



Nonlinear analysis and control of the uncertain micro-electro-mechanical system by using a fuzzy sliding mode control design

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ABSTRACT

This study analyzes the chaotic behavior of a micromechanical resonator with electrostatic forces on both sides and investigates the control of chaos. A phase portrait, maximum Lyapunov exponent and bifurcation diagram are used to find the chaotic dynamics of this micro-electro-mechanical system (MEMS). To suppress chaotic motion, a robust fuzzy sliding mode controller (FSMC) is designed to turn the chaotic motion into a periodic motion even when the MEMS has system uncertainties.

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1. Introduction

Nonlinearities exist ubiquitously in micro-electro-mechanical systems (MEMS). Examples include nonlinear springs and damping mechanisms [1], nonlinear resistive, inductive and capacitive circuit elements [2] and nonlinear surface, fluid, electric and magnetic forces [3]. Many researches have been conducted on various nonlinear dynamic phenomena, including bending of the frequency response curve and the jump phenomenon in MEMS resonators [4]. Nonlinearities may also cause chaotic behavior [5]. Modeling [6] has been used to predict the existence of chaotic motion in electrostatic MEMS. In one study [7], the chaotic motion of MEMS resonant systems close to the specific resonant separatrix was investigated under the corresponding resonant condition. An optimal linear feedback control strategy has been adopted [8] to reduce the chaotic motion of the system proposed in the former study [7] to a stable orbit. In a later investigation [9], the chaotic behavior of a micro-electro-mechanical oscillator was modeled by a version of the Mathieu equation and was studied both numerically and experimentally. Chaotic motion of a micro-electro-mechanical cantilever beam under both open and close loop control has also been reported [10].

This study develops a fuzzy sliding mode control (FSMC) scheme [11–13] that is designed to control chaos in a MEMS with system uncertainties. Firstly, the switching surface that is required to achieve chaos control is specified, and then a switching control law based on fuzzy linguistic rules is developed to generate a suitable chatter-free control signal for driving the error dynamic system such that the error state trajectories converge asymptotically to zero.

2. System description

Fig. 1 presents the electrostatically actuated micro-beam, where d is the initial width of the gap and z is the vertical displacement of the beam. An external driving force is applied as an electrical driving voltage on the resonator that causes

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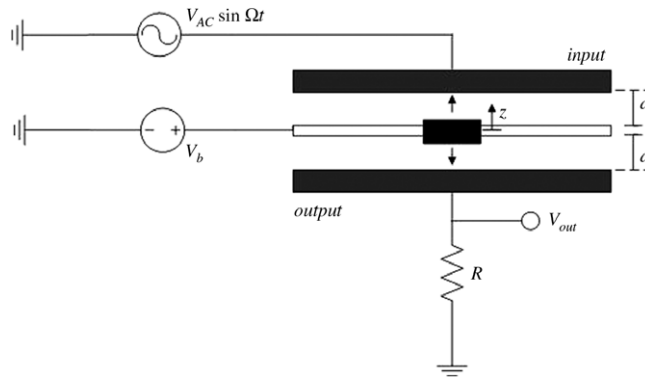


Fig. 1. A schematic diagram of the electrostatically actuated micromechanical resonator.

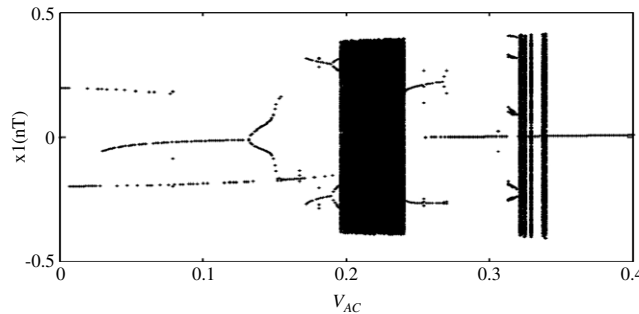


Fig. 2. The bifurcation diagram obtained by varying AC voltage V_{AC} from 0 to 0.4 V.

electrostatic excitation with a dc bias voltage between the electrodes and the resonator: $V_i = V_b + V_{AC} \cdot \sin \Omega t$, where V_b is the bias voltage and V_{AC} and Ω are the AC amplitude and frequency, respectively. The amplitude of the AC driving voltage is assumed to be much lower than the bias voltage, yielding the nondimensional equation of motion [14]:

$$\ddot{x} + \mu\dot{x} + \alpha x + \beta x^3 = \gamma \left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2} \right) + \frac{A}{(1-x)^2} \sin \omega \tau, \tag{1}$$

where the nondimensional variables x and ω are defined as

$$x = \frac{z}{d}, \quad \omega = \frac{\Omega}{\omega_0}, \quad A = 2\gamma \frac{V_{AC}}{V_b},$$

where ω_0 is the purely elastic natural frequency. Given the states $x_1 = x, x_2 = \dot{x}$ and $g(x) = \gamma \left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2} \right)$, this system can be transformed into the following nominal form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha x_1 - \beta x_1^3 - \mu x_2 + g(x_1) + \frac{A}{(1-x_1)^2} \sin \omega \tau. \end{cases} \tag{2}$$

This MEMS (2) exhibits complex dynamics and has been studied by Haghghi and Markazi [14] for values of V_{AC} in the range $0 < V_{AC} < 0.47$ and constant values of $\alpha = 1, \beta = 12, \gamma = 0.338, \mu = 0.01, V_b = 3.8$ and $\omega = 0.5$. Fig. 2 displays its bifurcation diagram. In this case, the qualitative behavior of the system is shown against a varying AC voltage from 0 to 0.4. When the AC voltage is increased from zero, periodic motion occurs around one of the center points. Fig. 3 presents the irregular motion that is exhibited by this system at $V_{AC} = 0.2$ V under initial conditions of $(x_1, x_2) = (0, 0)$. Fig. 3(b) reveals that the corresponding maximum Lyapunov exponent has a positive value, and so the MEMS trajectory is inferred to be in a state of chaotic motion at $V_{AC} = 0.2$ V. The following section examines the problem of the suppression of chaos of MEMS and introduces the FSMC to cope with this chaotic motion.

3. Robust fuzzy sliding mode control

Consider a chaotic MEMS of the form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha x_1 - \beta x_1^3 - \mu x_2 + g(x_1) + \frac{A}{(1-x_1)^2} \sin \omega \tau + \Delta f(x_1, x_2) + u, \end{cases} \tag{3}$$

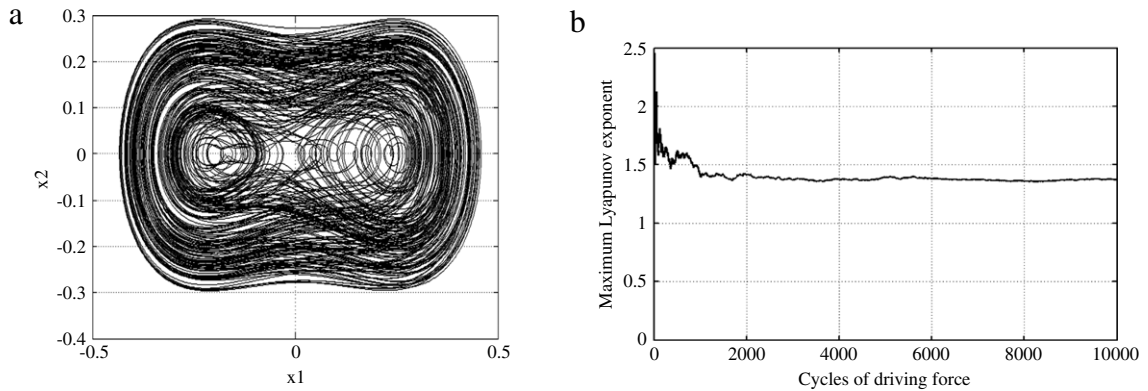


Fig. 3. Phase plane trajectory (a) and maximum Lyapunov exponent (b) of chaotic MEMS with $V_{AC} = 0.2$.

where $u \in R$ is the control input, and $\Delta f(y_1, y_2)$ is an uncertainty term that represents the unmodeled dynamics or structural variation of the system that is given by Eq. (3). Generally, the uncertainty is assumed to be bounded as follows:

$$|\Delta f(y)| \leq \rho,$$

where ρ is a positive constant.

If the tracking error states of the controlled systems are defined as $e_1 = x_1 - x_r$ and $e_2 = x_2 - \dot{x}_r$, then the dynamic equations of these errors can be obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -\alpha(e_1 + x_r) - \beta(e_1 + x_r)^3 - \mu(e_2 + \dot{x}_r) + g(e_1 + x_r) + \frac{A}{(1 - e_1 - x_r)^2} \sin \omega\tau + \Delta f(e_1 + x_r, e_2 + \dot{x}_r) + u, \end{cases} \quad (4)$$

where x_r is the desired trajectory and \dot{x}_r is the first derivative of x_r with respect to time. In the SMC field, the sliding surface is generally taken to be

$$s = e_2 + \lambda e_1, \quad (5)$$

where λ represents a real number. The existence of the sliding mode requires the following conditions to be satisfied [15]:

$$s = e_2 + \lambda e_1 = 0, \quad (6a)$$

and

$$\dot{s} = \dot{e}_2 + \lambda \dot{e}_1 = 0. \quad (6b)$$

Therefore, the equivalent control law is given by

$$u_{eq} = -\lambda e_2 + \alpha(e_1 + x_r) + \beta(e_1 + x_r)^3 + \mu(e_2 + \dot{x}_r) - g(e_1 + x_r) - \frac{A}{(1 - e_1 - x_r)^2} \sin \omega\tau - \Delta f(e_1 + x_r, e_2 + \dot{x}_r). \quad (7)$$

In the sliding mode, the error dynamics become

$$\dot{e}_1 = e_2 = -\lambda e_1, \quad (8a)$$

$$\dot{e}_2 = -\lambda e_2. \quad (8b)$$

If the parameter λ is assigned a positive value, then the stability of Eq. (8a) is assured, and $(e_1, e_2) \rightarrow 0$ as $t \rightarrow \infty$. Restated, the chaotic MEMS is asymptotically stabilized to a desired trajectory x_r . Notably, the rate of convergence to the sliding surface is governed by the value assigned to parameter λ .

Eq. (7) defines the output of the equivalent controller, and while the reaching law is given by

$$u_r = k_{fs} u_{fs}, \quad (9)$$

where k_{fs} is a normalization factor of the output variable and u_{fs} is the output of the FSMC [11], and is determined in accordance with the normalized outputs of the SMC, s and \dot{s} . Hence, the overall control signal, u , has the form

$$u = u_{eq} + u_r = u_{eq} + k_{fs} u_{fs}. \quad (10)$$

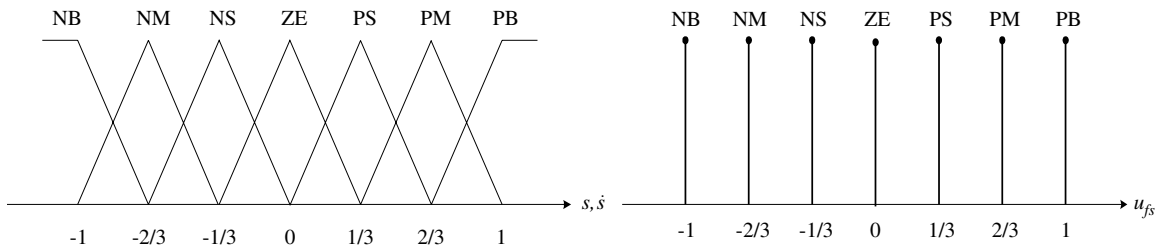


Fig. 4. Membership functions of the input–output variables for FSMC: (a) Membership function of s and \dot{s} ; (b) Membership function of u_{fs} .

Table 1
FSMC rule table.

u_{fs}		s						
		PB	PM	PS	ZE	NS	NM	NB
\dot{s}	PB	NB	NB	NB	NB	NM	NS	ZE
	PM	NB	NB	NB	NM	NS	ZE	PS
	PS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	NS	NM	NS	ZE	PS	PB	PB	PB
	NM	NS	ZE	PS	PB	PB	PB	PB
	NB	ZE	PS	PM	PB	PB	PB	PB

The fuzzy control rules are represented by the mapping of the input linguistic variables s and \dot{s} to an output linguistic variable u_{fs} :

$$u_{fs} = \text{FSMC}_1(s, \dot{s}), \tag{11}$$

where $\text{FSMC}_1(\cdot, \cdot)$ denotes the functional characteristics of the fuzzy linguistic decision scheme. Fig. 4(a) and (b) show the membership functions of the input linguistic variables (s and \dot{s}) and the output linguistic variable (u_{fs}), respectively. Table 1 is the corresponding fuzzy rule table [11].

In real-world applications, the system uncertainties $\Delta f(e_1 + x_1, e_2 + x_2)$ are unknown. The equivalent control input is therefore modified to

$$u_{eq} = -\lambda e_2 + \alpha(e_1 + x_r) + \beta(e_1 + x_r)^3 + \mu(e_2 + \dot{x}_r) - g(e_1 + x_r) - \frac{A}{(1 - e_1 - x_r)^2} \sin \omega \tau, \tag{12}$$

while the overall control input becomes

$$\begin{aligned} u &= u_{eq} + k_{fs} u_{fs} \\ &= -\lambda e_2 + \alpha(e_1 + x_r) + \beta(e_1 + x_r)^3 + \mu(e_2 + \dot{x}_r) - g(e_1 + x_r) - \frac{A}{(1 - e_1 - x_r)^2} \sin \omega \tau + k_{fs} u_{fs}. \end{aligned} \tag{13}$$

Let the Lyapunov function of the system be defined as $V = \frac{1}{2}s^2$. The first derivative of this system with respect to time can be expressed as

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s \cdot [\dot{e}_2 + \lambda \dot{e}_1] \\ &= s \cdot \left[-\alpha(e_1 + x_r) - \beta(e_1 + x_r)^3 - \mu(e_2 + \dot{x}_r) + g(e_1 + x_r) + \frac{A}{(1 - e_1 - x_r)^2} \sin \omega \tau \right. \\ &\quad \left. + \Delta f(e_1 + x_r, e_2 + \dot{x}_r) + u_{eq} + k_{fs} u_{fs} + \lambda e_2 \right] \\ &= s \cdot [\Delta f(e_1 + x_r, e_2 + \dot{x}_r) + k_{fs} u_{fs}] \\ &\leq |s| \rho - k_{fs} |s| = -(k_{fs} - \rho) |s|. \end{aligned} \tag{14}$$

If $k_{fs} > \rho$ is selected, then the reaching condition ($s\dot{s} < 0$ [15]) is always satisfied. Therefore, the uncertain MEMS (3) can be stabilized to a desired trajectory x_r .

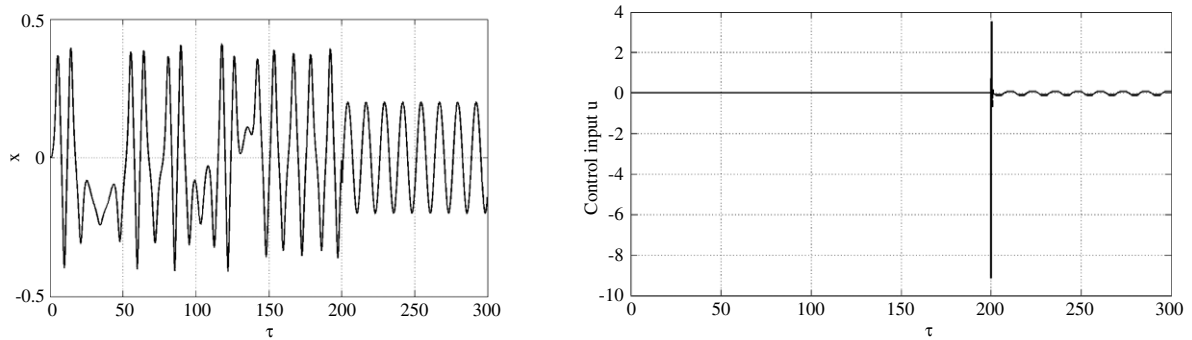


Fig. 5. Time responses of controlled MEMS system and variation of control action over nondimensional time τ . Note that control u is activated at $\tau = 200$.

4. Simulation results

This section describes a numerical simulation to demonstrate the feasibility and effectiveness of the proposed FSMC scheme in controlling the chaotic MEMS given by Eq. (3). In the solution procedure, Eq. (3) is solved using the 4th order Runge–Kutta algorithm with a time-step of 0.001. The parameters of the MEMS are $\alpha = 1$, $\beta = 12$, $\gamma = 0.338$, $\mu = 0.01$, $V_b = 3.8$, $V_{AC} = 0.2$ and $\omega = 0.5$, which, as shown in Section 2, give a rise to a chaotic state. The initial conditions are defined as $x_1(0) = 0$ and $x_2(0) = 0$. The goal is to control the position state x to track the desired trajectory $x_r = 0.2 \sin(0.5\tau)$. The uncertainty term $\Delta f(x_1, x_2) = -0.05 \sin(x_1)$, is assumed to be bounded by $|\Delta f(y_1, y_2)| \leq \rho = 0.05$. Consistent with Eq. (5) $\lambda = 2$ is selected to ensure a stable sliding mode, while $k_{fs} = 1$ is selected to satisfy the condition prescribed by Eq. (14), $k_{fs} > \rho$.

Fig. 5 presents the simulation results. It confirms that the chaotic MEMS can achieve a periodic state following activation of the control signal at $\tau = 200$. Additionally, the control input is chatter-free even though the overall system is subject to uncertainty.

5. Conclusion

This study discusses the chaotic motion of micromechanical resonators with electrostatic forces on both sides, and it shows that such a system will exhibit a complex behavior. Bifurcation that corresponds to transient chaotic behavior of the system and further increases of the AC voltage amplitude can lead to persistent chaotic motion. A fuzzy sliding mode control (FSMC) scheme is used to stabilize the chaotic motion. The simulation results verify the ability of the FSMC approach to control the chaotic MEMS through the application of a single control signal.

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