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Chaos suppression control of a coronary artery system with uncertainties by using variable structure control

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ABSTRACT

Nonlinear behavior and the chaos suppression problem are studied in a coronary artery chaotic system. Based on the variable structure control (VSC) theory, the sliding mode control scheme is used to design a chaos suppression controller in this study. A suitable sliding surface is selected to ensure a sliding mode motion of error states when the proposed control law is applied. As expected, the error state drives to zero with matched external uncertainties or into a predictable neighborhood of zero with mismatched external uncertainties. Therefore, suppressing the abnormal chaotic behavior of a coronary artery system to a normal unstable periodic orbit of the nominal coronary artery system can reduce the occurrence of heart disease. A modified continuous sliding mode controller is also proposed to avoid chatter. Illustrative examples are given to demonstrate the superiority of the proposed approach.

1. Introduction

Nonlinear science combined with biomedical engineering has yielded major advances in many areas of biological and medical research, including ecology, brain function, the nervous system, macromolecular theory, and pathological phenomena [1,2]. The coronary artery system is a biomathematical model of muscular blood vessels [3]. Due to the nonlinear differences between changes in vessel diameter, the behaviors of this system are highly complex in diseases such as myocardial infarction. Therefore, understanding its nonlinear behavior and suppressing undesired chaotic motion in coronary artery systems when it occurs are essential tasks.

Due to its importance, both the medical and engineering communities have intensively studied the coronary artery. Current work on the coronary artery involves the study blood flow dynamics, vessel wall mechanisms and the control of the motion states of the muscular vessel [4–7]. For chaotic control of the coronary artery system, the biomedical model of the chaotic coronary artery system must be synchronized with a prescribed chaotic or periodic system of a normal vessel. From the medical perspective, chaotic control requires synchronization of motion states of the vessel with pathological changes in the normal vessel so that treatment can be achieved. In 2006, Gong et al. used backstepping control technology to synchronize the master and slave coronary artery system. They showed that a spastic vessel can be synchronized with a normal vessel [8]. In 2011, Prof. Li developed an adaptive robust controller for tracking the control problem of chaotic coronary systems with dynamic uncertainties and unknown parameters. It found that a chaotic coronary artery system can be driven into the normal orbit [9].

The goal of this study was to synchronize the abnormal chaotic behavior of a slave coronary artery system with the normally unstable periodic orbit of the nominal coronary artery system even when they have different initial conditions and

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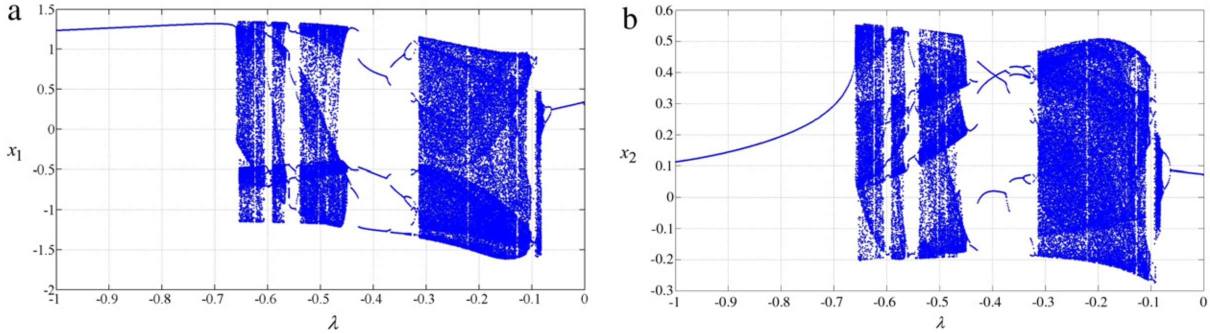

Fig. 1. The bifurcation diagrams obtained by varying parameter λ from -1 to 0.

external disturbances. The chaotic coronary artery system is then controlled with a variable structure control method by using a switching surface to suppress control coronary artery chaos. Without using approximation methods, the proposed sliding mode control law provides immunity to external disturbances, without chatter. Simulation results show that the proposed controller drives the abnormal system to synchronize with the normal system despite different initial conditions and external disturbances. It means that the control input u (e.g., nitroglycerin) is quickly absorbed; blood vessels dilate and increase blood supply to the heart muscle, which effectively relieves or eliminates angina symptoms. Therefore, the control of muscular vascular biological mathematical model is valid. In a disorder state and under any pre-specified suppression of unstable orbit in a nominal chaotic system, this study demonstrated that, for therapeutic purposes, a blood vessel spasm in a disorder state can theoretically be suppressed with normal blood vessels. The organization of this paper is as follows. The nonlinear behavior of coronary artery system presented in Section 3; the control problem is described in Section 3; a VSC design for chaos suppression is derived in Section 4; a modified continuous scheme for eliminating chattering phenomenon is presented in Section 5; finally, simulation results are presented in Section 6.

2. System description and nonlinear behavior analysis

The coronary artery is a heart muscle that supplies oxygen and nutrients to blood vessels, also known as muscle-type vessels. Obstructed transport of nutrients and oxygen to the heart due to a coronary artery obstruction can cause angina, myocardial infarction and other diseases, and the pathology of myocardial infarction suggests that coronary atherosclerosis and coronary artery spasm are common causes of morbidity.

The work by Ref. [7] showed that the coronary artery system can be mathematically modeled as

$$\begin{aligned} \dot{x}_1 &= -bx_1 - cx_2 \\ \dot{x}_2 &= -(b+1)\lambda x_1 - (c+1)\lambda x_2 + \lambda x_1^3 + E\cos\omega t \end{aligned} \tag{1}$$

where x_1 denotes the change in the inner radius of the vessel, x_2 is the pressure change in the vessel, t is the non-dimensional time variable, b, c and λ are the coronary artery system parameters, and $E\cos\omega t$ is a periodical disturbance term. A myocardial infarction is caused by coronary vascular atherosclerosis and coronary artery spasm. Coronary artery spasm is caused by a contraction of the epicedial artery in a transient conduction resulting in partial or complete occlusion of blood vessels, which then causes myocardial ischemia. Fig. 1 shows the bifurcation diagram of system (1) with initial condition (0.2, 0.2) and the parameters $b = 0.15$, $c = -1.7$, $E = 0.3$ and $\omega = 1$. The diagram shows the disorder and subharmonic motion in the system states of the coronary artery system during periods T, 2T, 3T, 5T and 6T ($T = 2\pi$). Fig. 2 reveals that the corresponding maximum Lyapunov exponent (MLE) [1] has positive values under some ranges of parameter λ. Therefore, the inferred trajectory of the coronary artery system is a state of chaotic motion with these λ. Fig. 3 shows the very complex dynamic behavior of the coronary artery system when $\lambda = -0.5$. Under certain conditions, chaos can be characterized by the static rheological characteristics of blood vessels caused by change in parameter λ, which represents vascular pathological changes in the blood vessels when descending into chaos.

From the mathematical perspective, the so-called vascular spasm is caused by blood vessels in a chaotic state. The causes of vascular spasm, which is a form of ischemic heart disease, include variant angina, unstable angina, acute myocardial infarction, and sudden vasomotor death. Such chaos immediately endangers health and must be controlled immediately. The following section examines the problem of suppressing chaos in the coronary artery system and then proposes the solution using VSC.

3. Control problem description

Consider the following uncertain coronary artery system:

$$\begin{aligned} \dot{x}_1 &= -bx_1 - cx_2 + d_1(x,t) \\ \dot{x}_2 &= -(b+1)\lambda x_1 - (c+1)\lambda x_2 + \lambda x_1^3 + E\cos\omega t + d_2(x,t) + u(t), \end{aligned} \tag{2}$$

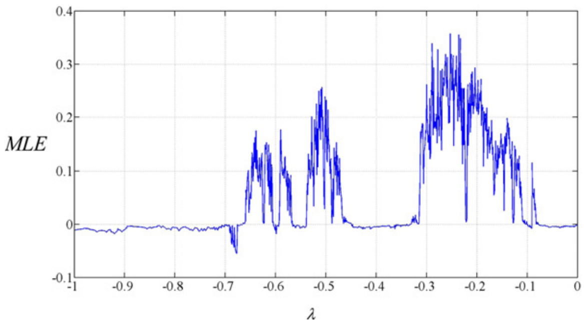

Fig. 2. The maximum Lyapunov exponents of coronary artery system with change of parameter λ.

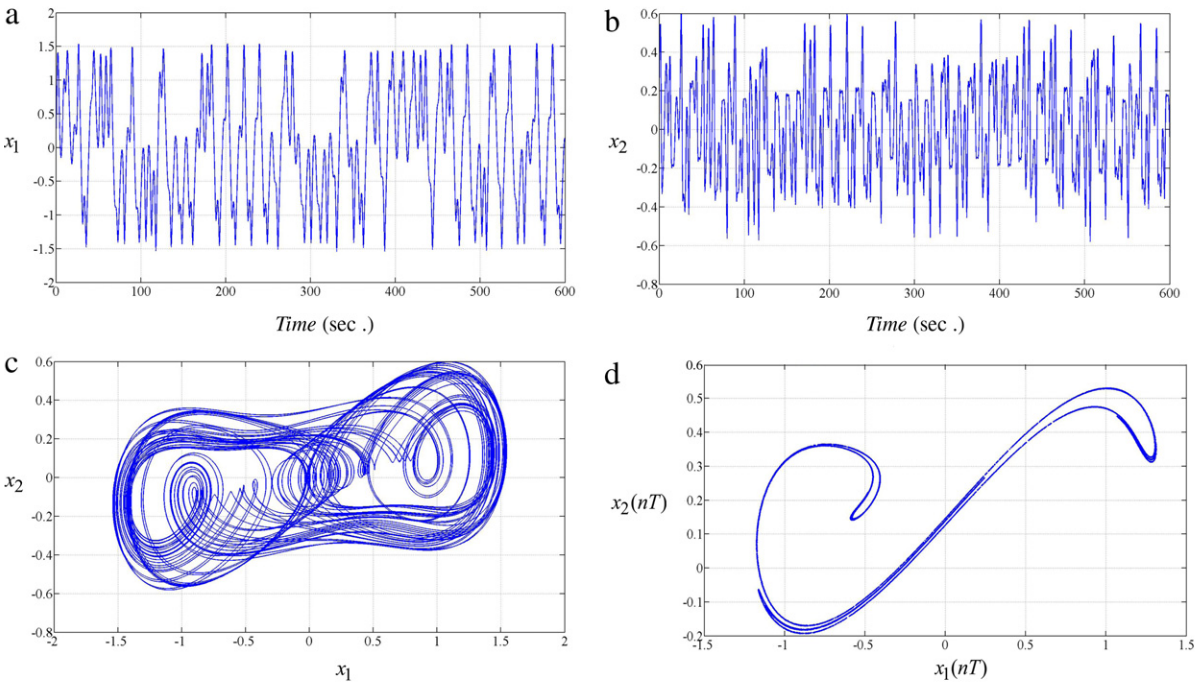

Fig. 3. Time responses (a), (b), phase plane trajectory (c) and poincarè map (d) with $\lambda = -0.5$.

where $x = [x_1, x_2]^T$ is the system state vector and $u(t)$ is the control input where controller u is the potency or dosage of the single and double nitrate isosorbide nitrate (isosorbide mononitrate and dinitrate) or the nitroglycerin nitroglycerin used for treating angina and other diseases, $d_i(x, t)$, $i = 1, 2$ are uncertain terms representing the un-modeled dynamics or structural variations of the system. Generally, the uncertainties d_{xi}, $i = 1, 2$ are assumedly bounded, i.e.,

$$|d_{xi}| \le \alpha_i, \quad i = 1, 2 \tag{3}$$

where $\alpha_i \ge 0$ are given. The dynamics of system (2) exhibit chaotic motion without control input. Further,

$$\begin{aligned} \dot{\tilde{x}}_1 &= -b\tilde{x}_1 - c\tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -(b+1)\lambda\tilde{x}_1 - (c+1)\lambda\tilde{x}_2 + \lambda\tilde{x}_1^3 + E\cos\omega t \end{aligned} \tag{4}$$

is the nominal system corresponding to system (2). System (4) can be either chaotic or non-chaotic, depending on the system parameters (4). Fig. 4 shows that the desired orbit is an unstable periodic trajectory of system (4). The control problem considered in this paper is that, for an unstable periodic orbit shown in Fig. 4(a) VSC law $u(t)$ is designed such that the resulting state responses of system (2) satisfies

$$\lim_{t\to\infty} \left\| x(t) - \tilde{x}(t) \right\| \to 0,$$

where $\|\cdot\|$ is the Euclidean norm of a vector.

The error states are then defined as

$$e_1 = x_1 - \tilde{x}_1; \qquad e_2 = x_2 - \tilde{x}_2. \tag{5}$$

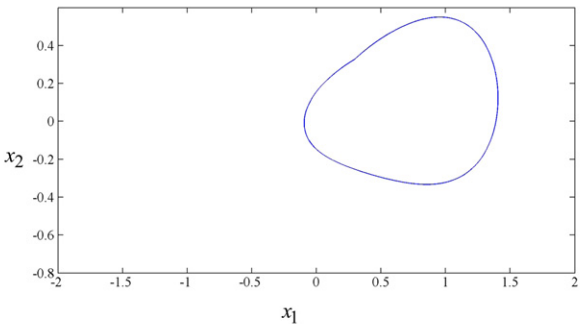

Fig. 4. The desired unstable periodic trajectory belong to the nominal system (4) (Fig. 2(c)) with $\lambda = -0.5$.

The dynamics of the error system are determined directly by Eqs. (2) and (4) as follows:

$$\dot{e}_1 = -be_1 - ce_2 + d_1 \tag{6a}$$

$$\dot{e}_2 = -(b+1)\lambda e_1 - (c+1)\lambda e_2 + \lambda(x_1^3 - \tilde{x}_1^3) + d_2 + u. \tag{6b}$$

The considered goal of this study is that, for any given coronary artery systems as (2) and (4), a VSC is designed such that the resulting tracking error can be driven to zero or into a predicable neighborhood of zero, i.e.,

$$\lim_{t\to\infty} |e_i| \le \gamma_i, \quad i = 1, 2 \tag{7}$$

where $\gamma_i \ge 0$ are constants depending on external uncertainties.

Therefore, this control goal for coronary artery systems with uncertainties is achieved in two major phases. First, an appropriate switching surface for the system must be selected such that the sliding motion on the manifold results in $\lim_{t\to\infty} |e_i| \le \gamma_i,\ i = 1, 2$ in any initial state and with any bounded external uncertainties. Second, the control law must be ensured the existence of the sliding mode.

4. Design of Chaos suppression controller

Consider the following switching function $s(t)$ corresponding to e_1 and e_2 in the error space:

$$s(t) = e_2 + \rho e_1 \tag{8}$$

where $s(t) \in R$ and ρ are design parameters that are easily determined later. In sliding mode, the system satisfies the following equations [10–12]:

$$\dot{s}(t) = \dot{e}_2 + \rho\dot{e}_1 = 0 \tag{9a}$$

and

$$s(t) = e_2 + \rho e_1 = 0. \tag{9b}$$

Therefore, the following sliding mode dynamics are

$$\dot{e}_1 = -(b - c\rho)e_1 + d_1 \tag{10a}$$

$$\dot{e}_2 = -(b+1)\lambda e_1 - (c+1)\lambda e_2 + \lambda(x_1^3 - \tilde{x}_1^3) + d_2 + u. \tag{10b}$$

Solving the differential equation (10a) for e_1 results in

$$e_1(t) = e^{-(b-c\rho)t}e_1(0) + \int_0^t e^{-(b-c\rho)(t-\tau)}d_1(\tau)d\tau. \tag{11}$$

The design parameters ρ are easily determined such that $b - c\rho > 0$. When the value for the ρ results in $b - c\rho > 0$, the bound for error state e_1 is

$$\begin{aligned} |e_1(t)| &= \left| e^{-(b-c\rho)t}e_1(0) + e^{-(b-c\rho)t}\int_0^t e^{(b-c\rho)\tau}d_1(\tau)d\tau \right| \\ &\le e^{-(b-c\rho)t}\,|e_1(0)| + \left(\max_{t_i\in[0,t]} |d_1(t_i)|\right)\left| e^{-(b-c\rho)t}\int_0^t e^{(b-c\rho)\tau}d\tau \right| \\ &\le e^{-(b-c\rho)t}\,|e_1(0)| + \alpha_1 \left| \frac{1 - e^{-(b-c\rho)t}}{b - c\rho} \right|. \end{aligned} \tag{12}$$

Eq. (12) with $(b - c\rho) > 0$ shows that

$$\lim_{t\to\infty} |e_1(t)| \le \gamma_1 = \frac{\alpha_1}{b - c\rho}. \tag{13}$$

Eq. (9b) also gives the bound for $e_2(t)$ at time $t \to \infty$:

$$\lim_{t\to\infty} |e_2(t)| = \lim_{t\to\infty} |\rho e_1(t)| \le \gamma_2 = \frac{\alpha_1 |\rho|}{b - c\rho}. \tag{14}$$

Since the above equations show that $e_1(t)$ and $e_2(t)$ converge to γ_1 and γ_2, there exist functions $\tilde{\gamma}_i(t),\ i = 1, 2$ such that

$$|e_i(t)| \le \tilde{\gamma}_i(t), \quad i = 1, 2 \tag{15}$$

and $\tilde{\gamma}_i(t) \to \gamma_i, i = 1, 2$, as $t \to \infty$. Thus, given any $\tilde{\gamma}_i > 0, i = 1, 2$, there exists a finite time t_1 such that $|e_i(t)| < \tilde{\gamma}_i, i = 1, 2$, for $t \ge t_1$.

Remark 1. Eqs. (13) and (14) show that, when the controlled nominal system, i.e., $d_1 = d_2 = 0$, is in sliding mode, the tracking error is driven to zero, *i.e.*, $\lim_{t\to\infty} |e_i| = 0, i = 1, 2$.

Although the error bound of the dynamics in sliding mode is ensured, a variable structure controller (VSC) is still needed to ensure that a sliding mode is available. Before presenting the controller scheme, the reaching condition of the sliding mode is given below. The motion of the sliding mode (8) is asymptotically stable if the following reaching condition holds true [11,12]

$$s\dot{s} < 0. \tag{16}$$

The $s(t)$ is then asymptotically stable in the switching surface and in sliding mode (8).

The proposed scheme for achieving the reaching condition indicated in Eq. (16) is

$$u(t) = ((b + 1)\lambda + b\rho)e_1 + ((c + 1)\lambda + c\rho)e_2 - \lambda(x_1^3 - \tilde{x}_1^3) - \beta \cdot k \cdot \operatorname{sign}(s), \quad \beta > 1 \tag{17}$$

where $k = \rho\alpha_1 + \alpha_2$.

The following theorem then proves that the proposed scheme (17) can drive the uncertain error in dynamic system (10) into sliding mode$s(t) = 0$.

Theorem 1. *For system (4), let the control $u(t)$ be as in (17). The error state of system (10) then converges to sliding mode $s(t) = 0$.*

Proof. Substituting Eqs. (6a), (6b), (8) and the control (17) into the derivative $s\dot{s}$

$$\begin{aligned} s\dot{s} &= s(\dot{e}_2 + \rho\dot{e}_1) \\ &= s[-(b + 1)\lambda e_1 - (c + 1)\lambda e_2 + \lambda(x_1^3 - \tilde{x}_1^3) + d_2 + u + \rho(-be_1 - ce_2 + d_1)] \\ &= s[d_2 + \rho d_1] - \beta k\,|s| \\ &\le (1 - \beta)k\,|s|\,. \end{aligned} \tag{18}$$

Since $\beta > 1$ has been selected in Eq. (17), one can conclude that the reaching condition ($s\dot{s} < 0$) is always satisfied. The proof is then complete. □

Remark 2. According to Eq. (18) the choice of β affects the convergence rate of $|s|$.

5. A modified continuous scheme without chattering

The continuous control law for avoiding chatter is

$$u(t) = ((b + 1)\lambda + b\rho)e_1 + ((c + 1)\lambda + c\rho)e_2 - \lambda(x_1^3 - \tilde{x}_1^3) - \beta \cdot k \cdot \frac{s}{|s| + \delta} \tag{19}$$

where $k = \rho\alpha_1 + \alpha_2, \beta > 1$ and $\delta(> 0)$ is sufficiently small. The following theorem then ensures that the proposed continuous scheme (19) drives the system (6) into a region arbitrarily close to the sliding mode $s(t) = 0$ without chatter.

Theorem 2. *Consider the error dynamics system (6). The error state of system (6) converges into an region arbitrarily close to $s(t) = 0$ if the control $u(t)$ is given by (19).*

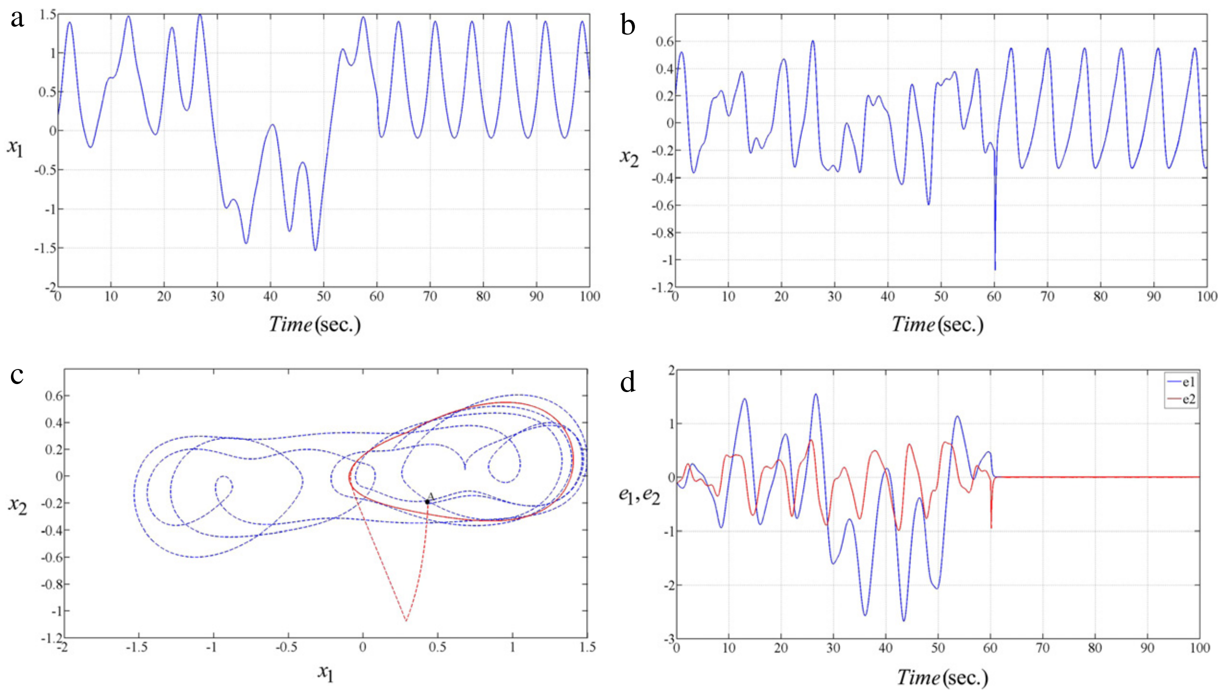

Fig. 5. The responses of coronary artery system with match disturbance $d_2 = 0.1\cos(3t)$: (a) time response of x_1, (b) time response of x_2, (c) phase plane trajectory of controlled system, and (d) error state responses.

Proof. Let the Lyapunov function of the system be $V = \frac{1}{2}s^2(t)$. Substituting Eqs. (6a), (6b), (8) and the control (19) into the derivative $s\dot{s}$ then obtains

$$\begin{aligned} s\dot{s} &= s(\dot{e}_2 + \rho\dot{e}_1) \\ &= s[-(b+1)\lambda e_1 - (c+1)\lambda e_2 + \lambda(x_1^3 - \tilde{x}_1^3) + d_2 + u + \rho(-be_1 - ce_2 + d_1)] \\ &= s[d_2 + \rho d_1] - \beta \cdot k \cdot \left(\frac{s^2}{|s| + \delta}\right) \\ &\le k\,|s| - \beta \cdot \mathrm{k} \cdot \left(|s| - \frac{|s|\,\delta}{|s| + \delta}\right). \end{aligned} \tag{20}$$

Since

$$\frac{\delta\,|s|}{|s| + \delta} \le \delta, \tag{21}$$

we have

$$\begin{aligned} s\dot{s} &\le (1-\beta)k\,|s| + \beta k\delta \\ &\le (1-\beta)k\left(|s| - \frac{\delta\beta}{\beta - 1}\right). \end{aligned} \tag{22}$$

Since $\beta > 1$ has been selected in (19), (22) implies that $\dot{V}(t) < 0$ whenever $|s| > \delta\frac{\beta}{\beta-1}$, i.e., $|s|$ is certain to converge to region $|s| < \delta\frac{\beta}{\beta-1}$. Since δ is a design parameter, a sufficiently small δ can be selected such that $|s|$ is arbitrarily bounded in the neighborhood of zero. The proof is complete. □

Remark 3. Chatter is then eliminated, and all results in the above section are available.

6. Numerical experiments

This section presents numerical experiments to demonstrate and confirm the performance of the present design. For the overall control system (1), the parameters are $b = 0.15$, $c = -1.7$, $\lambda = -0.5$, $E = 0.3$, $\omega = 1$ and $\rho = 3$, and the initial states of the controlled coronary artery system (2) are $x_1(0) = 0.2$, $x_2(0) = 0.2$.

Disturbance in the overall system is simulated first. Fig. 5(a)–(b) show the system time responses for the overall system with matched disturbance $d_2 = 0.1\cos(3t)$ and mismatched disturbance $d_1 = 0$. The responses show that the controlled

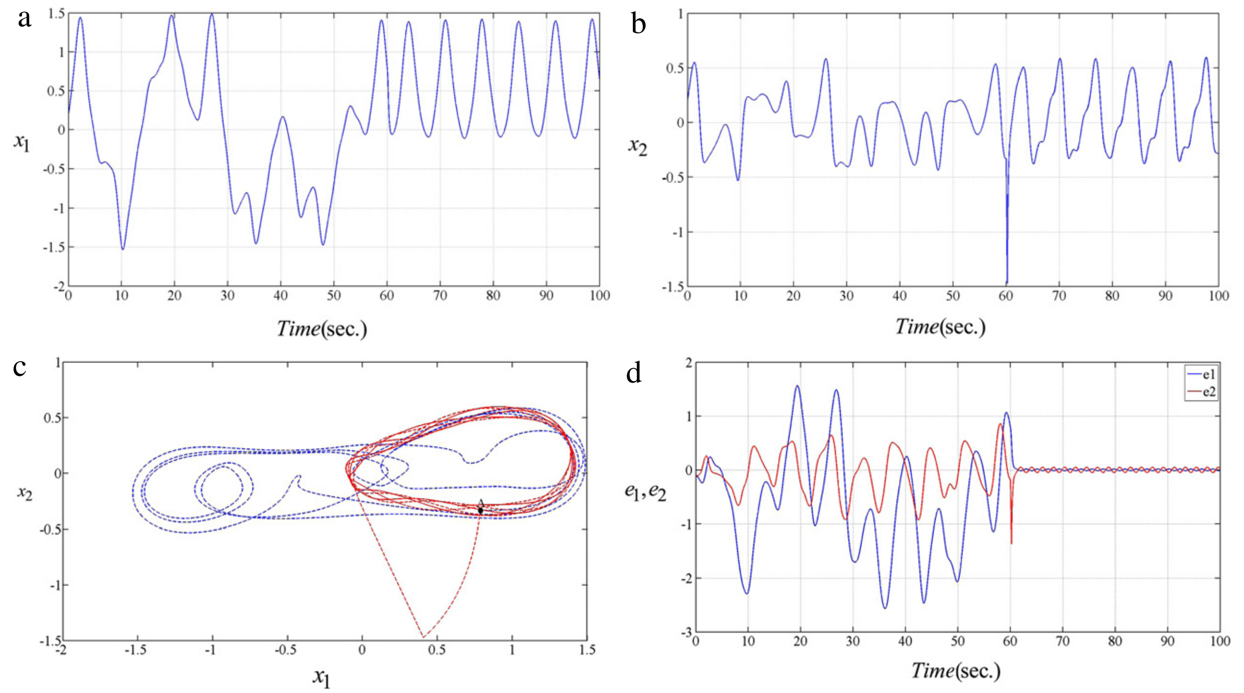

Fig. 6. The responses of coronary artery system with mismatch disturbance $d_1 = 0.1\cos(3t)$: (a) time response of x_1, (b) time response of x_2, (c) phase plane trajectory of controlled system, and (d) error state responses.

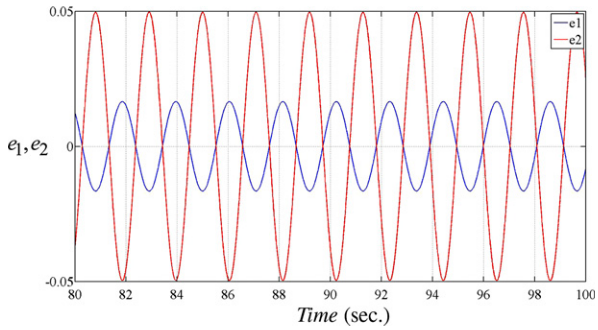

Fig. 7. The steady state error responses of controlled coronary artery system with mismatch disturbance $d_1 = 0.1\cos(3t)$.

coronary artery system (2) can be suppressed to an unstable periodic orbit of a nominal system (4) actively controlled for $t = 60$ s. Fig. 5(c)–(d) also show the phase plane trajectories and time responses of error states. The trajectories and error states show that the system error states are regulated to zero asymptotically, even under match disturbance in the overall system.

Fig. 6(a)–(d) show that, in the second simulation, the overall system responses are mismatched with disturbance $d_1 = 0.1\cos(3t)$ and matched with disturbance $d_2 = 0$. When control is active from $t = 60$ s, the mismatched disturbance d_1 causes the controlled system states to manifest a trajectory close to, but not synchronous with, the desired unstable periodic orbit of the nominal system (4). Fig. 7 clearly shows the time responses of error states in the time interval $t = 80 - 100$ s. A harmonic form of the error states e_1 and e_2 is clearly visible in the steady state, and they are all bounded in the ranges of $|d_1| \le \gamma_1 = \frac{\alpha_1}{b-c\rho} = 0.019$ and $|d_2| \le \gamma_2 = \frac{\alpha_1\rho}{b-c\rho} = 0.571$.

7. Conclusion

A variable structure control for chaos suppression control of coronary artery system is proposed. Based on the Lyapunov stability theory, a sliding mode controller is designed for regulating the error state vector to a desired point in the state space. In a coronary artery system with bounded disturbance, error states are then driven to zero or into a predicable neighborhood of zero in the steady state. The simulations confirm that the proposed method can solve synchronization problems in chaotic coronary artery systems. The derived controllers are sufficiently robust to maintain stability of the closed-loop system in the presence of uncertainties. The simulation results confirm that, since control input u (e.g., nitroglycerin) is quickly absorbed, blood vessels dilate and increase blood supply to the heart muscle, which effectively relieves or eliminates angina symptoms. Therefore, the VSC of muscular vascular biological mathematical model is valid. In a chaotic state and under any pre-specified suppression of unstable orbit in a nominal chaotic system, this study demonstrated that, for therapeutic purposes, a blood vessel spasm in a chaotic state can theoretically be suppressed with normal blood vessels.

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