectively. To achieve the desired temperature resolution with $0.1 \degree C$, the analog-todigital (A/D) converter of the controller requires at least 10-bit resolution. Aside from those, the isolated circuit must be inserted between the D/A converter and the controller in order to prevent the control signal from the noise caused by the compressor, the induction motor driver and the AC power source. This temperature controller generates appropriate nonnegative control signals from 0 VDC to 5 VDC via the digital-to-analog (D/A) converter, thus adjusting the cooling capability of the compressor. The proposed real-time control algorithm with the system sampling period of 10s was easily implemented on the DSP-based controller using standard C programming techniques and a C compiler. The process time delay was experimentally determined by $d = 4$.

4.2. Experimentation and discussion

The objective of the DSP-based control experiments for the oil-cooling process is to examine whether if the system response tracks the reference trajectory, and to check the efficacy of the

Fig. 4. (a) Experimental setup. (b) Block diagram of the oil-cooling control system with the proposed real-time SMRPC algorithm.

proposed method. The following three experiments were conducted to investigate the proposed method by tracking the desired model response with the following set-point trajectory:

 $u_c(k) =$ 23.0 °C, $0 \le k < 100$ 22.0 °C, $100 \le k < 200$ $21.0 °C$, $200 \le k < 300$ 20.0 °C, $300 \le k < 400$ $\overline{6}$ $\Big\}$ \vert

and the parameters of the reference model chosen as

$$
H_P = 5, \quad H_u = 1, \quad \lambda_w(0) = 0.001, \quad d_1 = -1.715
$$

$$
d_2 = 0.733, \quad n_0 = 0.018, \quad \lambda = 0.999
$$

The first experiment was conducted to examine whether the process output exactly follows the desired reference trajectory with no heat load. The set-point tracking response and the tracking errors are displayed in Figs. 5 and 6, respectively. The result in [Fig. 6](#page-7-0) indicates the tracking errors between the process output and desired model output almost remained within $+0.3$ °C. This result confirms that the proposed DSP-based controller works well under the condition of set-point changes with no load.

The second experiment was conducted to ensure the disturbance rejection capability of the proposed control in the presence of the heat load of 500 W applied at time instant $k = 0$. [Fig. 7](#page-7-0) illustrates the set-point tracking response under the heat

Fig. 5. DSP-based temperature tracking response under no load.

load. [Fig. 8](#page-7-0) depicts the tracking errors between the reference model and system response. The result in [Fig. 8](#page-7-0) shows that the proposed controller performs well for such a heat load.

Fig. 6. Temperature tracking errors under no load.

Fig. 7. DSP-based temperature tracking response under the fixed heat load of 500W.

The third experiment was to verify adaptive tracking and disturbance rejection abilities by changing the heat load from 500 to 1000W. The experimental result tracking response is depicted in Fig. 9, and the tracking errors of this experiment are presented in Fig. 10. The experimental results show that the weighted stochastic adaptive predictive controller is proven capable of giving satisfactory tracking and regulation performance under conditions of both set-point changes and load changes.

5. Conclusions

The paper has presented a stochastic model reference adaptive predictive temperature control with control weighting and integral action for an industrial oil-cooling process with VF driving. By approximating the process as a simple first-order model with time delay and including the least-squares parameter estimation method, the proposed control method has not only shown its adaptive set-point tracking ability to successfully meet the system specifications, but also provided a more pragmatic and useful real-time control algorithm for achieving high-precision

Fig. 8. Temperature tracking errors under the fixed heat load of 500 W.

Fig. 9. DSP-based temperature tracking response when the heat load was changed from 500 to 1000W after the 150th step.

Fig. 10. Temperature tracking errors when the heat load was changed from 500 to 1000W after the 150th step.

temperature tracking. Moreover, the proposed method can be extended to more general higher-order systems with time delay. The practical real-time control algorithm has been proposed and successfully implemented into a DSP-based controller. Through experimental results, the proposed control method has been shown to perform well under the set-point and load changes, as long as the system remains asymptotically stable. Due to the experience of the experiments in the industrial oil-cooling processes, it reminds the authors an important direction for future work that is to consider the simultaneous accurate temperature control and energy saving problem for developing a new structure of temperature controller.

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