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Particle swarm optimization with justification and designed mechanisms for resource-constrained project scheduling problem

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ABSTRACT

The studied resource-constrained project scheduling problem (RCPSP) is a classical well-known problem which involves resource, precedence, and temporal constraints and has been applied to many applications. However, the RCPSP is confirmed to be an *NP-hard* combinatorial problem. Restated, it is hard to be solved in a reasonable time. Therefore, there are many metaheuristics-based schemes for finding near optima of RCPSP were proposed. The particle swarm optimization (PSO) is one of the metaheuristics, and has been verified being an efficient nature-inspired algorithm for many optimization problems. For enhancing the PSO efficiency in solving RCPSP, an effective scheme is suggested. The justification technique is combined with PSO as the proposed justification particle swarm optimization (JPSO), which includes other designed mechanisms. The justification technique adjusts the start time of each activity of the yielded schedule to further shorten the makespan. Moreover, schedules are generated by both forward scheduling particle swarm and backward scheduling particle swarm in this work. Additionally, a mapping scheme and a modified communication mechanism among particles with a designed *gbest ratio* (GR) are also proposed to further improve the efficiency of the proposed JPSO. Simulation results demonstrate that the proposed JPSO provides an effective and efficient approach for solving RCPSP.

1. Introduction

Many applications involve scheduling notion, such as generating units planning of power plants (Saksornchai, Lee, Methaprayoon, Liao, & Ross, 2005), grid computing (Hou, Zhou, & Wang, 2006; Liu, Yang, Shi, Lin, & Li, 2005), control system (Park, Kim, Kim, & Kwon, 2002), food industrial (Simonov & Simonovov'a, 2002), network packet switching (Symington, Waddie, Taghizadeh, & Snowdon, 2003), classroom arrangement (Vejzovic & Humo, 2007) and manpower scheduling (Ohki, Morimoto, & Miyake, 2008). Generally, these problems commonly accompany the cost considerations related to certain constraints. A scheduling algorithm determines a schedule for a set of processes, satisfying the prerequisite constraints and minimizing cost. Scheduling problems differ markedly from case to case. One of the well studied scheduling problems is the resource-constrained project scheduling problem (RCPSP) (Hartmann, 2002); a variety of applications are part of RCPSP. RCPSP is a combinatorial optimization problem to schedule the activities such that the makespan (total completion time) of the schedule can be minimized, while satisfying given precedence constraint between the activities and resource constraint. The resource requirements of the scheduled activities per time unit do not exceed the given capacity limit of different types resources. However, the minimum makespan is hard to obtain since the inestimable situation of constraints. And RCPSP has been confirmed to be an *NP-hard* combinatorial problem (Blazewicz, Lenstra, & Rinooy Kan, 1983); it is hard to solve RCPSP in a reasonable time especially for large-scale scheduling problems. Restated, solving RCPSP requires considerable computation times for large instances.

Although there are some exactly algorithms such as branch-and-bound method (Brucker, Knust, Schoo, & Thiele, 1998; Jalilvand et al., 2005) is able to find optimal solutions of RCPSP. However, the execution time required is impractical when the number of activities increases. Comparatively, several priority-based heuristics (Buddhakulsomsiri & Kim, 2007; Li, Bettati, & Zhao, 1997) such as the latest finish time (LFT) and minimum slack (MSLK) (Edward & James, 1975), can solve RCPSP with shorter time, but they are hard to adapt to the constraints of problems dynamically. Hence, the sound solution is seldom obtained via heuristics.

Many studies solve the RCPSP by applying the metaheuristics-based schemes, such as genetic algorithm (GA) (Hartmann, 2002), simulated annealing algorithm (SA) (Bouleimen & Lecocq, 2003; Rutenbar, 1989), tabu search (TS) (Glover, 1989, 1990; Thomas & Salhi, 1998), ant colony optimization (ACO) (Lo, Chen, Huang, & Wu, 2008; Merkle, Middendorf, & Schmeck, 2002) and

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the particle swarm optimization (PSO) (Zhang, Li, & Tam, 2006), etc. The GA mimics the mechanism of natural selection as global evolution (Holland, 1987); then part of more superior solution is inherited via crossover operation, and increasing the diversity of solution via mutation process. Originally, simulated annealing was investigated by Kirkpatrick, Gelatt, and Vecchi (1983) as a stochastic method for combinatorial optimization problem. The optimal solution is a stable state when the thermal energy of the system minimized. The thermal energy is decreased by cooling down temperature parameter. Noteworthy, the SA applies a mechanism to avoid trapped on the local optimum by a probability during cooling down procedure. Tabu search is an approach proposed to prevent the search from sinking into the local minimum by recording the solutions which have been ever obtained. Therefore, the already obtained solutions in the following search can be avoided (Glover, 1989, 1990).

The ACO emulates the foraging behavior of ants (Dorigo & Gambardella, 1997). The ant left pheromone on the trail of the searched path from nest to the food source. The pheromone deposited on the way is for other ants to identify and communicate with each other. Additionally, the amount of pheromone is inverse proportional to the length of path; a large amount of pheromone is accumulated on the shorter path. The maximum amount of pheromone on the path can be regarded as an ant notification signal indicating where the shorter path is located at.

The particles swarm optimization (PSO) is first proposed by Kennedy and Eberhart (1995). In PSO, a swarm of particles spreads in the space and the position of a particle represents a solution of a dedicated problem. Each particle would move to a new position for the global optimal solution based on the global experience of the swarm and the individual experience of the particle. The PSO has been widely applied to solve the scheduling problems. Liu and Wang (2006) and Zhang, Sun, Zhu, and Yang (2008) solved flowshop scheduling problem (FSP) by means of the PSO, and Chen, Zhang, Hao, and Dai (2006) solved task scheduling in grid based on PSO. Zhang et al. (2006) used PSO to solve RCPSP; they showed that the PSO is applicable to various combinatorial problems and scheduling problems.

Besides the algorithm itself, some other schemes are combined with the algorithm to enhance the effectiveness and efficiency. There is a scheme named "justification" proposed by Valls, Ballest, and Quintanilla (2005), which is effective for improving the solution quality of the scheduling problems. The justification technique adjusts the start time of each activity in scheduling, and guarantees that the scheduling after justification is not worse even possible better than before one. Moreover, the efficiency of justification technique has been verified, it can apparently improve population-based algorithms such as GA while applying for RCPSP. In Valls et al. (2005), the justification implemented by double justify (DJ) applied to population-based algorithms, GA and SA have been tested, respectively, and the DJGA (GA applying DJ) and DJSA (SA applying DJ) outperform than all the state-of-the-art algorithms (such as GA, ACO). The performance evaluation comparison was also listed in Valls et al. (2005). Restated, the justification is able to promote the performance of population-based algorithms. Nevertheless, relatively few PSO studies with the combination of justification were devoted to solve RCPSP (no related literature was found). Hence, this study focuses on improving PSO algorithm based on the combination of PSO and justification for RCPSP, this proposed scheme is named justification particle swarm optimization (JPSO) herein.

Moreover, the suggested JPSO integrates two other designed mechanisms to further improve the efficiency, one is the mapping technique for enhancing the exploitation efficiency of justification, and the other is the adjusting ratio of communication topology of PSO for trade-off between exploration and exploitation. The simulation results demonstrate that both of these two schemes have significant improvement for solving RCPSP.

This article is organized as follows. Section 2 introduces the RCPSP. Section 3 presents the PSO. Section 4 presents the schemes of JPSO and how to solve RCPSP by JPSO. The simulated cases and results of experiments are displayed in Section 5. In Section 5, a complete comparative evaluation of the effectiveness and efficiency of the proposed JPSO algorithm as well as a comparison to other state-of-the-art approaches were presented. Finally, Section 6 presents the conclusions and discussions.

2. Resource-constrained project scheduling problem (RCPSP)

The scheduling problems have been applied in various fields. Among them, the resource-constrained project scheduling problem (RCPSP) is a general scheduling problem which involving activities need to be scheduled. Moreover, the RCPSP is confined to meet various constraints and achieves a certain objective. The studied RCPSP in this investigation is defined as follows:

1. The objective is to find the minimal *makespan* schedule.
2. There're $N+2$ activities, and each activity j has processing duration d_j ($j = 0, \ldots, N+1$). Meanwhile, activities are non-preemptive in the schedule. The activity 0 and activity $N+1$ are pseudo activities for indicating the start and end of schedule, respectively.
3. Activities have precedence constraint, let P_j be the set of immediate predecessors of activity j; the activity j cannot start to work until all of its immediate predecessors finished. Activity 0 is the source (start activity) that has no predecessors.
4. There are various renewable resources, constant amount renewable resources are provided at each time or period. Let

Table 1
30 activities case (j301_6) with precedence and resource requirement constraints.

Activity#	Successors			Activity#	Duration	Required resources			
						R 1	R 2	R 3	R 4
1	2	3	4	1	0	0	0	0	0
2	5	7	8	2	10	0	0	0	4
3	11			3	1	0	0	0	10
4	6	16		4	9	4	0	0	0
5	15	23		5	3	6	0	0	0
6	10	12		6	1	3	0	0	0
7	9	14	25	7	7	0	4	0	0
8	13			8	1	0	0	0	2
9	24			9	4	10	0	0	0
10	22			10	10	0	0	0	2
11	14	16	24	11	6	0	0	10	0
12	13	21		12	2	0	0	0	6
13	17	24	30	13	3	0	7	0	0
14	18			14	1	0	0	3	0
15	16	29		15	3	0	0	0	6
16	19			16	1	0	0	10	0
17	18			17	3	0	0	0	7
18	20	31		18	10	0	0	0	9
19	28			19	1	0	6	0	0
20	26			20	3	5	0	0	0
21	28			21	4	0	3	0	0
22	28			22	2	8	0	0	0
23	27			23	4	1	0	0	0
24	26	31		24	2	3	0	0	0
25	30			25	4	0	9	0	0
26	29			26	6	0	0	0	7
27	30			27	9	0	0	0	7
28	31			28	2	0	0	0	5
29	32			29	1	0	0	9	0
30	32			30	1	0	0	9	0
31	32			31	9	0	0	4	0
32				32	0	0	0	0	0
				Available resources		12	10	10	12

Q be a set of renewable resources with q types, and R_k ($k = 1, \ldots, q$) is the available amount of resource type k. Each activity j requires various resources $r_{j,1}, r_{j,2}, \ldots r_{j,q}$, where $r_{j,k}$ denotes the required amount of resource type k by activity j when activity j is processing. And the resource constraint confines the total amount of resources type k required by activities cannot exceed R_k at any time or period, such that $\sum_{j \in S(t)} r_{j,k} \leq R_k$, where the $S(t)$ is the set of activities to be processed at time or period t. The 30 activities example instance j301_6 of RCPSP in RCPSP is illustrated in Table 1.

3. The particle swarm optimization (PSO)

The particle swarm optimization (PSO) is first proposed by Kennedy and Eberhart (1995). It is a multi-agent general metaheuristic, and can be applied extensively in solving many complex problems. The PSO consists of a swarm of particles in the space; the position of a particle is indicated by a vector which presents a solution. PSO is initialized with a population of randomly positioned particles and searches for the best position with best fitness (usually minimum fitness).

In each generation or iteration, every particle moves to a new position and this new position is guided by a velocity (which is a vector), then the fitness corresponding to the new position of the particle would be calculated. Thus, the velocity plays an important role in searching solution with the better fitness. There are two experience positions are used in the PSO for updating the velocity; one is the global experience position of all particles, which memorizes the global best solution obtained through all particles; the other is each particle's individual experience, which memorizes the best position that particle has ever moved to. These two experience positions are used to determining the velocity.

Let an N dimension space (the number of dimension is typically concerned with the definition of problem) has M particles. For the ith particle ($i = 1, \ldots, M$), its position consists of N components $X_i = \{X_{i1}, \ldots, X_{iN}\}$, where X_{ij} is the jth component of the position. And the velocity of particle i is $V_i = \{V_{i1}, \ldots, V_{iN}\}$, particle individual experience is $L_i = \{L_{i1}, \ldots, L_{iN}\}$. Additionally, $G = \{G_1, \ldots, G_N\}$ represents the global best experience shared among all the particles. The updating of the jth component of the position and velocity of the ith particle are according to the following equation as shown in Eq. (1)

$$\begin{cases} V_{ij}^{new} = w \times V_{ij} + c_1 \times r_1 \times (L_{ij} - X_{ij}) + c_2 \times r_2 \times (G_j - X_{ij}) \\ X_{ij}^{new} = X_{ij} + V_{ij}^{new} \end{cases} \tag{1}$$

where w is an inertia weight used to determine the influence of the previous velocity to the new velocity. The c_1 and c_2 are learning factors used to derive how the ith particle approaches either closes to the individual experience position or the global experience position. Furthermore, the r_1 and r_2 are the random numbers uniformly distributed in $[0, 1]$, influencing the tradeoff between the global exploitation and local exploration abilities during search. There are many variations of PSO have been proposed, and one of them is named the "standard" PSO proposed by Bratton and Kennedy (2007) indicating that PSO can be significantly improved as required.

In the standard PSO, a constriction version of velocity update rule is suggested as shown in Eq. (2). Moreover, this velocity update rule is suggested for its stability as indicated in the "standard" PSO. Hence, this constriction velocity update rule is applied in this study

$$\begin{cases} V_{ij}^{new} = \chi \times (V_{ij} + c_1 \times r_1 \times (L_{ij} - X_{ij}) + c_2 \times r_2 \times (G_j - X_{ij})) \\ X_{ij}^{new} = X_{ij} + V_{ij}^{new} \end{cases} \tag{2}$$

where the χ is the constriction factor used for adjusting the velocity, where the values $\chi \approx 0.72984$ (0.73) and $c_1 = c_2 = 2.05$ are suggested in Bratton and Kennedy (2007).

The typical procedure of the PSO is shown as Table 2.

Table 2
The pseudo-code of PSO algorithm

```
PSO algorithm
Initialize
While End condition is not reached
  For each particle i in the swarm do
    Update position X_i^new using Eq. (2)
    Calculate particle's fitness f(X_i^new)
    Update L_i & G
  End for
End while
```

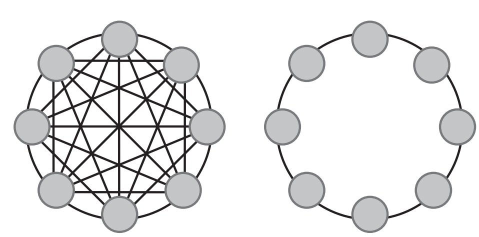
4. Justification particle swarm optimization (JPSO) for solving resource-constrained project scheduling problem (RCPSP)

4.1. Communication topology adjusting

There are two swarm communication topologies utilized in PSO (Bratton & Kennedy, 2007). One is the "gbest" topology as displayed in Fig. 1a which has been studied in most researches. The gbest is the global best model where every particle is able to acquire the information from others quickly because of it's fully connection with the others. However, the gbest's global communication ability usually leads to the premature convergence. The other is the "lbest" topology as displayed in Fig. 1b; the lbest has greatly attracted researcher's attention recently. The feature of lbest is the limited communication with others; every particle can just communicate with a part of swarm. Meanwhile, the lbest topology can be varied such as ring, star and Von Neumann neighborhood. Obviously, the lbest has slower convergence rate compared to the gbest.

Bratton and Kennedy (2007) observes that the gbest is usually resulting in better performance on simple unimodal problems than using lbest, since the situation of falling into local optimal is not happened frequently in such unimodal problems. However, the lbest surpasses the gbest in some evaluated functions, especially in the multimodal problems.

For the trade-off between gbest and lbest, a complement scheme is proposed to adjust the ratio of applying gbest and lbest in the process of PSO. In this study, there is a random variable named *gbest ratio* (GR) for the ratio of applying gbest in PSO, the GR denotes the probability of using gbest to update particle's velocity. Restated, a certain particle updates its velocity by gbest is determined by GR. Hence, the probability of lbest is (1-GR). Meanwhile, for simple implementation, the lbest topology is designed as the ring topology in this study as represented in Fig. 1b. Therefore, the velocity update rule can be defined as Eq. (3), where *rand* is a random variable uniformly distributed in $[0, 1]$, and the *better neighbor* (i) is the set of the neighbors of particle i with better performance (since the lbest topology is ring



(a) The gbest topology (b) The lbest topology

Fig. 1. Two topologies of PSO.

topology in the work, the number of the better neighbors is 2). Hence, in the velocity update rule, the global best experience G_j in Eq. (2) is replaced by Y_j as indicated in Eq. (3)

$$Y_j = \begin{cases} G_j, & rand < GR \\ X_{kj}, k \in better\ neighbor(i), & \text{otherwise} \end{cases}$$
$$V_{ij}^{new} = \chi \times (V_{ij} + c_1 \times r_1 \times (L_{ij} - X_{ij}) + c_2 \times r_2 \times (Y_j - X_{ij})) \tag{3}$$

4.2. Schedule generation schemes and encoding scheme

4.2.1. Schedule generation schemes (SGS)

Schedule generation schemes (SGS) (Hartmann & Kolisch, 1999) are usually used for generating the schedule of RCPSP, and there are two types of SGS, one is the serial schedule generation scheme (SSGS), and the other is parallel schedule generation scheme (PSGS). In this study, the SSGS is adopted, since the PSGS has been verified that it can only generates non-delay schedules, and the set of non-delay schedules is just a sub set of all schedules, hence the SSGS is suggested for RCPSP. In SSGS, the activity list $A = \{a_1, a_2, \ldots, a_{N+2}\}$ consisting of $N + 2$ activities is the solution of RCPSP, it is used for deciding the priorities of $N + 2$ activities. The a_1 is the activity which is the first start in schedule, i.e., the activity with the highest priority. Conversely, the a_{N+2} has the lowest priority and is the last activity in schedule. Furthermore, the activities order in the activity list A needs to meet the precedence constraint.

4.2.2. Encoding scheme

In PSO, the position of a particle is indicated by a vector which presents the solution of the investigated problem. And how to map the solution to the position vector, $\boldsymbol{X}$, is significant for PSO process. Since the RCPSP can be solved by using activity list A, when applying SSGS to generate schedule, hence the scheme of encoding activity list into the position vector $\boldsymbol{X}$ is necessary. The components of position vector are typically real number values. Noteworthy, the activity list is a permutation of activities, the permutation with the same value is not allowed. Thus, the key representation (Hartmann & Kolisch, 1999) is suitable for permutation type solution. For example, assume there are 5 keys correlated to 5 activities, and the position vector $\boldsymbol{X}$ with 5 components is given as following:

Key	1	2	3	4	5
X	0.5	0.6	0.15	0.9	0.2

After sorting $\boldsymbol{X}$ by decreasing order, the keys are also rearranged as following:

Key	4	2	1	5	3
X	0.9	0.6	0.5	0.2	0.15

Then the order of keys can be treated as the activity list, i.e., activity list $A = \{4,2,1,5,3\}$. Restated, the solution of RCPSP is generated by SSGS based on this activity list, and the activity list is encoded through the sorting operation on the components of the position vector.

4.3. Priority rule based heuristics for initial solution

Generally, for solving scheduling problem efficiently, the corresponding heuristics are often studied and integrated into metaheuristics in most researches.

In RCPSP, the latest finish time (LFT) heuristic (Edward & James, 1975) is often used for deciding the priority of activities. The LFT heuristic is applied in this study to give the higher priority to the activity which has the smaller latest finish time; the definition of the priority is displayed in Eq. (4). Meanwhile, in this study, the latest finish time (LFT) heuristic, which is also used for PSO to initialize the position vectors $\boldsymbol{X}$

$$p(j) = (\mathrm{LF}_j)^{-1} \tag{4}$$

where LF_j is the latest finish time of activity j, and the priority of activity j ($p(j)$) in activity list is inverse proportional to the LF_j.

For mapping the result of LFT to the initial position vectors $\boldsymbol{X}$, the $X(j)$ corresponding to the activity j can be assigned by $p(j)$ directly as listed in Eq. (5). In this way, the activity with higher priority would with higher value in position vectors $\boldsymbol{X}$, once the random key scheme is based on decreasing order, this activity can start earlier

$$X(j) = p(j) \tag{5}$$

Moreover, the LFT can be calculated based on the upper bound of scheduling completion time, $\overline{T}$ as follows:

$$\overline{T} = \sum_{j=0}^{N+1} d_j \tag{6}$$

Then, a traditional backward recursion computation is performed to determine LF_j for activity j. For example, an initial position vector is $\boldsymbol{X} = \{0.5, 0.6, 0.15, 0.9, 0.2\}$, where the component $X(j)$ is computed based on Eqs. (4) and (5). However, the drawback of LFT heuristic is without the consideration of resource constraint.

4.4. Forward–backward improvement

The forward–backward improvement (FBI) is a scheme that uses SGS to generate schedule by forward scheduling and backward scheduling, it has been proposed by Li and Willis (1992). The forward scheduling is applying SGS to ordinary precedence network when scheduling, somewhat differently, the backward scheduling would apply SGS to reversed precedence network. Hence, in backward scheduling, the start (source) activity is activity $N + 1$, and the end (drain) activity is activity 0. When given an activity list for SGS to generate schedule, it is possible that backward scheduling obtains schedule different from that using forward scheduling. Restated, the forward scheduling and backward scheduling would search for solutions in different searching area of the solution space. Some scheduling cases are appropriate by forward scheduling, and some scheduling cases are suitable using backward scheduling. Therefore, in some cases, the backward scheduling outperforms forward scheduling.

In this study, the FBI is involved in the JPSO as two particle swarms, these two particle swarms are labeled *forward* and *backward*, respectively. Once a certain particle belongs to the *forward* particle swarm, then the particle would generate schedule by forward scheduling at each iteration, and updating velocity via those particles in *forward* particle swarm, and vice versa. After all iteration finished, the best solution can be obtained from both particle swarms.

4.5. Justification and mapping

4.5.1. Justification

There is a simple and efficient scheme named “justification” proposed by Valls et al. (2005). The justification is simply adjusts the start time of each activity in scheduling for shorting the makespan. Restated, after justification, the makespan of the justified schedule would not larger than that before justification, but even possible shorter. Moreover, the efficiency of justification technique has been verified, it has been applied to GA for RCPSP as hybrid genetic algorithm (HGA) and performs well (Valls, Ballest, & Quintanilla, 2008).

Hence, in this study, one type of justification called double justification (DJ) is combined with the PSO as the proposed JPSO for solving RCPSP. In JPSO, the DJ is applied to double justify all solutions (schedules) obtained by particles (including *forward* and *backward* particles) in each iteration. The definitions of the justification can be shown as follows:

1. Right (Left) justification: for a schedule with more than one activity, sequencing these activities by finish time (start time) of activities in decreasing (increasing) order. At each step, the right (left) justification adjusts the start time S_i of the ith activity in sequence such that the adjusted start time $S_i' \geqq S_i (S_i' \leqq S_i)$ and make the S_i' as large (small) as possible (still meets constraints). When all activities have been justified, the justification is finished. Restated, after right (left) justification, the start time of activities is as late (early) as possible, and once the start (end) activity is later (earlier) than before justification, that is, the makespan of schedule is shortened by right (left) justification.
2. Double justification (DJ): for a schedule S after right justification, it can be denoted as S^R, and then left justifying the S^R, hence the schedule of double justification $\mathrm{DJ}(S) = (S^R)^L$ can be obtained. The makespan of S and DJ(S) can be denoted as $T(S)$ and $T(\mathrm{DJ}(S))$, respectively, and the most significant is that the $T(\mathrm{DJ}(S)) \leqq T(S)$ can be guaranteed.

Since the double justification (DJ) has double justifying operations, it can be treated as two extra schedules generated form source schedule. In Valls et al. (2005), the DJ combined population-based algorithms, SA and GA are simulated and evaluated. The DJ is simple to implement and can greatly improve the quality of the solution of RCPSP. Furthermore, the DJ is able to yield more schedules with less execution time.

4.5.2. Mapping

In JPSO, after a particle generates new solution (schedule), the double justification (DJ) would improve the solution quality. However, once this particle's solution is improved by DJ, the related position vector of this particle still remains unchanged as before DJ. And then, other particles cannot acquire the information about justified solutions while updating their velocity. Therefore, it is necessary to synchronize the justified solution with the corresponding position vector of particle. Restated, consistence between the new generated solution by particle and the justified solution by DJ is required.

In this study, this synchronization scheme is called "mapping". This mapping scheme maps the start time of each activities of justified solution to value of position vector of particle, the mapping function can be defined as Eq. (7):

$$X(j) = (S_j)^{-1} \tag{7}$$

where the S_j is the start time of activity j in justified solution, if the S_j is small, the $X(j)$ corresponding to the activity j would be large after mapping. Restated, a certain activity with small start time in justified solution indicates that this activity needs to start earlier. Hence, the corresponding component of position vector would be associated with a lager value, and then this activity has higher priority. This mapping scheme is used for mapping the double justified solution to the corresponding position of particle. The updated position information of particle after mapping can then be used to communicate with others immediately in the next iteration.

The proposed JPSO algorithm for RCPSP is summarized as shown in Table 3.

Table 3
The pseudo-code of JPSO algorithm

JPSO algorithm
Initialize the position vector of particles by LFT
While End condition is not met
For each particle i in the *forward or backward particle swarm* do
Update position X_i^{new} using Eq. (3) (the update rule with *gbest ratio* (GR))
Generating schedule S using position X_i^{new} and SSGS
Double justify S and *mapping* it to position X_i^{new}
Calculate particle's fitness $f(S)$
Update L_i & G
End for
End while

5. Experimental results and comparisons

In this section, for comparing the JPSO to the state-of-the-art heuristic metaheuristics, the instances in the well-known PSPLIB (Project Scheduling Problem Library http://129.187.106.231/psplib/) are used as the benchmark (Kolisch & Sprecher, 1997) and simulated. In PSPLIB benchmark, the number of RCPSP instances for 30, 60 activities are 480, respectively, and that for 120 activities is 600. Hence, there are total 1560 RCPSP instances are simulated in this study. For reasonable comparison with other studies, the termination condition for each instance in simulation is the limits of the number of generated solutions (schedules) such as 1000, 5000 and 50,000. Hence, the comparison among algorithms has no concern with the performance of the hardware, programming language and programming technique.

The simulation parameters are set as follows: once the heuristics for initial solution in Section 4.3 is not activated, and then the initial position and velocity of particles are random assigned. And the learning factors c_1 and c_2 are set to 2 due to the suggested value 2.05 in "standard" PSO (Bratton & Kennedy, 2007). The remaining parameters such as the number of particles and constriction factor χ are given either based on suggested value in "standard" PSO or by *trial-and-error* approach. Moreover, the value for designed *gbest ratio* (GR) is tested by *trial-and-error* approach.

For comparison, the quality of solution can be obtained via the deviation from the best so far. In Eq. (8), the DEV_i is the deviation between the obtained fitness and the "*best*" of instance i. The optimal makespans for all instances in 30 activities are known; hence the "*best*" in Eq. (8) is the optimal solution provided in PSPLIB. However, optimal makespans for some instances in 60 and 120 activities are unknown; then lower bounds (determined by the critical path) were used instead as the "*best*" in Eq. (8). Moreover, ADEV is the average deviation which is obtained by averaging the deviation of simulated instances as defined in Eq. (9)

$$\mathrm{DEV}_i = \frac{fitness_i - best_i}{best_i} \times 100\% \tag{8}$$

$$\mathrm{ADEV} = \frac{\sum_{i \in instances} \left(\frac{fitness_i - best_i}{best_i} \times 100\% \right)}{|instances|} \tag{9}$$

In the following paragraphs, the simulation results for instances of 30, 60 and 120 activities would be presented. Since the optimal solutions of 30 activities are known, the OPT ADEV (average deviations from optimal makespan) is used for comparison. However, the optimal solutions of 60 and 120 activities are still undetermined, the CP ADEV (average deviations from critical path) is used for comparison.

5.1. The 30 activities results

The RCPSP instances for 30 activities are labeled "J30". In J30, there are 480 instances and the optimal schedules are used for OPT ADEV (average deviations from optimal makespan) calculation. There are numerous tests with 1000, 5000 and 50,000 schedules limits were experimented, part of simulation results are displayed in Table 4. And the number of generated schedules, iterations (Iter.), constriction factor χ, gbest ratio (GR), number of particles in forward particle swarm (For.) and backward particle swarm (Back.) are also listed in Table 4.

Once the LFT heuristic and mapping schemes are adopted in simulation, they would be marked by "○", otherwise marked by "×". And the DJ column denotes whether the double justification is applied for each solution generated by particle in each iteration. The indication for "0" or "1" denoted that the DJ is "unused" or "used", respectively. Since the DJ would generate extra two schedules, the number of generated schedules can be calculated by Eq. (10)

$$\text{Generated schedules} = (\text{For.} + \text{Back.}) \times (1 + \text{DJ} \times 2) \times \text{Iter.} \leqq \text{schedule limit} \tag{10}$$

In Table 4, the no. 1–14 tests are limited by 1000 schedules. The LFT heuristic is evaluated by no. 1 and 2 tests. The χ is set to the suggested 0.73 for no. 1 and 2 tests. It can be found that when the LFT is adopted in no. 2 test, the OPT ADEV decreases apparently. Moreover, DJ was involved to test the effect on the performance. Whenever the DJ applied, only one-third iterations are needed to acquire the same number of schedules. Better performance of DJ was produced as indicated by the decreasing of OPT ADEV for no. 3 through no. 5 tests in Table 4. Furthermore, the proposed scheme including mapping scheme and forward–backward improvement (FBI) were further tested. In Table 4, the no. 6 through 14 tests reveal that the further better quality solution with decreased OPT ADEV is also obtained as expected. However, to evaluate the affection of χ on the solution, different $\boldsymbol{\chi}$ values were simulated; 0.4 and 0.5 seem to be the best χ for J30 based on the tests results, as displayed in Table 4. Restated, according to the above experiments results, the proposed JPSO with LFT heuristic, DJ and mapping schemes and FBI mechanism provides good performance for instances of J30.

The performance of most heuristic and schemes of JPSO are verified on the basis of tests with 1000 schedules on J30. Hence, experiments with 5000 schedules limit were performed based on the parameter setting in no. 13 test (the best parameter setting of tests with 1000 schedules) except for the possible best χ values and particles number. The simulation results are presented on no. 15 through 17 tests in Table 4.

In no. 18–27 tests, the number of generated schedules is set to 50,000. Hence, for verifying the effects of the gbest and lbest on trapping in local optima under numerous iterations, experiments were tested. The parameter GR is set to 1, 0.75, 0.5, 0.25 and 0, respectively, associated with the possible best χ values were tested for assessment. The best simulation result of OPT ADEV 0.04% is produced when χ = 0.4 and the GR = 0, i.e., the lbest seems to contribute better performance than the gbest for J30.

Moreover, for comparing the proposed JPSO with other state-of-art algorithms demonstrated in (Kolisch & Hartmann 2006), the 30 activities simulation results are also shown in Table 5.

In Table 5, different algorithms for comparisons have been introduced in Kolisch and Hartmann (2006), and they are ranked according to the OPT ADEV when schedule limit is 50,000. The JPSO proposed in this study presently ranks the 7th on the comparison table. Although the performance of JPSO is not the best apparently, the rating of JPSO is fairly good.

5.2. 60 activities results

The RCPSP instances for 60 activities are labeled "J60", and J60 also has 480 instances. However, the optimal solutions of J60 are unknown and the provided lower bound in PSPLIB would changes

Table 4
J30 simulation results.

J30:480 instances					Heuristic	Schemes			Particles		OPT ADEV (%)
No.	Schedules	Limit	Iter.	χ	LFT	DJ	Mapping	GR	For.	Back.	
1	1000	1000	50	0.73	×	0	×	1	20	0	1.69
2	1000	1000	50	0.73	○	0	×	1	20	0	1.13
3	960	1000	16	0.73	○	1	×	1	20	0	0.61
4	960	1000	16	0.6	○	1	×	1	20	0	0.61
5	960	1000	16	0.5	○	1	×	1	20	0	0.67
6	960	1000	16	0.73	○	1	○	1	20	0	0.50
7	960	1000	16	0.6	○	1	○	1	20	0	0.46
8	960	1000	16	0.5	○	1	○	1	20	0	0.35
9	960	1000	16	0.4	○	1	○	1	20	0	0.40
10	960	1000	16	0.73	○	1	○	1	10	10	0.52
11	960	1000	16	0.6	○	1	○	1	10	10	0.43
12	960	1000	16	0.5	○	1	○	1	10	10	0.34
13	960	1000	16	0.4	○	1	○	1	10	10	**0.29**
14	960	1000	16	0.3	○	1	○	1	10	10	0.38
15	4920	5000	41	0.6	○	1	○	1	20	20	0.20
16	4920	5000	41	0.5	○	1	○	1	20	20	**0.14**
17	4920	5000	41	0.4	○	1	○	1	20	20	0.16
18	49,920	50,000	416	0.5	○	1	○	1	20	20	0.07
19	49,920	50,000	416	0.5	○	1	○	0.75	20	20	0.06
20	49,920	50,000	416	0.5	○	1	○	0.5	20	20	0.06
21	49,920	50,000	416	0.5	○	1	○	0.25	20	20	0.06
22	49,920	50,000	416	0.5	○	1	○	0	20	20	0.09
23	49,920	50,000	416	0.4	○	1	○	1	20	20	0.17
24	49,920	50,000	416	0.4	○	1	○	0.75	20	20	0.14
25	49,920	50,000	416	0.4	○	1	○	0.5	20	20	0.13
26	49,920	50,000	416	0.4	○	1	○	0.25	20	20	0.07
27	49,920	50,000	416	0.4	○	1	○	0	20	20	**0.04**

Table 5
J30 comparison of different algorithms in Kolisch and Hartmann (2006) (OPT ADEV%)

Algorithm	SGS	Author(s)	Schedule limits		
			1000	5000	50,000
GA, TS – path relinking	Both	Kochetov and Stolyar	0.10	0.04	0.00
Scatter search – FBI	Serial	Debels et al.	0.27	0.11	0.01
GA – hybrid, FBI	Serial	Valls et al.	0.27	0.06	0.02
GA – FBI	Serial	Valls et al.	0.34	0.20	0.02
GA – forw.–backw., FBI	Both	Alcaraz et al.	0.25	0.06	0.03
GA – forw.–backw.	Serial	Alcaraz and Maroto	0.33	0.12	–
JPSO	**Serial**	**This study**	**0.29**	**0.14**	**0.04**
Sampling – LFT, FBI	Both	Tormos and Lova	0.25	0.13	0.05
TS – activity list	Serial	Nonobe and Ibaraki	0.46	0.16	0.05
Sampling – LFT, FBI	Both	Tormos and Lova	0.30	0.16	0.07
GA – self-adapting	Both	Hartmann	0.38	0.22	0.08
GA – activity list	Serial	Hartmann	0.54	0.25	0.08
Sampling – LFT, FBI	Both	Tormos and Lova	0.30	0.17	0.09
TS – activity list	Serial	Klein	0.42	0.17	–
Sampling – random, FBI	Serial	Valls et al.	0.46	0.28	0.11
SA – activity list	Serial	Bouleimen and Lecocq	0.38	0.23	–
GA – late join	Serial	Coelho and Tavares	0.74	0.33	0.16
Sampling – adaptive	Both	Schirmer	0.65	0.44	–
TS – schedule scheme	Related	Baar et al.	0.86	0.44	–
Sampling – adaptive	Both	Kolisch and Drexl	0.74	0.52	–
GA – random key	Serial	Hartmann	1.03	0.56	0.23
Sampling – LFT	Serial	Kolisch	0.83	0.53	0.27
Sampling – global	Serial	Coelho and Tavares	0.81	0.54	0.28
Sampling – random	Serial	Kolisch	1.44	1.00	0.51
GA – priority rule	Serial	Hartmann	1.38	1.12	0.88
Sampling – WCS	Parallel	Kolisch	1.40	1.28	–
Sampling – LFT	Parallel	Kolisch	1.40	1.29	1.13
Sampling – random	Parallel	Kolisch	1.77	1.48	1.22
GA – problem space	Mod. par.	Leon and Ramamoorthy	2.08	1.59	–

Table 6
J60 simulation results.

J60:480 instances					Heuristic	Schemes			Particles		CP ADEV (%)
No.	Schedules	Limit	Iter.	χ	LFT	DJ	Mapping	GR	For.	Back.	
1	960	1000	16	0.73	○	1	○	1	10	10	13.00
2	960	1000	16	0.6	○	1	○	1	10	10	12.90
3	960	1000	16	0.5	○	1	○	1	10	10	12.72
4	960	1000	16	0.4	○	1	○	1	10	10	12.10
5	960	1000	16	0.3	○	1	○	1	10	10	**12.03**
6	960	1000	16	0.2	○	1	○	1	10	10	12.27
7	4920	5000	41	0.5	○	1	○	1	20	20	12.12
8	4920	5000	41	0.4	○	1	○	1	20	20	**11.43**
9	4920	5000	41	0.3	○	1	○	1	20	20	11.67
10	49,920	50,000	416	0.4	○	1	○	1	20	20	11.41
11	49,920	50,000	416	0.4	○	1	○	0.75	20	20	11.29
12	49,920	50,000	416	0.4	○	1	○	0.5	20	20	11.12
13	49,920	50,000	416	0.4	○	1	○	0.25	20	20	**11.00**
14	49,920	50,000	416	0.4	○	1	○	0	20	20	**11.00**
15	49,920	50,000	416	0.3	○	1	○	1	20	20	11.66
16	49,920	50,000	416	0.3	○	1	○	0.75	20	20	11.66
17	49,920	50,000	416	0.3	○	1	○	0.5	20	20	11.57
18	49,920	50,000	416	0.3	○	1	○	0.25	20	20	11.41
19	49,920	50,000	416	0.3	○	1	○	0	20	20	11.48

with time as more studies involved. Hence, the lower bounds from critical path are used as the ideal standard. Restated, CP ADEV (average deviations from critical path) is calculated and as the basis of comparison. There are 19 test results with 1000, 5000 and 50,000 schedules limits for J60 displayed as shown in Table 6.

The performance of most heuristics and schemes has been verified in J30. In Table 6, the main tests of no. 1 through 9 tests focus on the best χ measurement. The possible best χ values are 0.3 and 0.4 as the experiment results as shown in Table 6. Moreover, the best GR was also estimated. The GR is set to 1, 0.75, 0.5, 0.25 and 0, respectively, associated with the possible best values were simulated for evaluation. The best CP ADEV 11.00% is obtained when the χ = 0.4 and GR = 0 (or GR = 0.25), i.e., for instances of J60, the lbest seems to provide better performance than the gbest.

As stated above, the J60 simulation results are also shown in Table 7, and these algorithms for comparisons have been also introduced in Kolisch and Hartmann (2006), and they are ranked according to the CP ADEV when schedules limit is 50,000. The JPSO proposed in this study ranks the 6th on the comparison table, it indicates that the JPSO can performs well for instances of J60.

5.3. 120 activities results

The RCPSP instances for 120 activities are labeled "J120", it has 600 instances. And as the same as J60, the CP ADEV (average deviations from critical path) can be the ideal standard for comparison. There are 14 test results with 1000, 5000 and 50,000 schedules limits for J120 illustrated as shown in Table 8.

In Table 8, the tests of no. 1 through 9 focus on the best χ measurement. The best χ value is 0.3. Moreover, the best GR was also estimated via tests of no. 10 through 14. The GR is set to 1, 0.75, 0.5, 0.25 and 0, respectively, and the best CP ADEV 32.89% is gained when the GR = 0.25. Restated, simulation results indicate that *lbest* seems to provide better performance again for instances of J120.

As J30 and J60, the J120 simulation results are also shown in Table 9, and these algorithms for comparisons have been also introduced in Kolisch and Hartmann (2006), and they are ranked based on the CP ADEV when schedules limit is 50,000. The JPSO proposed in this study is presently ranked the 7th on the comparison table. Restated, the proposed JPSO can solve the instances of J120 efficiently.

The experiment results are summarized as listed in Table 10. In Table 10, the best values of adjustable parameters χ and GR used for proposed JPSO are suggested.

6. Conclusions and discussion

This study proposes a novel JPSO scheme with designed mechanisms to solve the resource-constrained project scheduling problem (RCPSP). The suggested JPSO also involves the LFT heuristic, double justification (DJ), mapping, forward and backward particle swarms for forward–backward improvement (FBI), and adjusting communication topology by gbest ratio (GR). And those designed mechanisms of JPSO have been verified through the experiments.

Table 7
J60 comparison of different algorithms in Kolisch and Hartmann (2006) (CP ADEV%).

Algorithm	SGS	Author(s)	Schedule limits		
			1000	5000	50,000
Scatter search – FBI	Serial	Debels et al.	11.73	11.10	10.71
GA – hybrid, FBI	Serial	Valls et al.	11.56	11.10	10.73
GA, TS – path relinking	Both	Kochetov and Stolyar	11.71	11.17	10.74
GA – FBI	Serial	Valls et al.	12.21	11.27	10.74
GA – forw.–backw., FBI	Both	Alcaraz et al.	11.89	11.19	10.84
JPSO	**Serial**	**This study**	**12.03**	**11.43**	**11.00**
GA – self-adapting	Both	Hartmann	12.21	11.70	11.21
GA – activity list	Serial	Hartmann	12.68	11.89	11.23
Sampling – LFT, FBI	Both	Tormos and Lova	11.88	11.62	11.36
Sampling – LFT, FBI	Both	Tormos and Lova	12.14	11.82	11.47
GA – forw.–backw.	Serial	Alcaraz and Maroto	12.57	11.86	–
Sampling – LFT, FBI	Both	Tormos and Lova	12.18	11.87	11.54
SA – activity list	Serial	Bouleimen and Lecocq	12.75	11.90	–
TS – activity list	Serial	Klein	12.77	12.03	–
TS – activity list	Serial	Nonobe and Ibaraki	12.97	12.18	11.58
Sampling – random, FBI	Serial	Valls et al.	12.73	12.35	11.94
Sampling – adaptive	Both	Schirmer	12.94	12.58	–
GA – late join	Serial	Coelho and Tavares	13.28	12.63	11.94
GA – random key	Serial	Hartmann	14.68	13.32	12.25
GA – priority rule	Serial	Hartmann	13.30	12.74	12.26
Sampling – adaptive	Both	Kolisch and Drexl	13.51	13.06	–
Sampling – WCS	Parallel	Kolisch	13.66	13.21	–
Sampling – global	Serial	Coelho and Tavares	13.80	13.31	12.83
Sampling – LFT	Parallel	Kolisch	13.59	13.23	12.85
TS – schedule scheme	Related	Baar et al.	13.80	13.48	–
GA – problem space	Mod. par.	Leon and Ramamoorthy	14.33	13.49	–
Sampling – LFT	Serial	Kolisch	13.96	13.53	12.97
Sampling – random	Parallel	Kolisch	14.89	14.30	13.66
Sampling – random	Serial	Kolisch	15.94	15.17	14.22

Table 8
J120 simulation results.

J120:600 instances					Heuristic	Schemes			Particles		CP ADEV (%)
No.	Schedules	Limit	Iter.	χ	LFT	DJ	Mapping	GR	For.	Back.	
1	960	1000	16	0.73	○	1	○	1	10	10	39.30
2	960	1000	16	0.6	○	1	○	1	10	10	39.25
3	960	1000	16	0.5	○	1	○	1	10	10	38.96
4	960	1000	16	0.4	○	1	○	1	10	10	37.76
5	960	1000	16	0.3	○	1	○	1	10	10	**35.71**
6	960	1000	16	0.2	○	1	○	1	10	10	36.01
7	4920	5000	41	0.4	○	1	○	1	20	20	34.82
8	4920	5000	41	0.3	○	1	○	1	20	20	**33.88**
9	4920	5000	41	0.2	○	1	○	1	20	20	34.36
10	49,920	50,000	416	0.3	○	1	○	1	20	20	33.72
11	49,920	50,000	416	0.3	○	1	○	0.75	20	20	33.46
12	49,920	50,000	416	0.3	○	1	○	0.5	20	20	33.12
13	49,920	50,000	416	0.3	○	1	○	0.25	20	20	**32.89**
14	49,920	50,000	416	0.3	○	1	○	0	20	20	33.21

Table 9
J120 comparison of different algorithms in Kolisch and Hartmann (2006) (CP ADEV%)

Algorithm	SGS	Author(s)	Schedule limits		
			1000	5000	50,000
GA – hybrid, FBI	Serial	Valls et al.	34.07	32.54	31.24
GA – forw.–backw., FBI	Both	Alcaraz et al.	36.53	33.91	31.49
Scatter Search – FBI	Serial	Debels et al.	35.22	33.10	31.57
GA – FBI	Serial	Valls et al.	35.39	33.24	31.58
GA, TS – path relinking	Both	Kochetov and Stolyar	34.74	33.36	32.06
Population-based – FBI	Serial	Valls et al.	35.18	34.02	32.81
JPSO	Serial	This study	**35.71**	**33.88**	**32.89**
GA – self-adapting	Both	Hartmann	37.19	35.39	33.21
Sampling – LFT, FBI	Both	Tormos and Lova	35.01	34.41	33.71
Ant system	Serial	Merkle et al.	–	35.43	–
GA – activity list	Serial	Hartmann	39.37	36.74	34.03
Sampling – LFT, FBI	Both	Tormos and Lova	36.24	35.56	34.77
Sampling – LFT, FBI	Both	Tormos and Lova	36.49	35.81	35.01
GA – forw.–backw.	Serial	Alcaraz and Maroto	39.36	36.57	–
TS – activity list	Serial	Nonobe and Ibaraki	40.86	37.88	35.85
GA – late join	Serial	Coelho and Tavares	39.97	38.41	36.44
Sampling – random, FBI	Serial	Valls et al.	38.21	37.47	36.46
SA – activity list	Serial	Bouleimen and Lecocq	42.81	37.68	–
GA – priority rule	Serial	Hartmann	39.93	38.49	36.51
Sampling – adaptive	Both	Schirmer	39.85	38.70	–
Sampling – LFT	Parallel	Kolisch	39.60	38.75	37.74
Sampling – WCS	Parallel	Kolisch	39.65	38.77	–
GA – random key	Serial	Hartmann	45.82	42.25	38.83
Sampling – adaptive	Both	Kolisch and Drexl	41.37	40.45	–
Sampling – global	Serial	Coelho and Tavares	41.36	40.46	39.41
GA – problem space	Mod. par.	Leon and Ramamoorthy	42.91	40.69	–
Sampling – LFT	Serial	Kolisch	42.84	41.84	40.63
Sampling – random	Parallel	Kolisch	44.46	43.05	41.44
Sampling – random	Serial	Kolisch	49.25	47.61	45.60

Table 10
The performance of JPSO for RCPSP

Schedules	J30 (480 instances)			J60 (480 instances)			J120 (600 instances)		
	χ	GR	OPT ADEV (%)	χ	GR	CP ADEV (%)	χ	GR	CP ADEV (%)
1000	0.4	1	0.29	0.3	1	12.03	0.3	1	35.71
5000	0.5	1	0.14	0.4	1	11.43	0.3	1	33.88
50,000	0.4	0	0.04	0.4	0/0.25	11.00	0.3	0.25	32.89

The resulting ADEV is decreased after applying those schemes as indicated in the experiment results. Additionally, the resulting OPT ADEV and CP ADEV by applying JPSO are also compared with the top state-of-art algorithms (Kolisch & Hartmann, 2006).

The efficiency of proposed JPSO for solving the resource-constrained project scheduling problems ranks the 7th (on average) as demonstrated in the comparison Tables 5, 7 and 9. Hence, the performance of proposed JPSO is stable for instances of J30, J60 and J120 in PSPLIB. Restated, the suggested JPSO scheme is an effective and efficient algorithm for solving the class of RCPSP problems. Meanwhile, according to the simulation results, a small constriction factor is recommended for RCPSP instead of using suggested value 0.73 in Bratton and Kennedy (2007) when the proposed JPSO is applied, as indicated in Table 10. Moreover, the designed gbest ratio (GR) is suggested to be a lower value for situation with more schedules, such as 50,000 schedules instances, as demonstrated in Table 10.

Furthermore, some other important features of this study are summarized below.

- The DJ is simple to implement and can greatly improve the quality of the solution of RCPSP, i.e., shorten the resulting schedule *makespan*. And the DJ is able to yield more schedules with less execution time.
- The latest finish time (LFT) heuristic is used to initialize the position vectors $\boldsymbol{X}$. Therefore, worse solutions are excluded before starting JPSO process; and hence more good schedules can be obtained while under the schedules limit. Restated, increases the probability of finding optimal solutions.
- Additionally, forward–backward improvement applying forward and backward particle swarms increases the efficiency while searching for the optimal solution in different area in the solution space. Therefore, more optimal solutions can be yielded.

This study mainly focuses on the solving of RCPSP. In the future, the more complex conditions or situations could be considered, such as the preemptive activity setting, setup time between activities when they are in certain situation, and even the communication cost between two activities due to some relationship of them. Furthermore, there are many studies have been applied on the more general type RCPSP, i.e., the multi-mode resource-constrained project scheduling problem (MRCPSP). Hence, the MRCPSP is also the significant topic for the future study, the more schemes for improving the solution of RCPSP and MRCPSP is the main goal we are pursuing.

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