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Crew rostering with multiple goals: An empirical study

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ABSTRACT

An empirical crew rostering problem drawn from the customer service section of a department store in southern Taiwan is addressed in this paper. The service section established relevant service facilities and functions to provide services for customers as well as distinguished guests and visitors. The crew rostering task is concerned with assigning multi-functional workers to different types of job and scheduling working shifts for each worker within a given time horizon, where the available and demand workforce vary from one shift to another. The current crew rostering method is a seniority orientation method. In developing the roster under current method, lines of job are generated and then bids are taken in order of decreasing seniority. The most senior worker has the widest range of job lines from which to select so as to best satisfy his/her preference. Successive crew members bid for the remaining lines of job. The current method has some drawbacks. To overcome the drawbacks, this paper develops a problem-specific approach with three stages to deal with the crew rostering problem, making it more equitable and personalized for workers by considering the management goals concerning worker–job suitability, worker–worker compatibility and worker–shift fondness. Due to the vagueness of job characteristics and the personal attributes, fuzzy method is used to improve the evaluation results of suitability, compatibility and fondness. The utility similarities of fuzzy assessments with the linguistic grade of very good are used to measure the fitness grade for the management goals. A linear goal programming model is proposed to fulfill the "efficient assignment/match from the right" policy. The proposed approach ensures the right workers are assigned to the right jobs, the right workers are placed together in a job and the pleasing working shifts are given to the workers. An illustrative application demonstrates the implementation of the proposed approach.

1. Introduction

Crew rostering has become increasingly important as business becomes more service oriented and cost conscious in a global environment (Ernst, Jiang, Krishnamoorthy, & Sier, 2004). The general crew rostering problem concerns how to construct timetables for workers, in which the working and rest days are scheduled, so the predicted workload is met by considering the constraints deriving from the restriction rules, the conditions of tasks and the characteristics of the workers. In this paper, an empirical crew rostering problem drawn from the customer service section of a department store in southern Taiwan is addressed. The crew rostering problem addressed in this study is more complicated. It is concerned with assigning multi-functional workers to different types of job and scheduling the working and rest shifts for each worker over a planning week (i.e., from Sunday to Saturday). The constructed individual schedules, or rosters, for the workers reveal two aspects of information for each worker, which are: (1) the jobs assigned and (2) the working and rest shifts scheduled. The result of the working shifts scheduled for the workers is typically to construct a set of shift worker assignments required to meet workforce demand. Ernst et al. (2004) presented a review of staff scheduling and rostering. They pointed out, because of changing work environments and conditions, it is likely rostering algorithms will need to be more general in the future. Since in practice each case has its own business characteristics and problems that must be faced and solved, a specific rostering method should be developed for a case problem. For example, Lezaun, Perez, and de la Maza (2006) proposed a sequence of four types of integer programming problems to tackle a crew rostering problem in a public transport company. Chu (2007) dealt with a personnel planning problem in an international airport by decomposing it into duties generating phase (a GP planner), followed by GP scheduling and rostering phase. Topaloglu and Selim (2010) applied fuzzy set theory to a nurse scheduling problem to deal with uncertainties in the target values of the hospital management and nurses' preferences.

Early studies held the assumption each worker can only perform one type of task (e.g., see Buffa, Cosgrove, & Luce, 1976).

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However, recent studies often assume each worker is able to perform multiple types of tasks, viz., worker multi-functionality. This property of multi-functionality has important effects, such as organizing the workforce more efficiently and expending the labor cost more economically (Zulch, Rottinger, & Vollstedt, 2004). Corominas, Pastor, and Rodriguez (2006) argued working time flexibility (Oke, 2000) and the multi-functionality of workers helps mould production capacity more closely to demand. The problem of assigning multi-functional workers to different types of task falls into two categories. The first is each worker can perform any type of task, which is known as completely multi-functional. The second is each worker can perform a given subset of types of task. In the literature, it is common to assume worker's qualities or efficiencies are equal for all those that can perform given types of tasks. For example, Bergman (1994) emphasized the importance of the multi-functional hospital staff. In that case, each worker could perform a given subset of task types based on individual cross-training and experience. Campbell and Diaby (2002) conducted a study of allocating the cross-trained workers to the tasks of multi-department. They suggested the research direction of considering the re-assignment of workers to tasks (i.e., task rotation) within the shifts. Later, Tharmmaphornphilas and Norman (2004) discussed the advantages of task rotation for reducing worker fatigue and injuries. Corominas et al. (2006) addressed a problem of assigning different task types to completely multi-functional workers in a retail chain selling clothes.

In practice, different types of task require varying levels of relevant skills and characteristics, such as attention, reaction, communication, responsibility, and stability. The workers efficiencies are not necessarily equal for all workers to perform each of the task types. As Corominas et al. (2006) suggested, future research should relax the assumption each worker can perform a given type of task at equal efficiency. Therefore, the suitability for each worker to perform each of the task types should be evaluated. For the tasks requiring two or more workers to work together, the relationship among workers, or compatibility (Nussbaum et al., 1999), should also be considered to make an appropriate worker assignment. In the study of Nussbaum et al. (1999), compatibility has been referred to as the ability of the project team to work together based on working style. de Korvin, Shipley, and Kleyle (2002) stated committing members to teams as needed is critical to project success and effectively utilizing human resources.

Regarding the evaluation of suitability between worker and task type and relationships among workers, job placement or personnel selection methods can be referred and quoted. Since job placement problems often require group decision-making under multiple criteria and uncertain and imprecise data, fuzzy methods have been employed to obtain outstanding outcomes. For example, Liang and Wang (1994) developed a fuzzy algorithm for combining both test-oriented objective assessments and interview-oriented subjective assessments to obtain the final ranking values for candidates. The triangular fuzzy numbers (TFNs) were used to quantify the linguistic assessments of subjective ratings and criteria weightings, and then the fuzzy suitability indices were computed. Yaakob and Kawata (1999) also used fuzzy method to deal with the worker placement problem. The relationship among workers is considered to make an appropriate workers' assignment. de Korvin et al. (2002) considered the match between the skills possessed by each worker, the skills needed for each project phase, and rather flexible budget considerations. They developed the fuzzy construct of compatibility to measure the fit of a worker skills set to the goal set for each project phase.

Eiselt and Marianov (2008) showed their consideration for problem of errors committed in assignments. They presented a skill space to map the acquired skill levels of workers and the required skill levels of tasks. After feasible task assignments are determined, tasks are assigned to workers to minimize worker–task distances. Rothstein and Goffin (2006) pointed out adopting personality measures adequately could add value to job placement practices. Later, Dursun and Karsak (2010) argued many personal attributes considered for job placement, such as creativity, organizing ability and leadership, are vague and imprecise. Therefore, they proposed a fuzzy multi-criteria decision-making method using both linguistic and numerical scales to deal with the evaluation.

The rest of this paper is organized as follows. Section 2 describes the problem background. In Section 3, the proposed approach is presented. Section 4 provides an illustrative application. Section 5 discusses the results. Finally, conclusions are given in Section 6.

2. Problem description

2.1. Service workforce and jobs

The crew rostering problem addressed in this study concerns the multi-functional workers' assignment and shift scheduling for a customer service section of a department store in southern Taiwan. The workforce team consists of eight full-time workers, denoted by w_1–w_8, and four part-time workers under contract, denoted by w_9–w_{12}. w_1–w_8 are the section's own workers and referred to as regular workers, while w_9–w_{12} are self employed contractors and referred to as contract workers in this study. The service section established relevant service facilities and functions to provide service for customers as well as distinguished guests and visitors. According to the business characteristics, the providing services are classified to five types and referred to as jobs 1–5. Since a job indicates the work station, jobs 1–5 are denoted by s_1–s_5 for convenience. Table 1 shows the work stations/locations and the tasks included in each job. The workforce team is composed of multi-functional service workers, viz., each worker is able to perform s_1–s_5. Since the worker qualities and efficiencies are based on individual skill, knowledge, experience and cross-training effects, w_1–w_{12} are not necessarily equally able to perform s_1–s_5.

The service section carries on business 12 h per day 7 days a week all year round. The work time of 12 h is split into morning shift (denoted by t_1, from 10 am to 4 pm) and evening shift (denoted by t_2, from 4 pm to 10 pm). There are two 6-h shift types each day for the workers to be scheduled and the working shifts of the workers are scheduled week by week. The number of customer (as well as distinguished guests and visitors) varies from 1 day to another during the week, meaning the service workforce

Table 1
Jobs and tasks contained.

Job	Station/location	Service tasks included and should be performed
s_1	VIP lounge	Exclusive service for very important persons and customers
s_2	Desk for general service	Exchange paper money for subsidiary coins, lend baby carriages, look for lost child, call taxi from the special station, point the way for customers to sales place and relevant service
s_3	Shop floors/sales place	Accompany the manager on duty to go on a tour of inspection of the shop floors and sales place
s_4	Lobby and counters of leased departments	Provide reception and guidance for the distinguished guests and visitors, and assist in sales promotion for leased departments
s_5	Desk for member-card holder service	Handle the member-card applications, provide consultation for the rights and interests of the members of the store, and deal with customer complaints

Table 2
Workforce needed in a planning week.

Job	Shift	d_1	d_2	d_3	d_4	d_5	d_6	d_7
s_1	t_1	1	1	1	1	1	1	1
	t_2	2	1	1	2	1	2	2
s_2	t_1	2	1	1	1	1	1	1
	t_2	2	1	1	1	1	2	2
s_3	t_1	1	1	1	1	1	1	1
	t_2	1	1	1	1	1	1	1
s_4	t_1	1	1	1	1	1	1	1
	t_2	2	1	1	1	1	1	2
s_5	t_1	2	1	1	1	1	1	2
	t_2	2	1	1	1	1	1	2
Training courses arranged and special leave received				w_5 and w_7		w_2		

needs to vary from one shift to another, viz., flexible demand workforce. A rule of thumb shows the sales promotion programs, anniversary celebration, and major festivals and holidays increase the number of customers as well as distinguished guests and visitors.

To enhance workers' professional skill, knowledge and multifunctionality, the regular and contract workers are arranged to take part in a series of training courses. According to the Labor Standards Act of Taiwan, where a regular worker continues to work for one and the same employer or business entity for a certain period of time he or she must be granted special leave on an annual basis. Thus, the workforce available for assigning to job is also flexible. Table 2 illustrates, for example, the workers needed for each job as well as the training courses arranged and special leave received for a planning week in May 2010, where the 7 days in a week are denoted by d_1–d_7. The bottom row shows w_5 and w_2 were arranged to take part in training courses on d_3 and d_5, respectively. w_7 received 1 day of special leave on d_3. According to Table 2, the number of workers needed per shift for each job and the workers available for assignment in the planning week are known and used in developing the rosters. For example, the number of worker necessary for s_1 is one in t_1 on d_1 (simplified as (d_1, t_1)), while it is two in t_2 on d_1 (simplified as (d_1, t_2)). The latter indicates (d_1, t_2) is a rush period for s_1 (viz., in that period the service load of s_1 is heavy), therefore, two workers are necessary for performing the service tasks. For convenience, such a rush period (d_1, t_2) for s_1 is simplified as (d_1, t_2, s_1). All 12 workers are available for assignment on d_1, d_2, d_4, d_6 and d_7. Since w_5 and w_2 were arranged for training courses on d_3 and d_5, respectively, and w_7 received special leave on d_3, therefore, ten of the 12 workers (excluding w_5 and w_7) and eleven of the 12 workers (excluding w_2) are available for assignment on d_3 and d_5, respectively. These data constitute the coverage constraints (Topaloglu & Selim, 2010) in developing rosters.

2.2. Restriction rules

In practice, how restriction rules of crew rostering are established varies greatly depending on the labor act, job property, business characteristic and philosophy of each firm. For the study section, the following assignation rules and conditions, constituting the time related constraints (Topaloglu & Selim, 2010) in developing rosters, must be met:

(1) The basic working hours for a regular worker are 42 a week, equaling to seven working shifts a week. The time a regular worker takes part in training courses or obtains special leave is included in basic working hours, with each day counted as 12 h or two shifts. A regular worker cannot have overtime in excess of 12 h, or two shifts, a week.
(2) A contract worker will not have working hours in excess of 48 h, or eight shifts, a week. The training courses for contract workers are arranged in the shift off periods after the rosters are constructed. The time for contract workers to take part in training courses is gratuitous.
(3) For each shift, each worker (excluding those arranged for training courses or special leave) is a candidate to be assigned to a suitable job.
(4) Each worker may be scheduled to work one shift or two shifts a day. If a worker works two shifts a day, then the two job types must differ, which is known as job rotation, to reduce worker drabness or monotony.
(5) In a week, each worker should have at least two full rest days, besides the days scheduled to work and those arranged for training courses and special leave.

2.3. Current crew rostering method

On Wednesday, the director of service section collects the relevant data for next planning week from marketing and human resource departments to confirm the demand and available workforce. The relevant data includes the prediction about oncoming number of customer (as well as distinguished guests and visitors) and the workers who are arranged to take part in training courses or received special leave. Then, the crew rostering over the next planning week is conducted.

For convenience in worker assignment, it is common to assume worker qualities or efficiencies are equal for all those that can perform a given type of task. For example, in the study of Corominas et al. (2006), they assumed for each type of task, each worker could perform the task at equal efficiency. The current method used by the study section is analogous to the bidding systems (Ernst et al., 2004) based on the assumption all 12 workers can perform each of the five jobs at equal levels of quality and efficiency. Ernst et al. (2004) stated in the bidding systems, lines of work are generated and then bid for by crew members. Bids are normally taken in order of decreasing seniority, the most senior staff having the widest range of work lines from which to select so as to best satisfy their preferences. Successive crew members bid for the remaining lines of work.

2.4. Drawbacks of current method and improvement thinking

The current method is a seniority orientation view of the bidding systems and it has some drawbacks. The drawbacks along with the resulted disadvantageous consequences and the improvement thinking are elaborated here.

In addition to the discrimination against the workers with less work experience and qualifications, the suitability of each worker for each job, termed as worker–job suitability, the relationship between two workers, termed as worker–worker compatibility, and the fondness of working shifts for the workers, termed as worker–shift fondness, have not been considered. In fact, for the crew rostering problem under consideration, the worker–job suitability, worker–worker compatibility and worker–shift fondness internally influence the work satisfaction and then externally affect the service quality and efficiency. The drawbacks certainly result in disadvantageous consequences. Eiselt and Marianov (2008) argued errors committed in assignments can have far-reaching consequences, such as reduced efficiency due to absenteeism, lack of job satisfaction, formal grievances, and generally deteriorating labor relations.

Ernst et al. (2004) argued in many organizations, the people involved in developing rosters need decision support tools to help provide the right workers at the right time and at the right cost while achieving a high level of worker job satisfaction. Topaloglu and Selim (2010) pointed out a high quality schedule can lead to a more contented and thus more effective workforce. In the opinion of the manager of the study section, the worker–job suitability, worker–worker compatibility and worker–shift fondness certainly influence the work satisfaction. Therefore, improving these three items should be the management goals in developing rosters. The thinking for improving the achievement of management goals is elaborated as follows.

Different jobs contain different tasks requiring varying levels of relevant skills and characteristics, such as attention, reaction, communication, responsibility and stability. In fact, the service efficiencies and qualities differ for the 12 workers to perform each of the five jobs. Ill effects, such as inefficiency and lack of job satisfaction, will arise from unsuitable worker assignment (Eiselt & Marianov, 2008). Therefore, the suitability for each worker to perform each job should be evaluated.

For the tasks requiring two or more workers to work together, Nussbaum et al. (1999) and de Korvin et al. (2002) pointed out that compatibility should be considered to make an appropriate worker assignment because committing members to teams as needed is critical to work success and effectively utilizing human resources. Thereby, it is necessary to design an effective tool by considering compatibility for assigning workers to work together to reduce potential noncooperation or conflict.

Lezaun et al. (2006) indicated shift work is always uncomfortable and a nuisance. In some organizations shift work is one of the basic causes of discontentment and complaints among workers. Ernst et al. (2004) argued as the modern workplace becomes more complex and as enterprise bargaining agreements become more focused on the individual, rather than on a group or team, it is likely roster solutions will also need to cater to individual preferences. Thereby, to construct a well-designed work timetable for each worker by increasing roster personalization, the fondness of different working shifts for each worker should be detected and considered in developing rosters.

3. Proposed approach

To avoid the drawbacks of the current method, and more importantly, construct more equitable and personalized work timetables for workers to internally increase work satisfaction and then externally improve the service quality and efficiency, an effective approach is proposed for the study section to construct rosters with the best compromise solution of multiple management goals in terms of suitability, compatibility and fondness. For convenience, the momentous notations for the general situation (i.e., N workers, K jobs, a planning period contains L days, F shifts in each day and each worker has at least $\hat{\pi}$ full rest days over the planning period) are listed in Table 3. The conceptual flow of the proposed approach is depicted in Fig. 1, which fulfills the "efficient assignment/match from the right" policy with three stages.

In the first stage, fuzzy assessments along with the associated classifications for worker–worker compatibility, worker–job suitability and worker–shift fondness are conducted using the database about the descriptions and specifications of jobs and that about the characteristics and skills inventories of workers. Assessment of suitability of (w_i, s_k) is conducted by a decision-making group. Assessment of compatibility of (w_i, w_j) is conducted by w_i and w_j each other on a two-way manner. In the two-way assessment, the compatibility of (w_i, w_j) is assessed by w_i on the one hand

Table 3
Momentous notations.

Indices and input parameters
w_i and w_j: Worker i and j, $i, j = 1, \ldots, N$
W_R: A set of regular worker
W_C: A set of contract worker
s_k: Job k, $k = 1, \ldots, K$
d_ℓ: Day ℓ over a planning period, $\ell = 1, \ldots, L$
t_f: Working shift f in a day, $f = 1, \ldots, F$
t_R^U and t_R^L: The maximal and minimal numbers of working shift, respectively, for a regular worker to be assigned over a planning period
t_C^U: The maximal number of working shift for a contract worker to be assigned over a planning period
D_m: Decision-maker m, $m = 1, \ldots, M$, for assessing the worker–job suitability
$\lambda_{(q)}$: The importance weight of criterion r_q, $q = 1, \ldots Q$, for assessing the worker–job suitability
$\tilde{\pi}^m_{ik(q)}$: the quantified TFN rating of the suitability between w_i and s_k (simplified as suitability of (w_i, s_k) hereafter) under criterion r_q evaluated by D_m
$U_T(B)$: The total utility value of linguistic data B
$\widetilde{Y}_{ik}$: The weighted fuzzy assessment of the suitability of (w_i, s_k)
$U_S(\widetilde{Y}_{ik}, B)$: The utility similarity of $\widetilde{Y}_{ik}$ with B. Let $y_{ik} = U_S(\widetilde{Y}_{ik}, VG)$
Ω: A set of feasible combinations (w_i, s_k) with linguistic grades passing the suitability threshold
$\varpi^i_{(v)}$: The importance weight of criterion c_v, $v = 1, \ldots, V$, set by w_i for evaluating the compatibility of the other workers
$\tilde{\rho}^i_{j(v)}$: The quantified TFN rating of the compatibility of w_j under criterion c_v evaluated by w_i
$\widetilde{E}^i_j$: The weighted fuzzy assessment of the compatibility of w_j evaluated by w_i
$U_S(\widetilde{E}^i_j, B)$: The utility similarity of $\widetilde{E}^i_j$ with B. Let $e^i_j = U_S(\widetilde{E}^i_j, VG)$
e_{ij}: The two-way assessment of compatibility between w_i and w_j (simplified as compatibility of (w_i, w_j) hereafter) evaluated by w_i and w_j each other. Let $e_{ij} = e^i_j + e^j_i$
Π: A set of feasible combinations (w_i, w_j) with linguistic grades passing the compatibility threshold
$\widetilde{P}^i_{\ell f}$: The quantified TFN rating of the fondness of (d_ℓ, t_f) evaluated by w_i (simplified as fondness of (w_i, d_ℓ, t_f) hereafter)
$U_S(\widetilde{P}^i_{\ell f}, VG)$: The utility similarity of $\widetilde{P}^i_{\ell f}$ with VG. Let $p_{i\ell f} = U_S(\widetilde{P}^i_{\ell f}, VG)$
Ψ_i: A set of feasible combinations (w_i, d_ℓ, t_f) with linguistic grades passing the fondness threshold
$\widehat{W}$: A set of workers who are arranged to take part in training courses or receive special leave over the planning period
$\widehat{D}_i$: A set of days that w_i is arranged for training courses or special leave over the planning period
$\hat{\theta}_i$: Total number of days included in $\widehat{D}_i$
$\hat{\theta}_{i\ell} = 1$: If w_i takes part in training courses or receives special leave on d_ℓ; $\hat{\theta}_{i\ell} = 0$, otherwise
$\eta_{\ell fk}$: Number of workers required by s_k in (d_ℓ, t_f)
Φ: A set of (d_ℓ, t_f, s_k) indicating two workers are required, i.e., $\eta_{\ell fk} = 2$
Decision variables
$\theta_{i\ell fk}$: A binary variable. If w_i is assigned to s_k in (d_ℓ, t_f), then $\theta_{i\ell fk} = 1$; otherwise, $\theta_{i\ell fk} = 0$

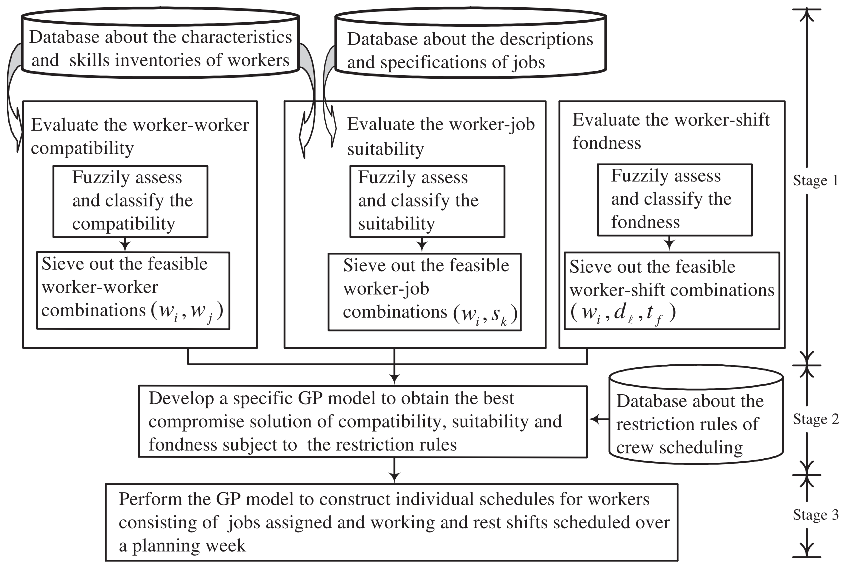

Fig. 1. The conceptual flow of proposed approach.

and by w_j on the other hand. Assessment of fondness of (d_ℓ, t_f) for w_i is conducted by w_i. According to the assessment and classification results, the feasible combinations of worker–job, worker–worker and worker–shift for constructing the effective rosters are then sieved out. In the second stage, a specific goal programming (GP) model is developed to construct the rosters, or individual schedules, for workers with the best compromise solution of multiple management goals subject to the restriction rules. In the third stage, the GP model is performed for generating rosters for workers that consist of jobs assigned and working and rest shifts scheduled over a planning week.

Table 4 shows the fuzzy scheme with seven linguistic judgment values used in this study, which is expanded from the fuzzy scheme with five linguistic judgment values proposed by Yaakob and Kawata (1999). In the fuzzy assessment and classification procedures, the ratings of suitability of (w_i, s_k), compatibility of (w_i, w_j) and fondness of (w_i, d_ℓ, t_f) are judged as linguistic variables. The judgment values of linguistic data are then quantified with TFNs, as shown in Table 4. The rationale for employing TFNs to capture the vagueness of the linguistic assessments is TFN is intuitively easy to use (Liang & Wang, 1994).

In column 3 of Table 4, the total utility value of linguistic data B, $U_T(B)$, $B = VG, G, \ldots, VP$, is calculated using the total utility function proposed by Chen (1985). In the author's study, the total utility values of TFNs are calculated with minimizing set and maximizing set to rank the TFNs. Since the total utility function (Chen, 1985) is easy to use and the application results are well (e.g., Hsieh & Chen, 1999; Lin, 2009), it is employed to obtain the values of $U_T(B)$ in the following analyses, and then $U_T(B)$ is used to derive the utility similarity of a TFN obtained from the assessment results. The calculation of $U_T(B)$ in this study is briefly described here. According to the fuzzy scheme of Table 4, the minimal value is set as $x_{\min} = 0$ and the maximal value as $x_{\max} = 22$. Then, the minimizing set is constructed as $\widetilde{H} = (x_{\min}, x_{\min}, x_{\max}) = (0, 0, 22)$ and maximizing set as $\widetilde{R} = (x_{\min}, x_{\max}, x_{\max}) = (0, 22, 22)$. Consider a TFN, say $\widetilde{X} = (x_a, x_b, x_c)$. The relationship for membership functions $\widetilde{H}$, $\widetilde{R}$ and $\widetilde{X}$ is depicted in Fig. 2. The left utility value and right utility value of $\widetilde{X}$, denoted by $U_H(\widetilde{X})$ and $U_R(\widetilde{X})$, respectively, are computed as follows:

$$U_H(\widetilde{X}) = \frac{x_{\max} - x_a}{(x_{\max} - x_{\min}) + (x_b - x_a)} \tag{1}$$

$$U_R(\widetilde{X}) = \frac{x_c - x_{\min}}{(x_{\max} - x_{\min}) - (x_b - x_c)} \tag{2}$$

Then, the total utility value of $\widetilde{X}$, denoted by $U_T(\widetilde{X})$, is calculated as

$$U_T(\widetilde{X}) = [U_R(\widetilde{X}) + 1 - U_H(\widetilde{X})]/2 \tag{3}$$

For the linguistic data G with a quantified TFN (18, 19, 20) in Table 4, for example, the left utility value and right utility value of G are computed by Eqs. (1) and (2) as follows:

$$U_H(G) = \frac{22 - 18}{(22 - 0) + (19 - 18)} = 0.174,$$
$$U_R(G) = \frac{20 - 0}{(22 - 0) - (19 - 20)} = 0.870.$$

Then, the total utility value of G, denoted by $U_T(G)$, is calculated by Eq. (3) as

$$U_T(G) = (0.870 + 1 - 0.174)/2 = 0.848.$$

Table 4
Linguistic variables and quantified TFNs.

Linguistic data (B)	TFN	Total utility value $U_T(B)$
Very good (VG)	(20,21,22)	0.935
Good (G)	(18,19,20)	0.848
Medium good (MG)	(13,16,18)	0.695
Fair (Fr)	(9,11,13)	0.5
Medium poor (MP)	(4,6,9)	0.305
Poor (P)	(2,3,4)	0.152
Very poor (VP)	(0,1,2)	0.065

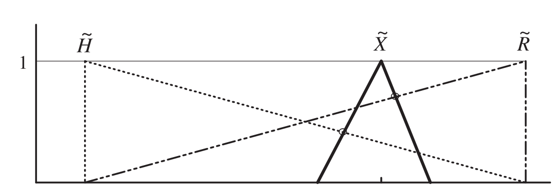

Fig. 2. The membership functions of $\widetilde{H}$, $\widetilde{R}$ and $\widetilde{X}$.

3.1. Fuzzy assessment of worker–job suitability

The suitability of (w_i, s_k) is assessed by a decision-making group as follows:

Step 1: Constitute a group consisting of M decision-makers, D_m, $m = 1, \ldots, M$, and select the appropriate Q criteria, r_1–r_Q, for assessing the worker–job suitability.

Step 2: Each decision-maker conducts pairwise comparisons for Q criteria with Saaty's 1–9 scale (Saaty, 1980) to show the relative importance of the criteria. Then, the geometric mean method suggested by Dyer and Forman (1992) is employed to include the decision-makers' judgments for determining the aggregate importance weights of Q criteria, $\lambda_{(q)}$, $q = 1, \ldots, Q$.

Step 3: To assess the suitability of (w_i, s_k), the decision-making group uses the fuzzy scheme shown in Table 4 to conduct the assessments under each criterion. Let $\tilde{\pi}^m_{ik(q)} = (\pi^m_{ik(q)a}, \pi^m_{ik(q)b}, \pi^m_{ik(q)c})$ be the quantified TFN of the linguistic rating assigned to the suitability of (w_i, s_k) under criterion r_q evaluated by D_m. Then, by applying the mean aggregation rule to pool the M decision-makers' judgments, the aggregate fuzzy assessment of the suitability of (w_i, s_k) under r_q is

$$\tilde{\pi}_{ik(q)} = \left(\sum_{m=1}^{M} \pi^m_{ik(q)a} \Big/ M, \sum_{m=1}^{M} \pi^m_{ik(q)b} \Big/ M, \sum_{m=1}^{M} \pi^m_{ik(q)c} \Big/ M \right) \tag{4}$$

The weighted fuzzy assessment of the suitability of (w_i, s_k), $\widetilde{Y}_{ik}$, can be obtained by incorporating $\tilde{\pi}_{ik(q)}$ with $\lambda_{(q)}$ as follows:

$$\widetilde{Y}_{ik} = (\lambda_{(1)} \otimes \tilde{\pi}_{ik(1)}) \oplus (\lambda_{(2)} \otimes \tilde{\pi}_{ik(2)}) \oplus \cdots \oplus (\lambda_{(Q)} \otimes \tilde{\pi}_{ik(Q)}) \tag{5}$$

Step 4: Compute the total utility value of $\widetilde{Y}_{ik}$, $U_T(\widetilde{Y}_{ik})$, and then use the utility similarity function proposed by Hsieh and Chen (1999) to calculate the utility similarity of $\widetilde{Y}_{ik}$ with linguistic data B, $U_S(\widetilde{Y}_{ik}, B)$. Using their utility similarity function, $U_S(\widetilde{Y}_{ik}, B)$ can be calculated as follows:

$$\begin{aligned} U_S(\widetilde{Y}_{ik}, B) &= \frac{U_T(\widetilde{Y}_{ik}) \times U_T(B)}{\max\{U_T(\widetilde{Y}_{ik}) \times U_T(\widetilde{Y}_{ik}), U_T(B) \times U_T(B)\}} \\ &= \frac{\min\{U_T(\widetilde{Y}_{ik}), U_T(B)\}}{\max\{U_T(\widetilde{Y}_{ik}), U_T(B)\}} \end{aligned} \tag{6}$$

Step 5: For classifying $\widetilde{Y}_{ik}$ to an appropriate linguistic grade, choose the corresponding best linguistic data of $\widetilde{Y}_{ik}$ with the largest utility similarity to it according to the values of $U_S(\widetilde{Y}_{ik}, B)$. Since the ideal linguistic grade is VG, the utility similarity of $\widetilde{Y}_{ik}$ with VG is selected to express the degree of suitability of (w_i, s_k). The degree of suitability, denoted and set as $y_{ik} = U_S(\widetilde{Y}_{ik}, VG)$, is used to measure the satisfaction grade for assigning w_i to s_k. The maximal value of y_{ik} is one in theory, which reveals the ideal situation.

Step 6: Set an appropriate threshold to screen the assessment results of suitability of (w_i, s_k). The worker–job combinations with suitability linguistic grades passing the threshold are sieved out and included in set Ω. These feasible worker–job combinations are used in the jobs placement for workers.

3.2. Fuzzy assessment of worker–worker compatibility

The compatibility of (w_i, w_j) is assessed by w_i and w_j each other on a two-way manner as follows:

Step 7: Each worker, say w_i, selects V criteria, c_1–c_V, which he or she pays close attention for assessing the compatibility between him or her and the other workers. w_i conducts the pairwise comparisons for V criteria with Saaty's 1–9 scale (1980) to obtain the importance weights of V criteria, $\varpi^i_{(v)}$, $v = 1, \ldots, V$.

Step 8: w_i uses the fuzzy scheme shown in Table 4 to assess the compatibility between him or her and w_j, $j = 1, \ldots, N$, $j \neq i$, under criterion c_v. Let $\tilde{\rho}^i_{j(v)} = (\rho^i_{j(v)a}, \rho^i_{j(v)b}, \rho^i_{j(v)c})$ be the quantified TFN of the linguistic rating for the compatibility of (w_i, w_j) under criterion c_v assigned by w_i. Then, the weighted fuzzy assessment of the compatibility of (w_i, w_j) evaluated by w_i, $\widetilde{E}^i_j$, can be obtained by incorporating $\tilde{\rho}^i_{j(v)}$ with $\varpi^i_{(v)}$ as follows:

$$\widetilde{E}^i_j = (\varpi^i_{(1)} \otimes \tilde{\rho}^i_{j(1)}) \oplus (\varpi^i_{(2)} \otimes \tilde{\rho}^i_{j(2)}) \oplus \cdots \oplus (\varpi^i_{(V)} \otimes \tilde{\rho}^i_{j(V)}) \tag{7}$$

Step 9: Compute the total utility value of $\widetilde{E}^i_j$, $U_T(\widetilde{E}^i_j)$, and then calculate the utility similarity of $\widetilde{E}^i_j$ with linguistic data B, $U_S(\widetilde{E}^i_j, B)$, as follows:

$$U_S(\widetilde{E}^i_j, B) = \frac{\min\{U_T(\widetilde{E}^i_j), U_T(B)\}}{\max\{U_T(\widetilde{E}^i_j), U_T(B)\}} \tag{8}$$

Step 10: For classifying $\widetilde{E}^i_j$ to an appropriate linguistic grade, choose the corresponding best linguistic data of $\widetilde{E}^i_j$ with the largest utility similarity to it according to the values of $U_S(\widetilde{E}^i_j, B)$. Since the ideal linguistic grade is VG, the utility similarity of $\widetilde{E}^i_j$ with VG is selected to express the degree of compatibility of w_j evaluated by w_i. The degree of compatibility, denoted and set as $e^i_j = U_S(\widetilde{E}^i_j, VG)$, is used to measure the satisfaction grade of w_i for working together with w_j. The maximal value of e^i_j is one in theory, revealing the ideal situation.

Step 11: Collect the assessment results of the compatibility of (w_i, w_j) evaluated respectively by w_i and w_j to form the two-way assessment outcomes. Calculate the two-way degree of compatibility of (w_i, w_j) as $e_{ij} = e^i_j + e^j_i$. Thus, e_{ij} reflects the two-way satisfaction grade for assigning w_i and w_j to work together. Both the maximal values of e^i_j and e^j_i are one in theory, and therefore the maximal value of e_{ij} is two.

Step 12: Set an appropriate threshold to screen the two-way assessment results of the compatibility of (w_i, w_j). The worker–worker combinations with two-way compatibility linguistic grades passing the threshold are sieved out and included in set Π. These feasible worker–worker combinations are considered for assigning together to the jobs requiring two workers.

3.3. Fuzzy assessment of worker–shift fondness

The fondness of (d_ℓ, t_f) for w_i is assessed by w_i in the following steps:

Step 13: Set guides for expressing the fondness grades of working shifts.

Step 14: Each worker, say w_i, conveys his or her fondness grades for the working shifts, excluding the shifts which are arranged for training courses or special leave. Then, the fuzzy scheme shown in Table 4 is used to quantify the fondness grades of shift (d_ℓ, t_f) evaluated by w_i as a TFN $\widetilde{P}^i_{\ell f}$. Compute the utility similarity of $\widetilde{P}^i_{\ell f}$ with linguistic data VG, $U_S(\widetilde{P}^i_{\ell f}, VG)$. Since the ideal linguistic grade is VG, the value of $U_S(\widetilde{P}^i_{\ell f}, VG)$ is selected to express the degree of fondness of (d_ℓ, t_f) for w_i. That is, set $p_{i\ell f} = U_S(\widetilde{P}^i_{\ell f}, VG)$ to express the degree of fondness of (w_i, d_ℓ, t_f), which measures the satisfaction grade for allocating w_i to work in (d_ℓ, t_f). The maximum value of $p_{i\ell f}$ is one in theory, which reveals the ideal situation.

Step 15: Set an appropriate threshold to screen the assessment results of fondness of (w_i, d_ℓ, t_f). For w_i, the worker–shift combinations with fondness linguistic grades passing the threshold are sieved out and included in set Ψ_i. These feasible worker–shift combinations are used to schedule w_i for working in his or her pleasing shifts.

3.4. Goal programming model

The GP formulation to construct the rosters for the problem in a general situation is proposed as follows:

$$\text{Min } \alpha G_s + \beta G_c + \gamma G_p \quad \text{(m0)}$$

$$\text{s.t. } \sum_{i=1}^{N}\sum_{\ell=1}^{L}\sum_{f=1}^{F}\sum_{k=1}^{K} y_{ik}\cdot\theta_{i\ell fk} + G_s = \sum_{i=1}^{N}\sum_{\ell=1}^{L}\sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} \quad \text{(m1)}$$

$$\sum_{(d_\ell,t_f,s_k)\in\Phi}\sum_{(w_i,w_j)\in\Pi} e_{ij}\cdot\theta_{i\ell fk}\cdot\theta_{j\ell fk} + G_c = \sum_{(d_\ell,t_f,s_k)\in\Phi}\sum_{(w_i,w_j)\in\Pi} 2\cdot\theta_{i\ell fk}\cdot\theta_{j\ell fk} \quad \text{(m2)}$$

$$\sum_{i=1}^{N}\sum_{\ell=1}^{L}\sum_{f=1}^{F} p_{i\ell f}\cdot\left(\sum_{k=1}^{K}\theta_{i\ell fk}\right) + G_p = \sum_{i=1}^{N}\sum_{\ell=1}^{L}\sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} \quad \text{(m3)}$$

$$\sum_{\ell=1}^{L}\sum_{f=1}^{F}\theta_{i\ell fk} = 0, \quad (w_i,s_k)\notin\Omega \quad \text{(m4)}$$

$$\sum_{k=1}^{K}\theta_{i\ell fk} = 0, \quad (w_i,d_\ell,t_f)\notin\Psi_i \quad \text{(m5)}$$

$$F\hat{\theta}_{i\ell} + \sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} \leqslant F, \quad \forall i,\ell \quad \text{(m6)}$$

$$\hat{\theta}_i = \sum_{d_\ell\in\hat{D}_i}\hat{\theta}_{i\ell}, \quad w_i\in\widehat{W} \quad \text{(m7)}$$

$$\hat{\theta}_i = 0, \quad w_i\notin\widehat{W} \quad \text{(m8)}$$

$$t_R^L \leqslant \sum_{\ell=1}^{L}\sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} + F\hat{\theta}_i \leqslant t_R^U, \quad w_i\in W_R \quad \text{(m9)}$$

$$\sum_{\ell=1}^{L}\sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} \leqslant t_C^U, \quad w_i\in W_C \quad \text{(m10)}$$

$$\sum_{i=1}^{N}\theta_{i\ell fk} = \eta_{\ell fk}, \quad \forall\ell,f,k \quad \text{(m11)}$$

$$\sum_{k=1}^{K}\theta_{i\ell fk} \leqslant 1, \quad \forall i,\ell,f \quad \text{(m12)}$$

$$\theta_{i\ell fk} + \theta_{j\ell fk} \leqslant 1, \quad (d_\ell,t_f,s_k)\in\Phi, \quad (w_i,w_j)\notin\Pi \quad \text{(m13)}$$

$$\sum_{f=1}^{F}\theta_{i\ell fk} \leqslant 1, \quad \forall i,\ell,k \quad \text{(m14)}$$

$$\sum_{\ell=1}^{L}\left[\left(1-\sum_{k=1}^{K}\theta_{i\ell 1k}\right)\cdot\left(1-\sum_{k=1}^{K}\theta_{i\ell 2k}\right)\cdot\ldots\cdot\left(1-\sum_{k=1}^{K}\theta_{i\ell Fk}\right)\right] \geqslant \hat{\pi} + \hat{\theta}_i, \quad \forall i \quad \text{(m15)}$$

$$\theta_{i\ell fk}\in\{0,1\}, \quad \forall i,\ \ell,f,k \quad \text{(m16)}$$

The objective function (m0) is a compromise solution for minimizing the deviations below the ideal values of management goals. The values of α, β and γ are set by the decision-maker to reflect the relative importance of each objective. Constraint (m1) represents a flexible goal in which the degree of suitability of all assignments, viz. the satisfaction grade for assigning w_i to s_k, may be below the ideal value. Constraint (m2) represents a flexible goal in which the degree of compatibility of all assignments, viz. the satisfaction grade for assigning w_i and w_j to work together, may be below the ideal value. Constraint (m3) represents a flexible goal in which the degree of fondness of all schedules, viz. the satisfaction grade for scheduling w_i to work in (d_ℓ, t_f), may be below the ideal value. Constraint (m4) prohibits w_i from working on s_k if (w_i, s_k) is not included in Ω. This is equivalent to fix the value of variables $\theta_{i\ell fk}$ to 0 for each $(w_i, s_k)\notin\Omega$. To prohibit w_i from working in (d_ℓ, t_f) if (w_i, d_ℓ, t_f) is not included in Ψ_i, Constraint (m5) set the value of variables $\theta_{i\ell fk}$ to 0 for each $(w_i, d_\ell, t_f)\notin\Psi_i$. In Constraint (m6), if w_i is arranged for training courses or special leave on d_ℓ (i.e., $\hat{\theta}_{i\ell}=1$ and $F\hat{\theta}_{i\ell}=F$), then the value of variables $\theta_{i\ell fk}$ is driven to be zero (i.e., w_i cannot be assigned to any jobs on d_ℓ); otherwise (i.e., $\hat{\theta}_{i\ell}=0$ and $F\hat{\theta}_{i\ell}=0$), the value of variables $\theta_{i\ell fk}$ can be as zero or one (i.e., w_i can be day off or assigned to any suitable jobs on d_ℓ). In Constraints (m7) and (m8), the number of days arranged for training courses or special leaves for $w_i\in\widehat{W}$ is calculated. For $w_i\notin\widehat{W}$, such number of days is set to zero. Constraint (m9) restricts the maximal and minimal numbers of working shifts for a regular worker, including the shifts scheduled to work and those arranged for training courses and special leave. In which 1 day for training courses or special leave is accounted for as F shifts. Constraint (m10) restricts the maximal number of working shifts for a contract worker. Constraint (m11) stipulates the actual number of workers assigned to a job in a shift should meet the required number. Constraint (m12) indicates each worker is assigned to one job at most in any shift. Constraint (m13) prohibits w_i from working together with w_j if (w_i, w_j) is not included in Π. In Constraint (m14), the number of shift w_i works at s_k on d_ℓ is one at most. This accomplishes the rule of job rotation in one working day. Constraint (m15) indicates each worker has at least $\hat{\pi}$ full rest days over the planning period, besides the days scheduled to work and those arranged for training courses and special leave. Constraint (m16) indicates $\theta_{i\ell kf}$ is a binary variable.

Constraints (m2) and (m15) of the GP formulation are clearly non-linear. In general, it is difficult to solve non-linear integer programming problem and in many cases only feasible but not optimal solutions can be obtained. In order to optimally solve the problem and generalize the proposed method to larger size problems, the non-linear Constraints (m2) and (m15) are further reformulated as linear constraints. Constraint (m2) is reformulated as following linear constraints:

$$0 \leqslant \theta_{i\ell fk} + \theta_{j\ell fk} - 2\pi_{ij\ell fk} \leqslant 1, \quad (d_\ell,t_f,s_k)\in\Phi, \quad (w_i,w_j)\in\Pi \quad \text{(m2}'\text{)}$$

$$\sum_{(d_\ell,t_f,s_k)\in\Phi}\sum_{(w_i,w_j)\in\Pi} e_{ij}\cdot\pi_{ij\ell fk} + G_c = \sum_{(d_\ell,t_f,s_k)\in\Phi}\sum_{(w_i,w_j)\in\Pi} 2\cdot\pi_{ij\ell fk} \quad \text{(m2}''\text{)}$$

$$\pi_{ij\ell fk}\in\{0,1\}, \quad (d_\ell,t_f,s_k)\in\Phi, \quad (w_i,w_j)\in\Pi \quad \text{(m2}'''\text{)}$$

Constraint (m2′) determines the value of $\pi_{ij\ell fk}$ to be either one or zero. If w_i and w_j are assigned to work together at s_k in (d_ℓ, t_f) (i.e., $\theta_{i\ell fk} = \theta_{j\ell fk} = 1$), the value of $\pi_{ij\ell fk}$ is determined as one; otherwise, the value of $\pi_{ij\ell fk}$ is determined as zero.

Constraint (m15) is reformulated as following linear constraints:

$$1 \leqslant \sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk} + F\cdot H_{i\ell} \leqslant F, \quad \forall i,\ell \quad \text{(m15}'\text{)}$$

$$\sum_{\ell=1}^{L} H_{i\ell} \geqslant \hat{\pi} + \hat{\theta}_i, \quad \forall i \quad \text{(m15}''\text{)}$$

$$H_{i\ell}\in\{0,1\}, \quad \forall i,\ \ell \quad \text{(m15}'''\text{)}$$

Constraint (m15′) determines the value of $H_{i\ell}$ to be either one or zero. If w_i is day off on d_ℓ $\left(\text{i.e., } \sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk}=0\right)$, the value of $H_{i\ell}$ is determined as one; otherwise $\left(\text{i.e., } 1\leqslant\sum_{f=1}^{F}\sum_{k=1}^{K}\theta_{i\ell fk}\leqslant F\right)$, the value of $H_{i\ell}$ is determined as zero.

4. Illustrative application

The data shown in Table 2 and restriction rules described in Section 2.2 are used as an empirical example to illustrate the implementation of the proposed approach.

Steps 1–2: Two professional staff appraised the suitability of each of the 12 workers for each of the five jobs. These two experts are the directors of service section and human resource department, denoted by D_1 and D_2, respectively. The used criteria include reaction capacity, communication skill, friendliness, attentiveness and stability, denoted by r_1–r_5, respectively. The pairwise comparisons for these criteria conducted by D_1 and D_2 are shown in Tables 5 and 6, respectively. Table 7 includes the judgments of D_1 and D_2 by the geometric mean method, where the last column lists the importance weights of the criteria.

Table 5
Pairwise comparison matrix for criteria conducted by D_1.

	r_1	r_2	r_3	r_4	r_5
r_1	1	0.3333	2	0.25	2
r_2	3	1	3	2	5
r_3	0.5	0.3333	1	0.3333	4
r_4	4	0.5	3	1	5
r_5	0.5	0.2	0.25	0.2	1

CR = 0.07 < 0.1.

Table 6
Pairwise comparison matrix for criteria conducted by D_2.

	r_1	r_2	r_3	r_4	r_5
r_1	1	0.5	2	1	4
r_2	2	1	3	3	4
r_3	0.5	0.3333	1	0.3333	5
r_4	1	0.3333	3	1	3
r_5	0.25	0.25	0.2	0.33333	1

CR = 0.08 < 0.1.

Table 7
Aggregate pairwise comparison matrix for criteria.

	r_1	r_2	r_3	r_4	r_5	$\lambda_{(q)}$
r_1	1	0.408	2	0.5	2.828	0.166
r_2	2.449	1	3	2.449	4.472	0.389
r_3	0.5	0.333	1	0.333	4.472	0.134
r_4	2	0.408	3	1	3.873	0.253
r_5	0.354	0.224	0.224	0.258	1	0.058

CR = 0.07 < 0.1.

Table 8
Assessments of suitability of (w_2,s_1).

	r_1	r_2	r_3	r_4	r_5
D_1	*MG* (13,16,18)	*Fr* (9,11,13)	*MP* (4,6,9)	*G* (18,19,20)	*Fr* (9,11,13)
D_2	*Fr* (9,11,13)	*MG* (13,16,18)	*Fr* (9,11,13)	*VG* (20,21,22)	*MP* (4,6,9)
Aggregate	(11,13.5,15.5)	(11,13.5,15.5)	(6.5,8.5,11)	(19,20,21)	(6.5,8.5,11)
$\lambda_{(q)}$	0.166	0.389	0.134	0.253	0.058

Weighted TFN $\widetilde{Y}_{21} = (12.16, 14.19, 16.03)$.

Table 9
Assessment results of suitability between w_2 and each of the jobs.

	s_1	s_2	s_3	s_4	s_5
w_2	0.675[a] *MG*[b]	0.513 *Fr*	0.432 *Fr*	0.865 *G*	0.096 *VP*

[a] Degree of suitability of (w_2,s_1).
[b] Classified linguistic grade of (w_2,s_1).

Step 3: The assessment results of the suitability of (w_2, s_1), for example, under r_1–r_5 are shown in Table 8. $\widetilde{Y}_{21}$ is obtained as (12.16,14.19,16.03).

Steps 4–5: For $\widetilde{Y}_{21} = (12.16, 14.19, 16.03)$, the total utility value of $\widetilde{Y}_{21}$ is computed as $U_T(\widetilde{Y}_{21}) = (0.6723 + 1 - 0.4095)/2 = 0.631$. Then, the values of utility similarity of $\widetilde{Y}_{21}$ with linguistic data *VG*–*VP* are calculated by Eq. (6) as follows:

$$U_S(\widetilde{Y}_{21}, VG) = \frac{\min\{U_T(\widetilde{Y}_{21}), U_T(VG)\}}{\max\{U_T(\widetilde{Y}_{21}), U_T(VG)\}} = \frac{\min\{0.631, 0.935\}}{\max\{0.631, 0.935\}} = 0.675,$$

$$U_S(\widetilde{Y}_{21}, G) = 0.745, \quad U_S(\widetilde{Y}_{21}, MG) = 0.908, \quad U_S(\widetilde{Y}_{21}, F) = 0.792,$$

$$U_S(\widetilde{Y}_{21}, MP) = 0.483, \quad U_S(\widetilde{Y}_{21}, P) = 0.241, \quad U_S(\widetilde{Y}_{21}, VP) = 0.103,$$

Since the largest value of utility similarity is $U_S(\widetilde{Y}_{21}, MG) = 0.908$, the linguistic grade of suitability of (w_2, s_1) is classified as *MG*. The degree of suitability of (w_2, s_1) is set as $y_{21} = U_S(\widetilde{Y}_{21}, VG) = 0.675$. Table 9 shows, for example, the assessment results, including degree of suitability and classified linguistic grades, of the combinations (w_2, s_k), $k = 1, \ldots, 5$.

Step 6: The threshold of suitability relation is set as linguistic grade of *Fr*. The results depicted in Table 9 show w_2 can be assigned to s_1, s_2, s_3 and s_4, whereas s_5 cannot. Therefore, the feasible worker–job combinations, (w_2, s_1), (w_2, s_2), (w_2, s_3) and (w_2, s_4), are included in Ω.

Steps 7–11: The compatibility evaluation is conducted in a two-way manner. For example, consider the compatibility between w_1 and w_2. On the one hand, w_2 uses his concerning two criteria (c_1 indicates the mutual understanding and c_2 indicates the tolerance in attitude), with importance weights $\varpi^2_{(1)} = 0.625$ and $\varpi^2_{(2)} = 0.375$, to evaluate the compatibility of (w_1, w_2). Table 10 depicts the assessment results. The degree of compatibility of (w_1, w_2) evaluated by w_2 is $e^2_1 = U_S(\widetilde{E}^2_1, VG) = 0.389$ and the associated linguistic grade classified by w_2 is *MP*. These are listed in the top cell of the first column of Table 11. On the other hand, w_1 uses his concerning three criteria (c_1 indicates the familiarity, c_2 indicates the unspoken consensus and c_3 indicates the tolerance in attitude), with importance weights $\varpi^1_{(1)} = 0.216$, $\varpi^1_{(2)} = 0.681$ and $\varpi^1_{(3)} = 0.103$, to evaluate the compatibility of (w_1, w_2). The degree of compatibility of (w_1, w_2) evaluated by w_1 is $e^1_2 = U_S(\widetilde{E}^1_2, VG) = 0.701$ and the associated linguistic grade classified by w_1 is *MG*. These are listed in the middle cell of the first column of Table 11. The two-way degree of compatibility of (w_1, w_2) is calculated as $e_{12} = e^1_2 + e^2_1 = 1.09$, which is listed in the bottom cell of the first column of Table 11.

Step 12: The threshold of worker–worker compatibility is set as the linguistic grade of *F*. The results shown in Table 11 show the two-way linguistic grades of the seven worker–worker combinations, (w_2, w_3), (w_2, w_5), (w_2, w_6), (w_2, w_7), (w_2, w_8), (w_2, w_{10}) and (w_2, w_{11}), pass the threshold. Under the threshold, w_3, w_5, w_6, w_7, w_8, w_{10} and w_{11} can perform a job together with w_2, whereas w_1, w_4, w_9 and w_{12} cannot. Therefore, the seven combinations are included in Π.

Table 10
Compatibility of (w_1,w_2) evaluated by w_2.

c_1	c_2	$\widetilde{E}^2_1$	$U_S(\widetilde{E}^2_1, B)$							Linguistic grade
			VG	*G*	*MG*	*Fr*	*MP*	*P*	*VP*	
P (2,3,4)	*MG* (13,16,18)	(6.13,7.88,9.25)	0.389	0.429	0.523	0.727	0.839	0.418	0.179	*MP*

Table 11
Two-way assessments of compatibility between w_2 and each of the other workers.

	w_1	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}
w_2	0.389[a]	0.836	0.227	0.965	0.456	0.534	0.483	0.764	0.587	0.671	0.404
	MP	*G*	*P*	*VG*	*Fr*	*Fr*	*Fr*	*MG*	*Fr*	*MG*	*MP*
	0.701[b]	0.431	0.985	1	0.854	0.907	0.744	0.159	1	0.478	0.756
	MG	*Fr*	*VG*	*VG*	*G*	*G*	*MG*	*P*	*VG*	*Fr*	*MG*
e_{i2}	1.09	1.267	1.212	1.965	1.31	1.441	1.227	0.923	1.587	1.149	1.16

[a] Degree of compatibility of (w_1, w_2) and the associated linguistic grade evaluated by w_2.
[b] Degree of compatibility of (w_1, w_2) and the associated linguistic grade evaluated by w_1.

Step 13: The guides for expressing fondness grades of working shifts are set as follows:

(1) In a planning week, there are 14 shifts available for assignment. For a regular worker, say w_i, the basic work hours for him or her are 42, equaling to seven shifts on duty. Suppose the number of days arranged for training courses or special leave for w_i in a planning week is x ($\geqslant 0$). These x days, counted as $2x$ shifts, are included in basic working hours and should be excluded from assigning w_i to any jobs. Thus, there are 14–2x shifts available for assigning w_i to the suitable jobs and the basic number of shifts w_i should be scheduled to work is 7–2x. To show the fondness grades of working shifts, w_i first indicates 7–2x linguistic grades of *VG* among the 14–2x shifts available for assignment. Then, each of the seven linguistic grades of *VG*–*VP* is indicated once among the remaining seven shifts available for assignment.
(2) To show the fondness grades of working shifts for a contract worker, he or she indicates each of the seven linguistic grades of *VG*–*VP* twice among the 14 shifts of a planning week.

Step 14: Table 12 shows, for example, the fondness grades of 12 working shifts conveyed by w_2, besides two shifts arranged for training courses. The basic number of shifts w_2 should be scheduled to work is five. According to the expressing guides set in Step 13, w_2 first conveys five linguistic grades of *VG* to express his pleasing shifts. Then, each of the seven grades, *VG*–*VP*, is conveyed among the remaining seven shifts. The fondness grade of (d_4, t_1), for example, conveyed by w_2 is *VP*. This linguistic grade is quantified as a TFN $\widetilde{P}^2_{41} = (0, 1, 2)$, and then the associated degree of fondness is calculated as $p_{241} = U_S(\widetilde{P}^2_{41}, VG) = 0.069$.

Step 15: The threshold of fondness grade is set as linguistic grade of *MP*. The results depicted in Table 12 show 10 shifts pass the threshold. Therefore, these 10 feasible worker–shift combinations, (w_2, d_1, t_1), (w_2, d_1, t_2), (w_2, d_2, t_2), (w_2, d_3, t_1), (w_2, d_3, t_2), (w_2, d_4, t_2), (w_2, d_6, t_1), (w_2, d_6, t_2), (w_2, d_7, t_1) and (w_2, d_7, t_2), are included in Ψ_2. (w_2, d_2, t_1) and (w_2, d_4, t_1) are not included in Ψ_2, which exclude w_2 from working in (d_2, t_1) and (d_4, t_1).

Table 12
Fondness grades of working shifts evaluated by w_2.

	d_1	d_2	d_3	d_4	d_5	d_6	d_7
t_1	*VG*	*P*	*VG*	*VP*	Training course	*VG*	*VG*
t_2	*VG*	*MG*	*Fr*	*VG*	Training course	*MP*	*G*

Table 13
Individual schedules for w_2 and w_5.

	d_1		d_2		d_3		d_4		d_5		d_6		d_7	
	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2
w_2	s_2	s_4						s_4	*	*	s_1	s_4	s_4	s_1
w_5	s_5				*	*			s_5	s_1	s_2	s_5	s_5	s_1

*Training courses arranged.

To construct the individual schedules for w_1–w_{12} by the proposed GP model, some preliminary data are set as $N = 12$, $K = 5$, $L = 7$, $F = 2$, $t^U_R = 9$, $t^L_R = 7$, $t^U_C = 8$, $\hat{\pi} = 2$, $W_R = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8\}$ and $W_C = \{w_9, w_{10}, w_{11}, w_{12}\}$ to conform them to the current assignation rules and conditions of the study section. Table 2 establishes the following data:

$$\widehat{W} = \{w_2, w_5, w_7\},$$
$$\widehat{D}_2 = \{d_5\}, \quad \widehat{D}_5 = \{d_3\} \quad \text{and} \quad \widehat{D}_7 = \{d_3\},$$
$$\hat{\theta}_{i\ell} = 1, \quad \text{if } (i, \ell) = (2,5), (5,3), (7,3); \quad \hat{\theta}_{i\ell} = 0, \text{ otherwise},$$
$$\hat{\theta}_2 = \hat{\theta}_5 = \hat{\theta}_7 = 1; \quad \hat{\theta}_i = 0, \quad i \neq 2, 5, 7,$$

$$\eta_{\ell fk} = 2,$$
$$\text{if } (\ell, f, k) = (1,2,1), (1,1,2), (1,2,2), (1,2,4), (1,1,5), (1,2,5), (4,2,1), (6,2,1), (6,2,2), (7,2,1), (7,2,2), (7,2,4), (7,1,5), (7,2,5),$$

$$\eta_{\ell fk} = 1, \quad \text{otherwise},$$

$$\Phi = \{(d_1, t_2, s_1), (d_1, t_1, s_2), (d_1, t_2, s_2), (d_1, t_2, s_4), (d_1, t_1, s_5), (d_1, t_2, s_5), (d_4, t_2, s_1), (d_6, t_2, s_1), (d_6, t_2, s_2), (d_7, t_2, s_1), (d_7, t_2, s_2), (d_7, t_2, s_4), (d_7, t_1, s_5), (d_7, t_2, s_5)\}.$$

By using the linear GP model with $\alpha = \beta = \gamma = 1$ to obtain the optimal solution, the individual schedules constructed for w_2 and w_5, for example, are shown in Table 13. The individual schedules reveal two aspects of information, which are the jobs assigned and the working and rest shifts scheduled. For example, w_2 is assigned to s_2 in (d_1, t_1) and to s_4 in (d_1, t_2), which conform to the restriction rule of job rotation on d_1. w_2 takes two full rest days on d_2 and d_3 and takes one rest shift in (d_4, t_1), while w_5 takes two full rest days on d_2 and d_4 and takes one rest shift in (d_1, t_2), which conform to the restriction rule of at least two full rest days. In the planning week, the number of working shifts for w_2 is nine, including seven shifts to work at s_1, s_2 and s_4 and two shifts to take part in training courses. Thus, besides the basic working time of seven shifts, w_2 has overtime of two shifts.

5. Discussion

The proposed linear GP model can construct effective rosters by adopting the feasible worker–job, worker–worker and worker–shift combinations as well as minimizing the deviations below the ideal values of management goals. Consider the individual schedules shown in Table 13. In Step 6, (w_2, s_1), (w_2, s_2), (w_2, s_3) and (w_2, s_4) are differentiated as feasible worker–job combinations, in which (w_2, s_1), (w_2, s_2) and (w_2, s_4) have a higher degree of suitability (see Table 9). Therefore, w_2 is assigned to work at s_1, s_2 and s_4 in the roster. Since (w_2, w_5) is included in Π in Step 12 and it has a higher two-way degree of compatibility (see column 4 of Table 11), they are assigned to work together at s_1 in (d_7, t_2). In Step

Table 14
Computational results.

Problem size	CPU time (minute:second)			Number of iteration		
	Min.	Max.	Average	Min.	Max.	Average
$N = 20$, $K = 8$	00:04	00:24	00:10	8344	48,650	21,351
$N = 25$, $K = 10$	07:02	16:48	10:51	558,982	1,864,645	1,055,067
$N = 30$, $K = 12$	09:25	18:07	12:34	746,188	2,246,365	1,176,696
$N = 34$, $K = 14$	17:49	52:24	31:09	808,279	2,587,717	1,202,726
$N = 36$, $K = 15$[a]	>60:00	>60:00	>60:00	>2,694,296	>2,694,296	>2,694,296

[a] The used solver is unable to get optimal solutions for the test instances within 60 min. The solving process is interrupted when the CPU time reaches 60 min.

15, Ψ_2 does not contain (w_2, d_2, t_1) and (w_2, d_4, t_1). Thereby, (d_2, t_1) and (d_4, t_1) are naturally scheduled as rest shifts for w_2. The individual schedule for w_5 shows he is assigned to work at s_1, s_2 and s_5. This is caused by (w_5, s_1), (w_5, s_2), (w_5, s_3) and (w_5, s_5) are included in Ω and (w_5, s_1), (w_5, s_2) and (w_5, s_5) have a higher degree of suitability.

Note that in Table 11, the two-way degree of compatibility of (w_2, w_4) is 1.212 (see bottom cell of column 3) and that of (w_2, w_{11}) is 1.149 (see bottom cell of column 10). The former is higher than the latter. In Step 12, the higher one, (w_2, w_4), is excluded from Π while the lower one, (w_2, w_{11}), is included in Π under the threshold of Fr. It may be seen somewhat strange, but the reason is due to the linguistic grade of w_4 evaluated by w_2 is classified as P. From the viewpoint of w_2, to work together with w_4 is uncomfortable or unpleasant. Thus, w_2 and w_4 are excluded from the consideration of being assigned to work together. In this study, the two-way evaluation for worker–worker compatibility is an important scheme that can fulfill the "efficient assignment/match from the right" policy.

The illustrated example is a real-life instance with $N = 12$ workers, $K = 5$ jobs, $L = 7$ days, and $F = 2$ working shifts per day. Although the section manager expresses that scaling organization up is not considered in the foreseeable future, some larger size instances are still generated to assess the computational efficiency of the proposed approach. Ten instances are generated for each of five different sizes at ($N = 20$, $K = 8$), ($N = 25$, $K = 10$), ($N = 30$, $K = 12$), ($N = 34$, $K = 14$) and ($N = 36$, $K = 15$). Eveborn and Ronnqvist (2004) argued, in the case of a great many variables and constraints, the optimization problem for the integer problem may be considered as NP-complete, and cannot be solved in an economical time. In order to reduce the number of binary variables, the variables $\theta_{i\ell fk}$ for which $(w_i, s_k) \notin \Omega$ are omitted. By using Lingo software on a PC with Intel (R) Core (TM) i7 CPU processor, the computational results of the generated instances are shown in Table 14. The branch-and-bound algorithm is used to solve the instances. Iterations represent the branch-and-bound nodes generated. The computational results show that the linear GP model can be performed to obtain optimal solutions within 53 min within problem size at $N = 34$ and $K = 14$. When the problem size is the triple of current size (i.e., $N = 12 \times 3 = 36$ and $K = 5 \times 3 = 15$), the used solver is unable to get an optimal solution within 60 min.

For applying the proposed approach in practice, there are some matters requiring attention. First, in the GP model, the multiple objectives are combined using the weights α, β and γ. The values of weights can be set by the decision-maker using proper methods, such as pairwise comparison shown in Tables 5–7. Second, the worker–shift fondness should be reassessed for each of the planning periods. Third, since either hiring new workers or firing existing ones can occur in the long term and the workers are arranged to take part in training courses, the worker–job suitability and worker–worker compatibility should be periodically reassessed. For example, reassess them one season or half a year later. Fourth, the thresholds for sieving out the feasible combinations of (w_i, s_k), (w_i, w_j) and (w_i, d_ℓ, t_f) to include in the sets of Ω, Π and Ψ_i, respectively, should be properly selected and flexibly adjusted, to provide a sufficient number of feasible combinations for obtaining the effective rosters.

Regarding the advantages or convenience of the proposed approach, Corominas et al. (2006) pointed out that in earlier studies several authors presented a hierarchical scheme for workforce organization problems consisting of three phases: (1) planning, (2) scheduling and (3) allocation. Once a schedule has been assigned to each worker, the assignment of tasks to multi-functional workers is done during phase (3). Compared with early studies, the proposed approach has a convenience. That is, instead of separating phases (2) and (3), these two phases can be simultaneously completed by the proposed GP model once phase (1) is complete (i.e., the coverage and time related constraints as shown in Sections 2.1 and 2.2 are determined). Compared with the current method, the proposed approach has some advantages. First, instead of using the seniority orientation bidding systems, the rosters are fairly constructed for the workers. Thus, the rosters are more equitable for workers. Second, the worker–job suitability, worker–worker compatibility and worker–shift fondness are considered in developing rosters. Thus, the rosters are more personalized for workers.

Applying the proposed approach certainly requires some computational efforts, but these can be easily implemented with the well-known powerful packages such as Excel and LINGO. In the opinions of the section manager, implementing the proposed approach requires minor outside help. Therefore, the proposed approach constitutes a potential tool to develop effective rosters for the study section.

6. Conclusions

This paper deals with an empirical crew rostering problem drawn from the customer service section of a department store in southern Taiwan. A problem-specific approach with three stages is developed. In stage 1, the fuzzy assessments along with the associated classifications for worker–worker compatibility, worker–job suitability and worker–shift fondness are conducted. The feasible combinations of worker–job, worker–worker and worker–shift for constructing the workers' rosters are then sieved out from the assessment results. The utility similarities of fuzzy assessments with the linguistic grade of very good are used to measure the fitness grade for the crew rostering results. This stage contributes to the literature by revealing the fuzzy method can be effectively used in the suitability, compatibility and fondness evaluations. In stage 2, the linear GP model is developed to fulfill the "efficient assignment/match from the right" policy for better satisfying the management goals. In stage 3, the linear GP model is performed to construct individual schedules for workers, consisting of the jobs assigned and the working and rest shifts scheduled over a planning week. The rosters constructed are more equitable and personalized work timetables for the workers to internally increase work satisfaction and then externally improve service quality and efficiency.

An illustrative application demonstrates the implementation of the proposed approach. The experimental results show that the linear GP model can be performed to obtain optimal solutions within 53 min within problem size at $N = 34$ and $K = 14$. Therefore, the proposed approach is a value-added and easily performed crew rostering tool. It can be used to ensure the right workers are assigned to the right jobs, the right workers are placed together in a job and pleasing working shifts are scheduled for the workers.

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