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ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 8 (5): 743-749, 2009
ISSN 1812-5638

Robust Controller Design for Synchronization of Two Chaotic Circuits

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Abstract: This study present a robust algorithm to synchronize, under the master/slave configuration, a class of piecewise linear chaotic circuits based on sliding mode control. The synchronization objective is to obtain identical synchronization between the master and slave systems in spite of the existence of external disturbances and structural variations. A switching surface is adopted such that it becomes easy to ensure the stability of the error dynamics in the sliding mode. Then a Sliding Mode Controller (SMC) is derived to guarantee the occurrence of the sliding motion, even when the system is undergoing external disturbance and structural variations. This controller renders the closed loop system robust with respect to matched bounded disturbances and to terms produced by structural variations. The advantages of this method can be summarized as: (1) it is a systematic procedure for chaos suppression, (2) it can be applied to a variety of chaotic systems whether it contains uncertainties or not, (3) this controller is robust to external disturbance and (4) there is no chattering in controller, so it is easy to implement. Numerical simulations have verified the effectiveness of this method.

Key words: Sliding mode control, synchronization, chaos, sprott circuits

INTRODUCTION

In the recent years, chaos synchronization problems have attracted increasing attention since the pioneering work of Carroll and Pecora (1990). Synchronization can be defined as a phenomenon where two or more appropriately coupled systems undergo resembling evolution in time. Chaos synchronization can be applied in the vast areas of physics and engineering systems; for example, many techniques of controlled synchronization have been applied to synchronize chaotic circuits to develop private communication systems. In this application, the objective is to encode or encrypt information through a chaotic signal that will be sent to a receiver, where a chaotic system is synchronized to re-create the information. Many deep theories and control methods have been developed to achieve chaos synchronization. For example, adaptive control (Liao and Tsai, 2000; Feki, 2003), variable structure control (Zhang *et al*., 2004; Yau, 2004), optimal control (Tian and Yu, 2000), digital redesign control (Guo *et al*., 2000), backstepping control (Zhang *et al*., 2004), fuzzy control (Yau and Shieh, 2008) etc.

On the other hand, simplicity is always a desirable characteristic to consider in a practical implementation. As shown by Sprott (2000), a practical chaos generator can be constructed with a very simple circuit where the nonlinear term is a piecewise linear function. At the same time, the piecewise linear nature of the system simplifies the analysis because in this case the nonlinear system can be reformulated as a variable structure system consisting of linear parts with a given switching logic. There are some proposals to synchronize this kind of chaotic systems, see for example (Bai *et al*., 2002), where two Sprott circuits are synchronized using a feedback linearization and the Open-Plus-Closed-Loop (OPCL) control techniques, respectively. In these works, the synchronization is achieved provided no external disturbances or structural perturbations are present. These conditions are not realistic in practice due to the tolerance in electronic components and devices; therefore, a robustness analysis to ensure stability and convergence to zero of the error dynamics must be performed.

The purpose of this study lies in the development of a SMC for synchronizing the state trajectories of two identical chaotic sprott circuits. A switching surface, which makes it easy to guarantee the stability of the error dynamics in the sliding mode, is first proposed. And then, based on this switching surface, a SMC is derived to guarantee the occurrence of the sliding motion, even when the system is undergoing external disturbance and

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system structural variations. Finally, we present the numerical simulation results to illustrate the effectiveness of the proposed control scheme. In theory, we can attain asymptotic identical synchronization in spite of the existence of this kind of disturbances. However, in practice, as a result of a discontinuous coupling signal, there will be a small chattering component in the synchronization errors. Nevertheless, in many applications this error may be negligible.

Throughout this study, it is noted that, |w| represents the absolute value of w and sign(s) is the sign function of s, if s > 0, sign(s) = 1; if s = 0, sign(s) = 0; if s<0, sign(s) = -1.

SYNCHRONIZATION PROBLEM FORMULATION

In this study, the Sprott circuits (Sprott, 2000) are defines by:

$$\dddot{x} + a\ddot{x} + \dot{x} = f_i(x) \tag{1}$$

where, (•) means time derivatives of the variable x and $f_i(x)$ can take one of the following forms:

$$\begin{aligned} &f_1(x) = |x| - 2, \\ &f_2(x) = -6 \cdot \max(x, 0) + 0.5, \\ &f_3(x) = 1.2x - 4.5 \cdot \mathrm{sign}(x), \\ &f_4(x) = -1.2x + 2 \cdot \mathrm{sign}(x). \end{aligned} \tag{2}$$

Let, the system states x_1, x, $x_2 = \dot{x}$ and $x_3 = \ddot{x}$, the system Eq. 1 can be transformed to a nominal form as follows:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = x_3 \\ &\dot{x}_3 = -x_2 - ax_3 + f_i(x_1) \end{aligned} \tag{3}$$

If $f_4(x)$ is selected and the value of a is 0.6, then the Sprott system exists chaotic behavior (Sprott, 2000) with initial conditions of $[x_1(0), x_2(0), x_3(0)] = [1, 0, 2]$. The complex time responses and phase plane trajectory are shown in Fig. 1a-d.

Consider two coupled chaotic circuits as follows: Master circuit system:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = x_3 \\ &\dot{x}_3 = -x_2 - ax_3 + f_i(x_1) \end{aligned} \tag{4}$$

Slave circuit system:

$$\begin{aligned} &\dot{y}_1 = y_2 \\ &\dot{y}_2 = y_3 \\ &\dot{y}_3 = -y_2 - ay_3 + d(t) + \Delta\xi(y_1) + f_i(y_1) + u \end{aligned} \tag{5}$$

where, d(t) is a external disturbances and $\Delta\xi(y_1)$ is the structural variation of $f_i(y_1)$. In general, the uncertain term $\Delta\xi$ and the disturbance term are assumed bounded, i.e.:

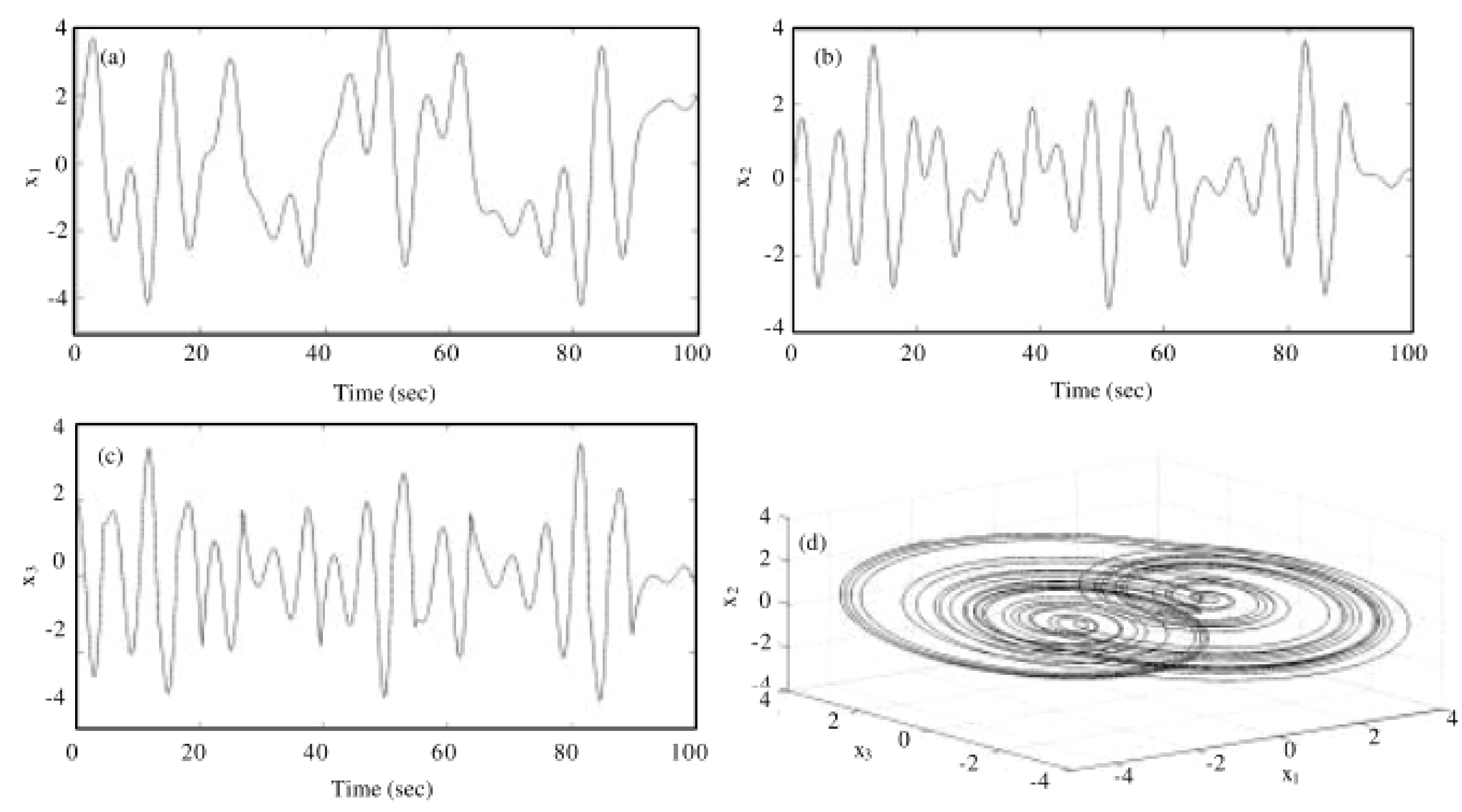

Fig. 1: (a-d) The time responses and chaotic attractor of the master Sprott circuit

$$|\Delta\xi(y_1)| < \alpha \text{ and } |d(t)| < \beta \tag{6}$$

where, α, β are positive.

In the following, we will consider the synchronization of two identical Sprott systems and give an explicit and simple procedure to establish a robust sliding mode controller to cope with the external disturbance and structural variations appearing in the slave system such that:

$$\lim_{t\to\infty}\|x(t) - y(t)\| \to 0 \tag{7}$$

where, $\|\cdot\|$ is the Euclidean norm of a vector.

SLIDING MODE CONTROLLER DESIGN

Let us define the synchronization errors between the master system Eq. 4 and the slave system Eq. 5 as follows:

$$e_1 = y_1 - x_1,\ e_2 = y_2 - x_2,\ e_3 = y_3 - x_3 \tag{8}$$

then the dynamics of the error system is determined, directly from subtracting Eq. 4 from 5, as follows:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -e_2 - ae_3 + f_i(y_1) - f_i(x_1) + d(t) + \Delta\xi(y_1) + u \end{aligned} \tag{9}$$

The considered goal of this study is that for any given chaotic circuits as Eq. 4 and 5, a SMC is designed in spite of the external disturbance and structural variations, such that the asymptotical stability of the resulting error system Eq. 9 can be achieved in the sense that:

$$\|e(t)\| \to 0 \text{ as } t \to \infty \tag{10}$$

where, $e(t) = [e_1, e_2, e_3]$.

As a sequence, using the sliding mode control method to control the chaotic circuits involves two basic steps: (1) selecting an appropriate switching surface such that the sliding motion on the sliding mode is stable and ensures $\lim_{t\to\infty}\|e(t)\| = 0$ and (2) establishing a robust control law which guarantees the existence of the sliding mode $s(t) = 0$.

To ensure the asymptotical stability of the sliding mode, a switching surface s(t) in the error space is defined as follows:

$$s(t) = e_3(t) + c_2e_2(t) + c_1e_1(t) = e_3 + KE(t) \tag{11}$$

where, $s(t) \in R$, $E = [e_1\ e_2]^T \in R^{2\times1}$ and $K = [c_1\ c_2] \in R^{1\times2}$ is a design parameter vector which can be easily determined later. For the existence of the sliding mode, it is necessary and sufficient that:

$$s(t) = e_3 + KE = 0 \tag{12}$$

and

$$\dot{s}(t) = \dot{e}_3 + K\dot{E} = 0 \tag{13}$$

Therefore, by Eq. 9 and 12, the following sliding mode dynamics can be obtained in a matrix form of:

$$\dot{E} = (A - BK)E = \hat{A}E \tag{14}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

It is so easy to check that $\text{Rank}[B \vdots AB] = 2$. Therefore, (A, B) is controllable which means that there always exists a parameter vector K such that the maximum real part eigenvalue of (A-AK) is negative, that is, $\text{Max}[\text{Re}(\lambda(\hat{A}))] < 0$. Furthermore, we can easily assign the system performance in the sliding mode just by selecting an appropriate matrix K using any pole assignment method. Meanwhile, it is worthy of note that these eigenvalues of matrix $\hat{A}$ are also relative to the speed of system response.

Having established an appropriate switching surface, the next step is to design a SMC scheme to drive the error system trajectories onto the switching surface $s(t) = 0$. Before stating the scheme of the controller, the hitting condition of the sliding mode is given below (Slotine and Li, 1991).

Lemma 1: The motion of the sliding mode Eq. 11 is asymptotically stable, if the following hitting condition is held:

$$s(t)\dot{s}(t) < 0 \tag{15}$$

Proof: Let $V(t) = 0.5s^2(t)$ be the Lyapunov function. According to Lyapunov stability theory, condition Eq. 11 ensures that:

$$\dot{V}(t) = s(t)\dot{s}(t) < 0 \tag{16}$$

Then, s(t) is toward the sliding surface and the sliding mode Eq. 11 is asymptotically stable.

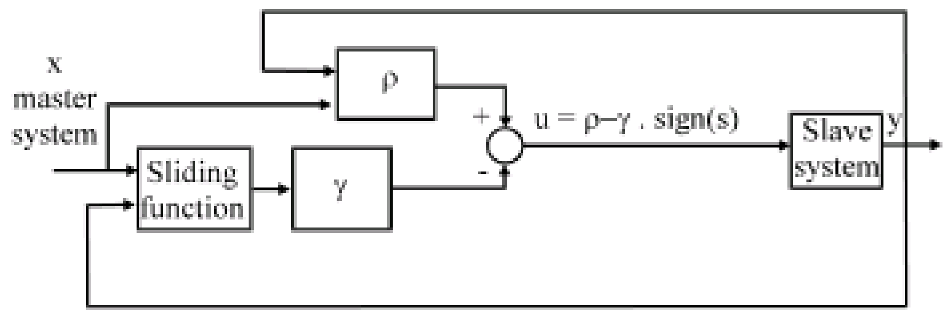

Fig. 2: Diagram of the SMC scheme

The current SMC design is stated in the following theorem.

Theorem 1: For the controlled Sprott circuit given in Eq. 5, if the control input u(t) is suitably designed as:

$$u(t) = \rho(x_1, y_1, e_2, e_3) - r(\alpha+\beta)\cdot \text{sign}(s(t)); r > 1 \tag{17}$$

where, $\rho = -(c_1 - 1)e_2 - (c_2 - a)e_3 - f_1(y_1) + f_1(x_1)$, then the hitting condition Eq. 15 of the sliding mode is satisfied, i.e., the trajectory of the error dynamics system converges to the switching surface s(t).

Remark: The controllers in Eq. 17 demonstrate discontinuous control laws and the phenomenon of chattering will appear. In order to eliminate the chattering, the controller is modified as:

$$u(t) = \rho(x_1, y_1, e_2, e_3) - r(\alpha+\beta)\cdot \frac{s}{|s| + \delta}; r > 1$$

where, δ is a sufficiently small design constant. Therefore, the controllers can be implemented in real word system and the SMC control scheme is shown in Fig. 2. We further to show that the sliding condition can be achieved in the following theorem.

Proof: Substituting Eq. 9, 11 and 17 into the derivative $s(t)\dot{s}(t)$, we get the following result:

$$\begin{aligned} \dot{V} = s\cdot\dot{s} &= s\left[\dot{e}_3 + c_2\dot{e}_2 + c_1\dot{e}_1\right] \\ &= s\left[-e_2 - ae_3 + f_1(y_1) - f_1(x_1) + d(t) + \Delta\xi(y_1) + u + c_2e_3 + c_1e_2\right] \\ &= s\left[(c_1 - 1)e_2 + (c_2 - a)e_3 + f_1(y_1) - f_1(x_1) + d(t) + \Delta\xi(y_1) + \rho - r(\alpha+\beta)\cdot \text{sign}(s)\right] \\ &\le |s|\cdot(|d(t)| + |\Delta\xi(y_1)|) - r(\alpha+\beta)\cdot|s| \\ &\le (1 - r)\cdot(\alpha+\beta)\cdot|s| \end{aligned} \tag{18}$$

Since, α and β are positive and r>1 has been specified in Eq. 17, it can be concluded that $s\dot{s} < 0$. Furthermore, according to Lemma 1, s(t) will converge to zero finally. Hence, the proof is completed.

The following theorem is introduced to guarantee the stability of the closed-loop error system.

Theorem 2: The closed-loop error system Eq. 9 driven by the controller u(t) expressed in Eq. 17 is asymptotically stable in the large.

Proof: When the error system Eq. 9 is driven by the control input u(t) given in Eq. 17, the trajectory of the error dynamics system converges to the sliding mode s = 0, as previously discussed in Theorem 1. Thus, the equivalent error dynamics system in the sliding mode is obtained as shown in Eq. 14. As discussed previously, in Eq. 14, the values of $\text{Max}[\text{Re}(\lambda(\hat{A}))]<0$ is specified to guarantee the asymptotical stability of the error system. Consequently, the asymptotical stability of the closed-loop error system is also ensured. The theorem is therefore proved.

NUMERICAL SIMULATIONS

Here, simulation results are presented to demonstrate and verify the performance of the present design. The 4th order Runge-Kutta algorithm was used to obtain the numerical solutions of systems Eq. 4 and 5 with a time grid of 0.0001. The parameters a = 0.6 and $f_4(x)$ are chosen in the simulation to ensure the existence of chaos for the derive system Eq. 4. The initial states of the derive system Eq. 4 are $[x_1(0), x_2(0), x_3(0)] = [1\ 0\ 2]$ and initial states of the response system Eq. 5 are $[y_1(0), y_2(0), y_3(0)] = [-1\ 2\ 1]$. The external disturbance and structural variation term in response system Eq. 5 are defined as d(t) = 0.5 · cist and $\Delta\xi(y_1) = (0.1\times\sin t)$, respectively. Thus $|d(t)|\le 0.5 = \alpha$ can be obtained. The time responses and phase plane trajectory of slave system are shown in Fig. 3a-d. It can be seen that the absolute value of y_1 is less than 4, that is $|\Delta\xi(y_1)| = |(0.1\times\sin t)y_1| \le 0.4 = \beta$. As mentioned in sliding mode controller design, the proposed design procedure may be obtained as follows:

Step 1: According to Eq. 14, we select $K = [c_1, c_2] = [10, 10]$ to result in a stable sliding mode. Therefore, the switching surface equation is:

$$s(t) = e_3 + 10e_2 + 10e_1 \tag{19}$$

Step 2: According to Eq. 17, we select r = 2>1 to guarantee the existence of the sliding motion. Therefore, the control input is:

$$u(t) = \rho(x_1, y_1, e_2, e_3) - r(\alpha+\beta)\cdot \text{sign}(s(t)); r = 2 > 1 \tag{20}$$

In order to eliminate the chattering, the controller is modified as:

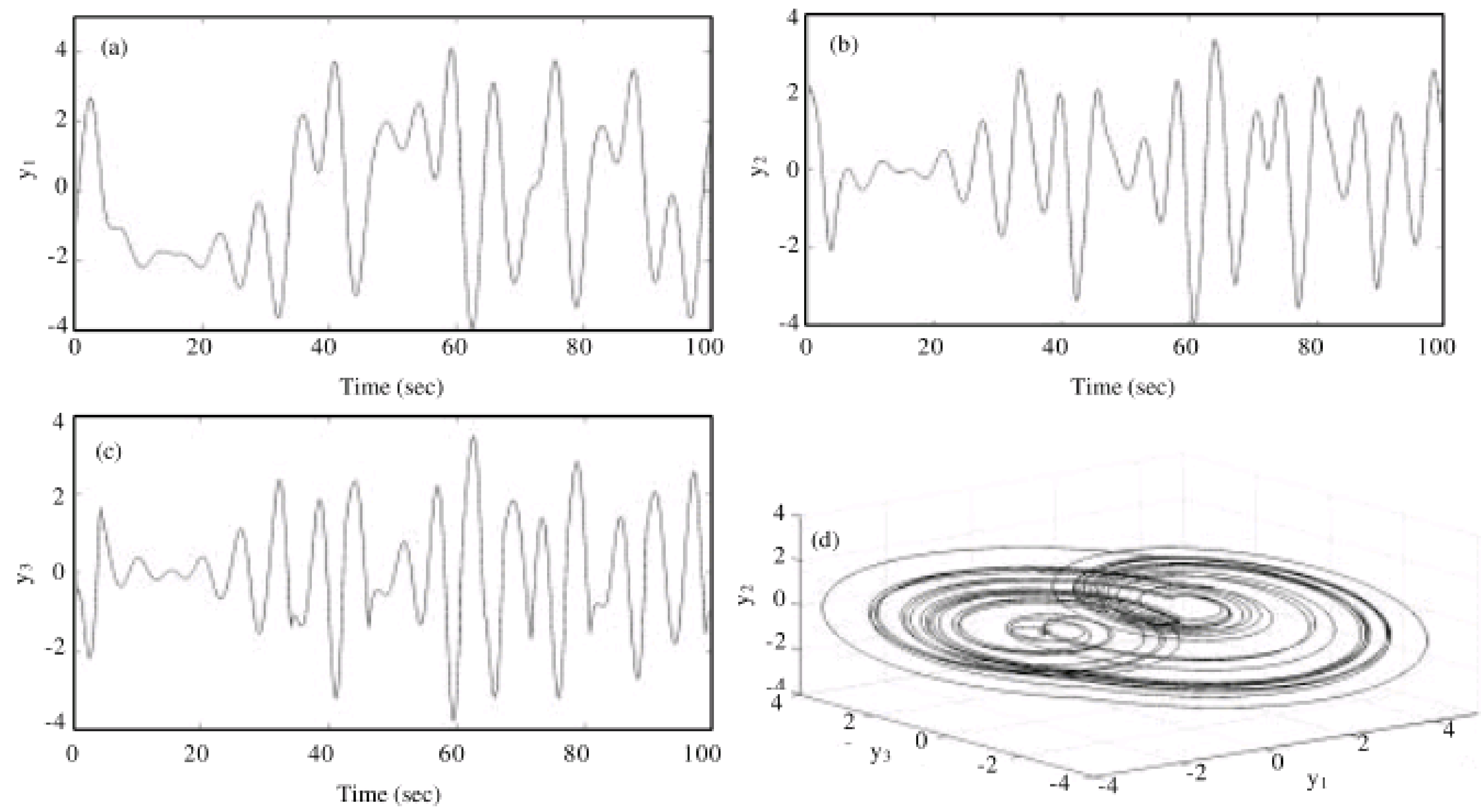

Fig. 3: (a-d) The time responses and chaotic attractor of the uncontrolled slave Sprott circuit (with u = 0)

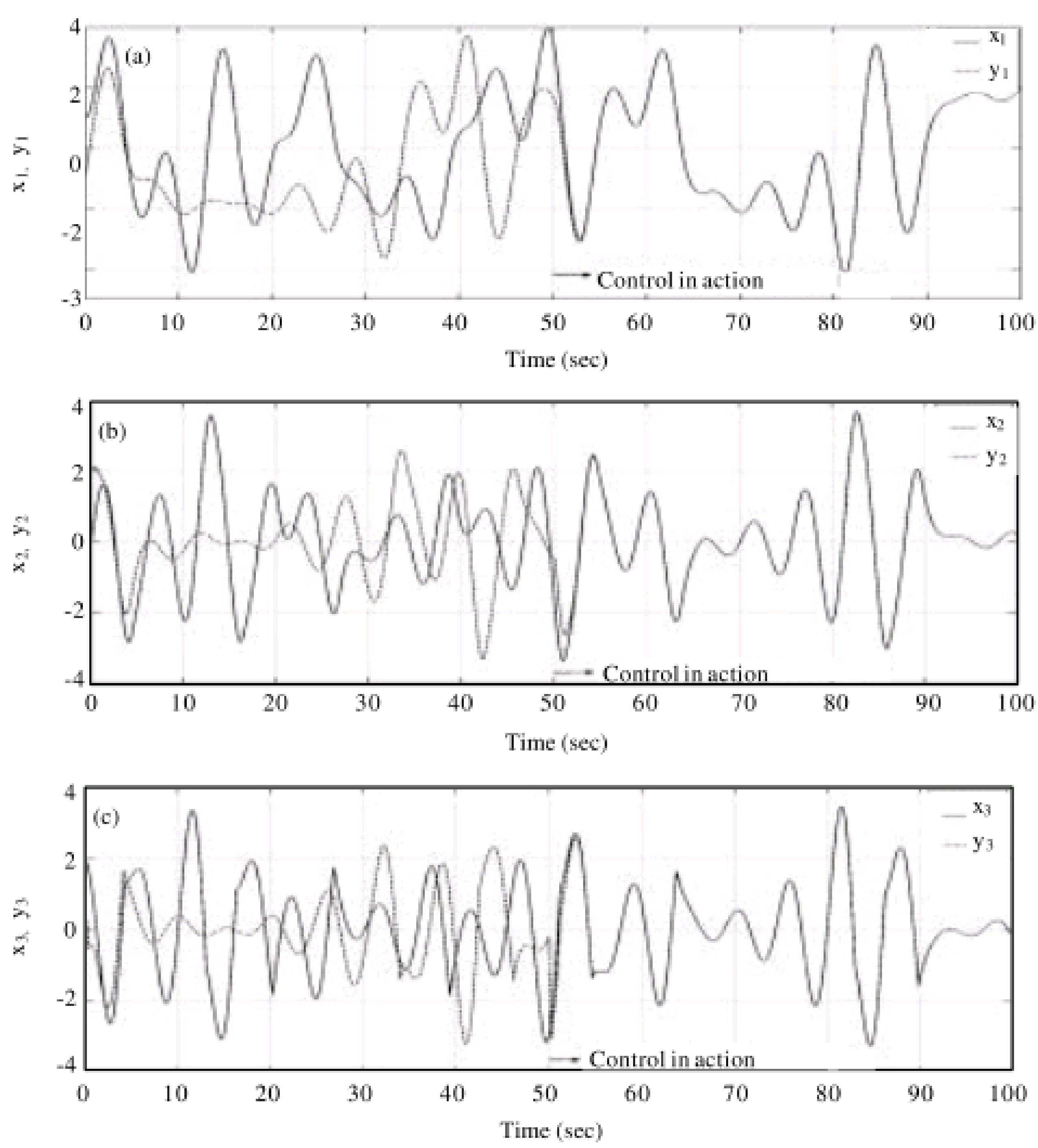

Fig. 4: The state responses of controlled Sprott circuits: master and slave system outputs are x_1, x_2, x_3 (solid) and y_1, y_2, y_3 (dashed), respectively. (a-c) Control u(t) is activated at t = 50 sec

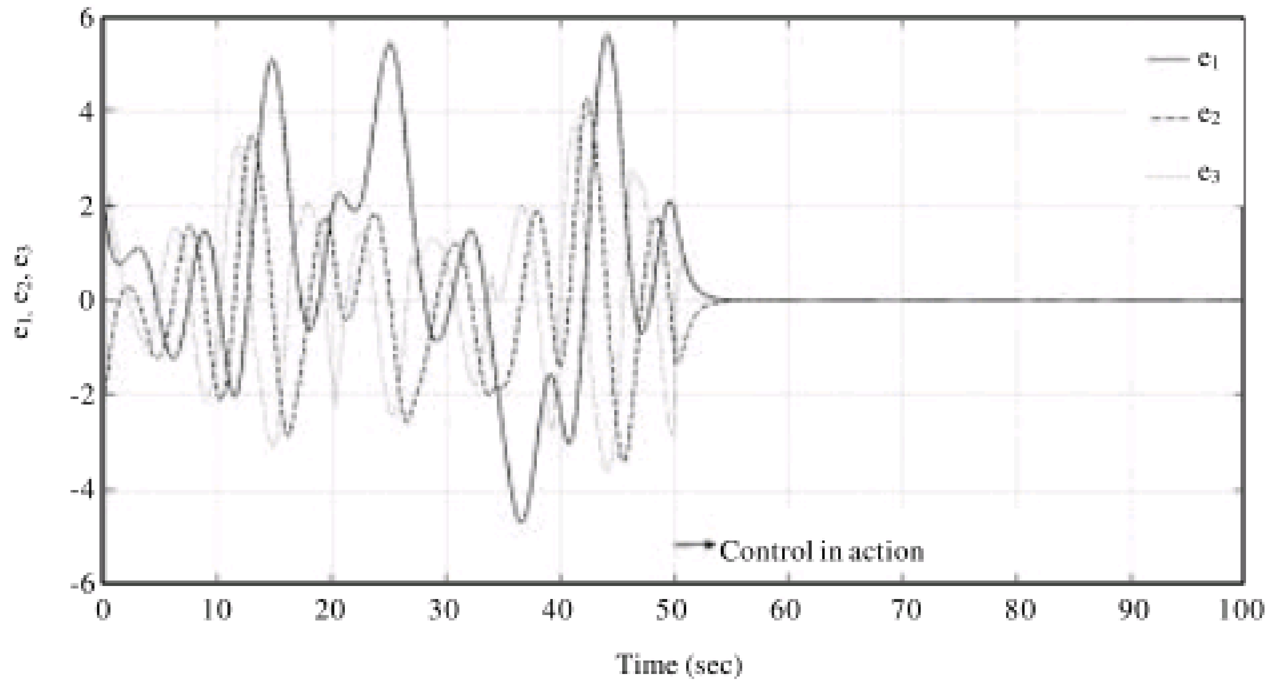

Fig. 5: Time responses of synchronization errors. Control u(t) is activated at t = 50 sec

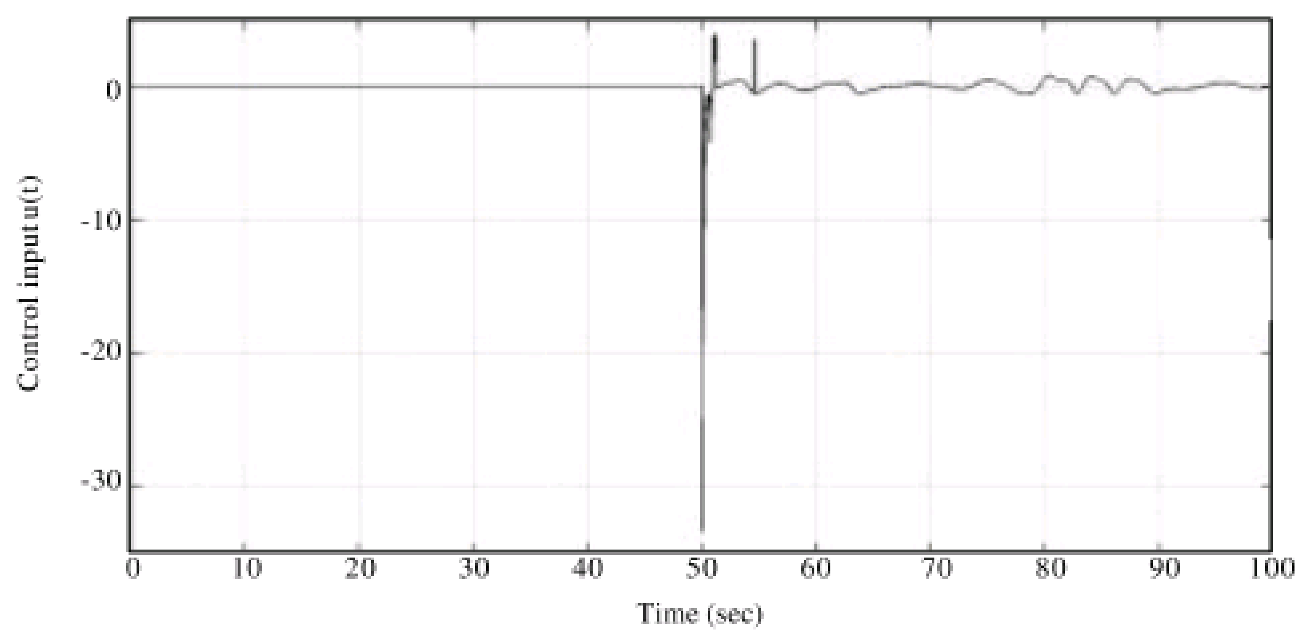

Fig. 6: Variation of control action over time. Control u(t) is activated at t = 50 sec

$$u(t) = \rho(x_1, y_1, e_2, e_3) - r(\alpha + \beta) \cdot \frac{s}{|s| + \delta}; r = 2 > 1,\ \delta = 0.05 \qquad (21)$$

The simulation results are shown in Fig. 4-6 under the proposed SMC Eq. 21. Figure 4a-c show the state responses for the controlled master-slave chaotic Sprott circuits. Figure 5 and 6 show, respectively, the error state time responses and control input of the controlled master-slave Sprott circuits. From the simulation result, it shows that the trajectories of master system states synchronize to slave system states and the synchronization error converges to zero after the control is activated. Thus, the proposed SMC works well and two chaotic Sprott circuits from different initial values are indeed achieving chaos synchronization even when external disturbance and structural variation are present.

CONCLUSION

In this study, we have proposed an algorithm to synchronize two piecewise linear chaotic systems called Sprott systems. The condition to apply this algorithm is that the perturbations satisfy the matching conditions. In theory, this algorithm guarantees a zero steady-state synchronization error; however, due to sign function in the control input signal, in practice the chattering in control force is impracticable to implement. In order to eliminate the chattering, the controller is modified to a continuous function. The simulation results show that the proposed nonlinear control enables stabilization of synchronization error dynamics to zeros asymptotically in spit of external disturbance and structural variation. This method can also be easily extended to a general class of chaotic systems.

ACKNOWLEDGMENT

The financial support of this research by the National Science Council of the ROC, under Grant No. NSC 96-2221-E-269-010-MY2 is greatly appreciated.

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